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A note on decomposition numbers for general linear groups and symmetric groups

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In [5] James proved theorems on the decomposition numbers, for the general linear groups and symmetric groups, involving the removal of the first row or column from partitions. In [1] we gave different proofs of these theorems based on a result valid for the decomposition numbers of any reductive group. (I am grateful to J. C. Jantzen for pointing out that the Theorem in [1] may also be derived from the universal Chevalley group case, which follows from the proof of 1·18 Satz of [6] – the analogue of equation (1) of the proof being obtained by means of the natural isometry (with respect to contravariant forms) between a certain sum of weight spaces of a Weyl module $V(\lambda)$ of highest weight λ and the Weyl module corresponding to λ for the Chevalley group determined by the subset of the base involved.) However, we have recently noticed that this result for reductive groups, even when specialized to the case of GL_n, gives a substantial generalization of James's Theorems. This generalization, which we give here, is an expression for the decomposition number $[\lambda : \mu]$ for a pair of partitions λ, μ whose diagrams can be simultaneously cut by a horizontal (or vertical) line so as to leave the same number of nodes above the line (or to the left of the line for a vertical cut) in both cases. Cutting between the first and second rows gives James's principal of row removal ([5], theorem 1) and cutting between the first and second column gives his principle of column removal ([5], theorem 2). Another special case of our horizontal result, involving the removal of bottom rows of a pair of partitions, is stated in [7], Satz 8.

1. *Partitions*

By a partition we mean a sequence $\lambda = (\lambda_1, \lambda_2, \ldots)$ of non-negative integers with $\lambda_1 \geqslant \lambda_2 \geqslant \ldots$ and $\lambda_m = 0$ for all large m. We say that λ is a partition of n if the sum of the components of λ is n and we say that λ is a partition into m parts if $\lambda_{m+1} = 0$. If λ is a partition of n into m parts we identify λ with the element $(\lambda_1, \lambda_2, \ldots, \lambda_m)$ of $\Lambda^+(m, n)$, defined in [3], p. 36.

Let K be an infinite field of arbitrary but fixed characteristic. We have, for any $\lambda \in \Lambda^+(m, n)$, the Weyl module $V_{\lambda,K}$ and the irreducible module $F_{\lambda,K}$ for the general linear group $GL_m(K)$ (see [3], p. 65 and p. 45). For partitions λ and μ of n with $\lambda_{m+1} = \mu_{m+1} = 0$ we denote by $[\lambda : \mu]$ the multiplicity of $F_{\mu,K}$ as a composition factor of $V_{\lambda,K}$. We adopt the convention that if λ and μ are the zero partition then $[\lambda : \mu] = 1$. By theorem 6·6*e* of [3], $[\lambda : \mu]$ does not depend on the choice of m, nor does $[\lambda : \mu]$ change if K is replaced by a different infinite field of the same characteristic (remark (i), section 3·5 of [3]).

Let λ and μ be partitions of n and k a positive integer. We say that the pair (λ, μ) admits a horizontal k-cut if $\lambda_1 + \lambda_2 + \ldots + \lambda_k = \mu_1 + \mu_2 + \ldots + \mu_k$. Equivalently,

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drawing a horizontal line between the k and $(k+1)$th row in the diagram $[\lambda]$ ([4], p. 8) of λ and in the diagram of μ, the pair (λ, μ) admits a horizontal k-cut if and only if the number of nodes above (or below) the line is the same in both diagrams. We say that (λ, μ) admits a vertical k-cut if $c_1 + c_2 + \ldots + c_k = d_1 + d_2 + \ldots + d_k$ where the conjugate partition (see [4], p. 9) of λ is $\lambda' = (c_1, c_2, \ldots)$ and $\mu' = (d_1, d_2, \ldots)$. Thus the pair (λ, μ) admits a vertical k-cut if and only if the pair (μ', λ') admits a horizontal k-cut.

