Macroeconomic Dynamics, **23**, 2019, 2338–2359. Printed in the United States of America.
doi:10.1017/S1365100517000712

ASSET BUBBLES AND ECONOMIC GROWTH UNDER ENDOGENOUS MARKET STRUCTURE

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By considering a simple endogenous growth model, we propose a new theoretical channel through which the presence of asset bubbles can promote economic growth. In the model economy, long-lived value-maximizing firms continuously improve the quality of their specific products through in-house research and development (R&D), while simultaneously new firms enter into the market. The key feature is endogenous market structure: The number of firms is endogenously determined, which leads to variation in firm size measured in terms of the scale of production at the level of an individual firm. The presence of asset bubbles unambiguously gives rise to larger firms. This allows in-house R&D expenditure to be spread over the greater numbers of goods that the firms produce, which can increase incentives to undertake in-house R&D.

Keywords: Asset Bubbles, Endogenous Growth, Firm Size, Overlapping Generations

1. INTRODUCTION

When asset bubbles emerge, they are accompanied by higher economic growth and a consumption boom [e.g., Martin and Ventura (2012)].[1] Theoretically, the seminal study by Tirole (1985) shows the existence of a rational deterministic bubble on an asset in an economy with overlapping generations. However, incorporating Tirole's notion into a standard growth model, like the AK model, leads to the conflicting result that the presence of asset bubbles increases consumption but slows economic growth as higher consumption absorbs resources and crowds out investment [e.g., Saint-Paul (1992), Grossman and Yanagawa (1993), King and Ferguson (1993), and Futagami and Shibata (2000)]. On the other hand, several

The paper was previously circulated with the title "Growth effect of bubbles in a non-scale endogenous growth model with in-house R&D." A part of this work was conducted while visiting the Institute of Economics, Academia Sinica. Financial support from the Strategic Young Researcher Overseas Visits Program for Accelerating Brain Circulation (JSPS) is gratefully acknowledged. I would like to express my sincere thanks to the two anonymous referees, Koichi Futagami, Ching-Chong Lai, Been-Lon Chen, and Pietro Peretto for beneficial discussions and suggestions. I would also like to thank Katsunori Yamada, Kei Nanamiya, Takayuki Oishi, and the participants at the 2013 Annual Autumn Meeting of the Japanese Economic Association at the University of Kanagawa, the 2014 Asia Meeting of the Econometric Society in Taipei, the 2015 Summer Workshop on Economic Theory in Otaru, and KIER-joint workshop at Kochi-Tech University. This work was supported by JSPS KAKENHI Grant Number 15H06524. All remaining errors are mine. Address correspondence to: Kizuku Takao, Department of Economics, Aomori Public University, 153-4 Aza Yamazaki, Oaza Goushizawa Aomori City, Aomori 030-0196, Japan; e-mail: kizukutakao@gmail.com.

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https://doi.org/10.1017/S1365100517000712 Published online by Cambridge University Press

recent studies have proposed a theory that can resolve the conflict, taking credit market imperfections and heterogeneous firms' efficiencies into account [e.g., Farhi and Tirole (2012), Martin and Ventura (2012), Kunieda (2014), Kunieda and Shibata (2016), and Hirano and Yanagawa (2017)]. Their main insight is that asset bubbles reallocate resources from inefficient to efficient firms and this reallocation can promote economic growth.

The purpose of the present paper is to propose a new theoretical channel through which the presence of asset bubbles can promote economic growth by considering a simple endogenous growth model. Our theory differs from the preceding studies referred to in the earlier paragraph in that the model economy is characterized by a perfect credit market and homogeneous firms' efficiencies. The key feature is that there are two-dimensional investment opportunities: quality improvement of existing goods (vertical innovation) and the creation of a new product (horizontal innovation). This causes the goods market to be endogenous. Here, monopolistic competition prevails in the goods market. In line with much of the macroeconomic literature of monopolistic competition, the term "firm" will be used to refer to the producer or product line of individual goods. The model has an endogenous market structure in which the number of firms is endogenously determined. This leads to variation in the scale of production (henceforth called "firm size") at the level of the individual firm for any given aggregate market size. A variation in firm size affects the intensity of investment for upgrading the quality of existing goods. Our main conclusion is that the presence of asset bubbles unambiguously gives rise to larger firms, which can have a positive effect on quality growth.