For a partition λ and a positive integer k we denote by $\lambda^t(k)$ the top k-part $(\lambda_1, \lambda_2, \ldots, \lambda_k, 0, 0, \ldots)$ of λ and by $\lambda^b(k)$ the bottom k-part $(\lambda_{k+1}, \lambda_{k+2}, \ldots)$. We denote by $\lambda^l(k)$ the left k-part $(x_1, x_2, \ldots)$ where x_i is the minimum of λ_i and k and by $\lambda^r(k)$ the right k-part $(y_1, y_2, \ldots)$ where y_i is $\lambda_i - k$ if $\lambda_i \geqslant k$ and 0 otherwise.

Example 1. For $\lambda = 5321$ we have $\lambda^t(2) = 53$, $\lambda^b(2) = 21$. Also, from the picture below, one sees that $\lambda^l(2) = 2^31$, $\lambda^r(2) = 31$.

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The purpose of this note is to explain the observation that for a pair of partitions (λ, μ) of n admitting a horizontal (resp. vertical) k-cut we have

$$[\lambda : \mu] = [\lambda^t(k) : \mu^t(k)] . [\lambda^b(k) : \mu^b(k)]$$

(resp.
$$[\lambda : \mu] = [\lambda^l(k) : \mu^l(k)] . [\lambda^r(k) : \mu^r(k)]).$$

Example 2. $[642^21 : 5^221^3] = [64 : 5^2] . [2^21 : 21^3]$ since $(642^21, 5^221^3)$ admits a horizontal 2-cut.

Example 3. $[5432^21 : 4^23^21^3] = [2^51 : 2^41^3] . [321 : 2^21^2]$ since $(5432^21, 4^23^21^3)$ admits a vertical 2-cut. The picture here is

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* * | * *            * * | * *
* * | *              * * | *
* * |                * * | *  .
* * |                *   |
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Note that further reductions may be obtained in Examples 2 and 3 by using again the k-cut principles (or theorems 1 and 2 of [5]).

2. *The results for the general linear groups*

THEOREM 1. *Suppose that λ and μ are partitions of n such that the pair (λ, μ) admits a horizontal k-cut. Then*

$$[\lambda : \mu] = [\lambda^t(k) : \mu^t(k)] . [\lambda^b(k) : \mu^b(k)].$$

Proof. We take K to be algebraically closed. As in [1], p. 233 we identify an n-tuple of integers $\tau = (\tau_1, \tau_2, \ldots, \tau_n)$ with a character of the maximal torus of $GL_n(K)$ consisting of diagonal elements. By (3·1) of [1] we have that $[\lambda : \mu]$ is the composition

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multiplicity $[Y(\lambda): L(\mu)]$, where $Y(\lambda)$ is an induced module in the sense of algebraic group theory and $L(\mu)$ is the simple $GL_n(K)$-module of highest weight μ. By p. 233 of [**1**] we also have $[Y(\lambda): L(\mu)] = [Y(\lambda^*): L(\mu^*)]$ where $\lambda^* = i_\Delta(\lambda)$, $\mu^* = i_\Delta(\mu)$. Thus we have

$$[\lambda: \mu] = [Y(\lambda^*): L(\mu^*)] \tag{1}$$

where $\lambda^* = (-\lambda_n, -\lambda_{n-1}, \ldots, -\lambda_1)$, $\mu^* = (-\mu_n, -\mu_{n-1}, \ldots, -\mu_1)$. Now, by the hypotheses, $\lambda^* - \mu^* \in \mathbb{Z}\Omega$, where $\Omega = \Delta \backslash \{\epsilon_{n-k} - \epsilon_{n-k+1}\}$ (see [**1**], p, 233). Thus, by (1) above and the theorem of section 2 of [**1**],

$$[\lambda: \mu] = [Y_\Omega(\lambda^*): L_\Omega(\mu^*)], \tag{2}$$

i.e. $[\lambda: \mu]$ is equal to the composition multiplicity of $L_\Omega(\mu^*)$ in $Y_\Omega(\lambda^*)$ as a G_Ω-module (see [**1**], p. 228 for the notation). The subgroup G_Ω of $GL_n(K)$ consists of the matrices

$$\begin{pmatrix} P & 0 \\ 0 & Q \end{pmatrix}$$

with $P \in GL_{n-k}(K)$, $Q \in GL_k(K)$. Thus we identify G_Ω with $GL_{n-k}(K) \times GL_k(K)$. The set X_Ω (p. 228 of [**1**]) consists of n-tuples of integers $\xi = (\xi_1, \xi_2, \ldots, \xi_n)$ such that $\xi_1 \geqslant \xi_2 \geqslant \ldots \geqslant \xi_{n-k}$ and $\xi_{n-k+1} \geqslant \xi_{n-k+2} \geqslant \ldots \geqslant \xi_n$. For each $\xi \in X_\Omega$ we have a simple $GL_k(K)$-module $L(\xi^t(k))$ of highest weight $\xi^t(k) = (\xi_1, \ldots, \xi_k)$ and a simple $GL_{n-k}(K)$-module $L(\xi^b(k))$ of highest weight $\xi^b(k) = (\xi_{k+1}, \ldots, \xi_n)$. Moreover the Kronecker product $L(\xi^b(k)) \boxtimes L(\xi^t(k))$ is a simple $G_\Omega = GL_k(K) \times GL_{n-k}(K)$-module of highest weight $\alpha(\xi) = (\xi_{k+1}, \ldots, \xi_n, \xi_1, \ldots, \xi_k)$. Thus we have

$$L_\Omega(\alpha(\xi)) \cong L(\xi^b(k)) \boxtimes L(\xi^t(k)). \tag{3}$$

For $\tau = (\tau_1, \tau_2, \ldots, \tau_n) \in X_\Omega$ we have the induced $GL_k(K)$-module $Y(\tau^t(k))$, which has lowest weight $(\tau_k, \tau_{k-1}, \ldots, \tau_1)$ and the induced $GL_{n-k}(K)$-module $Y(\tau^b(k))$ which has lowest weight $(\tau_n, \tau_{n-1}, \ldots, \tau_{k+1})$. It follows that the Kronecker product

$$Y(\tau^b(k)) \boxtimes Y(\tau^t(k))$$

is a G_Ω-module with simple B_Ω ([**1**], p. 228) socle of weight $(\tau_n, \ldots, \tau_{k+1}, \tau_k, \ldots, \tau_1)$ (see (1·5·2) of [**2**]). However $Y_\Omega(\alpha(\tau))$ (where $\alpha(\tau) = (\tau_{k+1}, \ldots, \tau_n, \tau_1, \ldots, \tau_k)$) has a simple B_Ω-socle of the same weight $(\tau_n, \ldots, \tau_{k+1}, \tau_k, \ldots, \tau_1)$ and $Y_\Omega(\alpha(\tau))$ has the same dimension as $Y(\tau^b(k)) \boxtimes Y(\tau^t(k))$ (this follows from (2·2·8) of [**2**]), so that

$$Y_\Omega(\alpha(\tau)) \cong Y(\tau^b(k)) \boxtimes Y(\tau^t(k)) \tag{4}$$

as G_Ω-modules, by (1·5·4) of [**2**].

Let $\xi \in X_\Omega$. It follows from (3) that, for every rational $GL_{n-k}(K)$-module V and every rational $GL_k(K)$-module W, the composition multiplicity $[V \boxtimes W: L_\Omega(\alpha(\xi))]$ of $L_\Omega(\alpha(\xi))$ in the G_Ω-module $V \boxtimes W$ is the product of the composition multiplicity $[V: L(\xi^b(k))]$ of the $GL_{n-k}(K)$-module $L(\xi^b(k))$ in V and the composition multiplicity $[W: L(\xi^t(k))]$ of the $GL_k(K)$-module $L(\xi^t(k))$ in W. In particular, from (4) we have

$$[Y_\Omega(\alpha(\tau)): L_\Omega(\alpha(\xi))] = [Y(\tau^b(k)): L(\xi^b(k))] . [Y(\tau^t(k)): L(\xi^t(k))]$$

for $\tau, \xi \in X_\Omega$. Applying this to (2) we obtain

$$[\lambda: \mu] = [\tilde{\lambda}: \tilde{\mu}] [\hat{\lambda}: \hat{\mu}] \tag{5}$$

where $\tilde{\lambda} = (-\lambda_n, \ldots, \lambda_{n-k+1})$, $\tilde{\mu} = (-\mu_n, \ldots, -\mu_{n-k+1})$, $\hat{\lambda} = (-\lambda_k, \ldots, -\lambda_1)$ and $\hat{\mu} = (-\mu_k, \ldots, -\mu_1)$. Applying (1) now to $GL_{n-k}(K)$ with λ replaced by $\tilde{\lambda}$ and μ