More specifically, to capture endogenous market structure, we consider a production economy, along the lines of Peretto (1998, 2007a, 2007b, 2011) and Dinopoulos and Thompson (1998),[2] in which long-lived value-maximizing firms continuously improve upon the quality of their specific product through in-house research and development (R&D), while simultaneously new firms enter into the market. In the steady-state equilibrium, the aggregate economy is fragmented by endogenous product proliferation and the main source of economic growth is derived from quality growth. The advantage of the model is that we can eliminate the well-known undesirable scale effect [e.g., Jones (1995)], while keeping the policy effect property supported by the recent and growing empirical literature [e.g., Laincz and Peretto (2006), Ha and Howitt (2007), and Ang and Madsen (2011)]. The model is also consistent with empirical evidence concerning industrial organization [e.g., Adams and Jaffe (1996), Cohen and Klepper (1996), and Pagano and Schivardi (2003)].

We introduce the above-mentioned production structure into a continuous-time overlapping generations model developed by Weil (1989). More specifically, we assume that economic agents live forever but have no intergenerational altruism and new generations are born at a constant rate.[3] Initially, there exists an intrinsically useless asset (fiat money). Along with many other related studies, we focus on the case in which bubbles attach to only an intrinsically useless asset.[4] The reason for the presence of asset bubbles in equilibrium is the same as that laid out in

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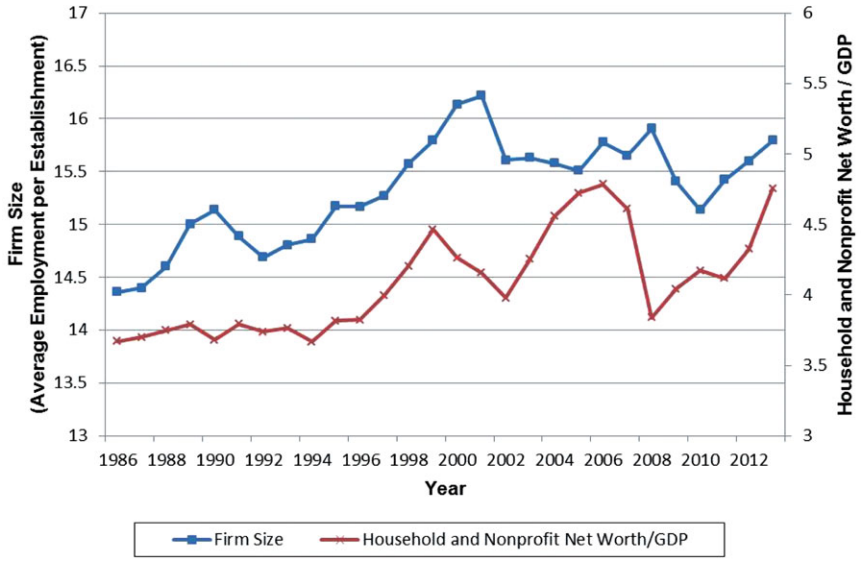


FIGURE 1. Firm size and the ratio of household and nonprofit net worth to GDP (1986–2013, US).

Tirole (1985). Due to an absence of intergenerational altruism, one generation can transfer an intrinsically useless asset to another generation in a pyramid scheme only if price appreciation of an intrinsically useless asset is equal to the return on a real asset (equity) whose value is backed by its fundamental value.

The intuitive reason why the presence of asset bubbles can promote economic growth in the present framework is as follows. If asset bubbles emerge, households think that they are wealthier and thus want to consume more. This leads to a higher reservation interest rate of households in the asset market. To satisfy this higher rate, return on a real asset (equity) must increase under the no-arbitrage restriction among assets. This requires product proliferation to be impeded so that demand for an individual good increases because return on equity is proportional to profits. In short, the presence of asset bubbles gives rise to larger firms. This allows in-house R&D expenditure to be spread over more units of goods that firms produce, thus creating a positive incentive to conduct in-house R&D (the cost spreading effect). If the positive cost spreading effect on in-house R&D resulting from larger firm size exceeds the usual negative effect resulting from higher interest rates, the intensity of in-house R&D increases, thus promoting economic growth. In addition, the lower product proliferation provides economic resources for higher consumption by households.