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replaced by $\tilde{\mu}$ gives $[\tilde{\lambda}\colon \tilde{\mu}] = [\lambda^b(k)\colon \mu^b(k)]$. Similarly $[\hat{\lambda}\colon \hat{\mu}] = [\lambda^t(k)\colon \mu^t(k)]$ so that (5) becomes

$$[\lambda\colon \mu] = [\lambda^t(k)\colon \mu^t(k)].[\lambda^b(k)\colon \mu^b(k)].$$

THEOREM 2. *Suppose that λ and μ are partitions of n such that the pair (λ, μ) admits a vertical k-cut. Then*

$$[\lambda\colon \mu] = [\lambda^l(k)\colon \mu^l(k)].[\lambda^r(k)\colon \mu^r(k)].$$

Proof. There is no further representation theory: the theorem is deduced from Theorem 1 using a horizontal cut corresponding to the vertical k-cut. For partitions η and τ the multiplicity $[\eta\colon \tau]$ is 0 unless $\eta \trianglerighteq \tau$ (where $\trianglerighteq$ is the partial order defined in [4], p. 8). Moreover $\lambda \trianglerighteq \mu$ if and only if $\lambda^l(k) \trianglerighteq \mu^l(k)$ and $\lambda^r(k) \trianglerighteq \mu^r(k)$ (take conjugates and use [4], p. 9, line 16). Thus we may assume $\lambda \trianglerighteq \mu$.

Let $\lambda' = (c_1, c_2, \ldots)$ and $\mu' = (d_1, d_2, \ldots)$. Since $\lambda \trianglerighteq \mu$, $\mu' \trianglerighteq \lambda'$, so we have

$$d_1 + \ldots + d_{k-1} \geqslant c_1 + \ldots + c_{k-1}, \quad d_1 + \ldots + d_{k+1} \geqslant c_1 + \ldots + c_{k+1}$$

and, as (λ, μ) admits a vertical k-cut, $d_1 + \ldots + d_k = c_1 + \ldots + c_k$. We deduce

$$d_k \leqslant c_k \quad \text{and} \quad d_{k+1} \geqslant c_{k+1}. \tag{1}$$

Let $s = d_k$. If $s = 0$, then $\mu_1 < k$, so that $\mu^l(k) = \mu$ and $\mu^r(k)$ is the zero partition. Also, in this case, $d_k \geqslant d_{k+1}$ implies that $d_{k+1} = 0$, so that $c_{k+1} = 0$ by (1). Thus $\lambda^l(k) = \lambda$ and $\lambda^r(k)$ is the zero partition so that the the theorem holds trivially. Thus we may assume $s > 0$.

Note that we have the following:

$$\left.\begin{array}{l} \text{if } i \leqslant k \text{ then } c_i \geqslant d_k \text{ and } d_i \geqslant d_k; \\ \text{if } i > k \text{ then } c_i \leqslant d_k \text{ and } d_i \leqslant d_k. \end{array}\right\} \tag{2}$$

For if $i \leqslant k$ then $c_i \geqslant c_k \geqslant d_k$, by (1), and if $i > k$, $c_i \leqslant c_{k+1} \leqslant d_{k+1} \leqslant d_k$, by (1). Of course if $i \leqslant k$ then $d_i \leqslant d_k$ and if $i > k$ then $d_i \leqslant d_k$ since $\mu' = (d_1, d_2, \ldots)$ is a partition.

Now imagine drawn in the diagrams of λ and μ a horizontal line, between the d_kth and (d_k+1)th row and a vertical line between the kth and $(k+1)$th column. By (2), the number of nodes above the horizontal line and to the left of the vertical line is kd_k in both $[\lambda]$ and $[\mu]$. However, the pair (λ, μ) admits a vertical k-cut so the number of nodes to the left of the vertical lines is the same for $[\lambda]$ and $[\mu]$. Therefore the number of nodes to the left of the vertical line and below the horizontal line is the same for $[\lambda]$ and $[\mu]$. By (2), there are no nodes in $[\lambda]$ or $[\mu]$ which are below the horizontal line and to the right of the vertical line. Thus the number of nodes below the horizontal line is the same for $[\lambda]$ as it is for $[\mu]$, that is to say (λ, μ) admits a horizontal $s = d_k$-cut. By Theorem 1 we have

$$[\lambda\colon \mu] = [\lambda^t(s)\colon \mu^t(s)].[\lambda^b(s)\colon \mu^b(s)].$$

By (2) the diagram of $\lambda^t(s)$ and the diagram of $\mu^t(s)$ both begin with k-columns of length d_k. Applying James's principle of column cancellation (theorem 2 of [5] or theorem 2 of [1]) k times, we have $[\lambda^t(s)\colon \mu^t(s)] = [\alpha\colon \beta]$, where $\alpha = (\lambda_1 - k, \ldots, \lambda_s - k)$, $\beta = (\mu_1 - k, \ldots, \mu_s - k)$. Hence we have

$$[\lambda\colon \mu] = [\alpha\colon \beta].[\lambda^b(s)\colon \mu^b(s)]. \tag{3}$$

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We now calculate $[\lambda^l(k):\mu^l(k)].[\lambda^r(k):\mu^r(k)]$. By (2), $\lambda^l(k) = (k, k, \ldots, k, \lambda_{s+1}, \lambda_{s+2}, \ldots)$ and $\mu^l(k) = (k, k, \ldots, k, \mu_{s+1}, \mu_{s+2}, \ldots)$. Thus by the principle of row cancellation ([5], theorem 1 or theorem 1 of [1]) or by Theorem 1 above,

$$[\lambda^l(k):\mu^l(k)] = [(\lambda_{s+1}, \lambda_{s+2}, \ldots):(\mu_{s+1}, \mu_{s+2}, \ldots)] = [\lambda^b(s):\mu^b(s)].$$

By (2) we have $\lambda^r(k) = (\lambda_1 - k, \lambda_2 - k, \ldots, \lambda_s - k, 0, 0, \ldots)$ and $\mu^r(k) = (\mu_1 - k, \mu_2 - k, \ldots, \mu_s - k, 0, 0, \ldots)$, i.e. $\lambda^r(k) = \alpha$ and $\mu^r(k) = \beta$. We now have

$$[\lambda^l(k):\mu^l(k)].[\lambda^r(k):\mu^r(k)] = [\lambda^b(s):\mu^b(s)].[\alpha:\beta]. \tag{4}$$

Combining (3) and (4) gives

$$[\lambda:\mu] = [\lambda^l(k):\mu^l(k)].[\lambda^r(k):\mu^r(k)],$$

as required.

3. *The results for the symmetric groups*

Let $G(n)$ denote the symmetric group on n symbols. Let S_λ denote the Specht module for $G(n)$, corresponding to the partition λ of n, defined over an arbitrary field K of characteristic p. (In the notation of [3], p. 90, we have $S_\lambda = KG\{C(T)\}[R(T)]$, for a bijective λ tableau T.) For λ column p-regular let D_λ be the unique irreducible submodule of S_λ. The following results are obtained as in section 4 of [5].

THEOREM 3. *Suppose that λ and μ are partitions of n with μ column p-regular. Assume that $\lambda \trianglerighteq \mu$ and that (λ, μ) admits a horizontal k-cut. Let*

$$\alpha = (\lambda_1 - \lambda_k, \lambda_2 - \lambda_k, \ldots, \lambda_{k-1} - \lambda_k, 0)$$

and $\beta = (\mu_1 - \lambda_k, \mu_2 - \lambda_k, \ldots, \mu_{k-1} - \lambda_k, \mu_k - \lambda_k)$. Then the composition multiplicity of D_μ in S_λ (as $G(n)$-modules) is the product of the composition multiplicity of D_β in S_α (as $G(\lambda_1 + \ldots + \lambda_k - k\lambda_k)$-modules) and the composition multiplicity of $D_{\mu^b(k)}$ in $S_{\lambda^b(k)}$ (as $G(n - \lambda_1 - \ldots - \lambda_k)$-modules).

Proof. Notice that $\mu^b(k)$ is column p-regular. Since $\lambda \trianglerighteq \mu$ and (λ, μ) admits a horizontal k-cut we have $\lambda_k \leqslant \mu_k$ and $\lambda_{k+1} \geqslant \mu_{k+1}$ (see the proof of Equation (1) in the proof of Theorem 2). Thus β has non-negative entries and

$$\mu_k - \lambda_k \leqslant \mu_k - \lambda_{k+1} \leqslant \mu_k - \mu_{k+1}$$

so that β is also column p-regular. Every field is a splitting field for the symmetric group so we may assume that K is infinite. The result now follows from Theorem 1 above, theorem 2 of [5] and theorem 6·6g of [3].

In the same way we obtain from Theorem 2:

THEOREM 4. *Suppose that λ and μ are partitions of n with μ column p-regular and suppose that (λ, μ) admits a vertical k-cut. Let m be the sum of the lengths of the first k-columns of $[\lambda]$. Then the composition multiplicity of D_μ in S_λ (as $G(n)$-modules) is the product of the composition multiplicity of $D_{\mu^l(k)}$ in $S_{\lambda^l(k)}$ (as $G(m)$-modules) and the composition multiplicity of $D_{\mu^r(k)}$ in $S_{\lambda^r(k)}$ (as $G(n-m)$-modules).*

Let $\bar{S}_\lambda$ denote the dual of S_λ and, for λ p-regular, let $\bar{D}_\lambda$ be the unique irreducible submodule of $\bar{S}_\lambda$. We have $\bar{S}_\lambda \cong S_{\lambda'} \otimes K_s$ and, for λ p-regular, $\bar{D}_\lambda \cong D_{\lambda'} \otimes K_s$ ([3], pp. 99 and 100), where K_s is the one-dimensional module affording the sign representation of $G(n)$. We deduce Theorem 5 and Theorem 6 from Theorem 4 and Theorem 3, respectively.

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THEOREM 5. *Suppose that λ and μ are partitions of n with μ p-regular and assume that (λ, μ) admits a horizontal k-cut. Let $m = \lambda_1 + \lambda_2 + \ldots + \lambda_k$. Then the composition multiplicity of $\overline{D}_\mu$ in $\overline{S}_\lambda$ (as $G(n)$-modules) is the product of the composition multiplicity of $\overline{D}_{\mu^{t(k)}}$ in $\overline{S}_{\lambda^{t(k)}}$ (as $G(m)$-modules) and the composition multiplicity of $\overline{D}_{\mu^{b(k)}}$ in $\overline{S}_{\lambda^{b(k)}}$ (as $G(n-m)$-modules).*

THEOREM 6. *Suppose that λ and μ are partitions of n with μ p-regular. Assume that $\lambda \trianglelefteq \mu$ and that (λ, μ) admits a vertical k-cut. Let $\lambda' = (c_1, c_2, \ldots)$, let $\mu' = (d_1, d_2, \ldots)$, let α be the partition whose diagram has column lengths $c_1 - c_k, \ldots, c_{k-1} - c_k$ and let β be the partition whose diagram has column lengths $c_1 - c_k, \ldots, c_{k-1} - c_k$ and let β be the partition whose diagram has column lengths $d_1 - c_k, \ldots, d_{k-1} - c_k, d_k - c_k$. Then the composition multiplicity of $\overline{D}_\mu$ in $\overline{S}_\lambda$ (as $G(n)$-modules) is the product of the composition multiplicity of $\overline{D}_\beta$ in $\overline{S}_\alpha$ (as $G(c_1 + c_2 + \ldots + c_k - kc_k)$-modules) and the composition multiplicity of $\overline{D}_{\mu^{r(k)}}$ in $\overline{S}_{\lambda^{r(k)}}$ (as $G(n - c_1 - \ldots - c_k)$-modules).*

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