In fact, comovement of asset price and firm size is empirically observed. Figure 1 plots firm size and the ratio of household and nonprofit net worth to gross domestic product (GDP) in the United States for the period 1986–2013. Here, the proxy for firm size is average employment per establishment, along the lines of Laincz

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and Peretto (2006), and the boom-and-bust cycle of asset prices is indicated by the movement of the ratio of household and nonprofit net worth to GDP, along the lines of Martin and Ventura (2012).[5] It is widely recognized that the United States has experienced at least two episodes of asset bubbles, one of which corresponds to the period 1996–2000 (the dot-com bubble) and the other to the period 2002–2006 (the US housing bubble) [e.g., Basco (2014)]. Figure 1 illustrates that, for the corresponding periods, firm size moved in the same direction (with time lags) as the rise and fall of the ratio of household and nonprofit net worth to GDP. After the asset bubble burst in 2007, the ratio of net worth to GDP has begun increasing once again in the most recent period, and this is also followed by an increase in firm size. Indeed, the correlation coefficient between the ratio of household and nonprofit net worth to GDP and one year lag of firm size for the period 1986–2013 is equal to 0.776.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 characterizes the market equilibrium and the dynamical system. Section 4 analyzes the steady-state equilibrium as well as comparing the steady-state equilibrium with and without asset bubbles. Section 5 discusses the sensitivity of the results to the model specification and whether our theoretical implication holds in alternative model setups. Last, we conclude in Section 6.

2. THE MODEL

In this section, we establish a continuous-time overlapping-generations version of the model specified by Peretto (2011).[6] We assume that there exists an initial and intrinsically useless asset, that is, fiat money. The economy is closed and consists of the final goods sector, the intermediate goods sector, and households. In the intermediate goods sector, there are two-dimensional investment opportunities: in-house R&D for quality improvement of existing intermediate goods and the creation of new intermediate goods. Time is continuous and is denoted by $t \geq 0$.

2.1. The Final Goods Sector

Perfect competition prevails in the final goods sector. The (real) price of final goods is set to be numeraire. Final goods are consumed by households and are used to produce intermediate goods, to invest in quality improvement of existing intermediate goods, and to invest in the creation of new intermediate goods. The final goods, Y_t, have the production function

$$Y_t = \int_0^{N_t} X_{it}^{\theta} \left[Z_{it}^{\alpha} \bar{Z}_t^{1-\alpha} \left(\frac{L_t}{N_t} \right) \right]^{1-\theta} di, \quad 0 < \alpha, \theta < 1, \tag{1}$$

where N_t is the variety of intermediate goods (the number of intermediate goods firms), X_{it} is the input of intermediate good $i \in [0, N_t]$ (produced by firm i), and L_t is the input of labor. Z_{it} is the quality of intermediate good i and $\bar{Z}_t \equiv \int_0^{N_t} \frac{1}{N_t} Z_{jt} dj$

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is the average quality across intermediate goods, capturing the effects of both knowledge spillover and labor-augmenting technology.[7] This specification of final goods production function is a slightly modified version of that in Aghion and Howitt (1998).[8] The parameter $\alpha \in (0, 1)$ determines each firm's incentives for in-house R&D to upgrade the quality of intermediate goods. Diminishing returns on in-house R&D are required so that we have a symmetric equilibrium. Dividing L_t by N_t eliminates any productivity gains resulting from product proliferation in the reduced form of (1) as shown later in (20). The justification for this is explained below.

Solving the optimization problem, we have

$$X_{it} = \left(\frac{\theta}{p_{it}^I}\right)^{1/(1-\theta)} \left[Z_{it}^{\alpha} \bar{Z}_t^{1-\alpha} \left(\frac{L_t}{N_t}\right)\right] \tag{2}$$

$$L_t = \left(\frac{1-\theta}{w_t}\right)^{\frac{1}{\theta}} N_t^{-(1-\theta)/\theta} \left[\int_0^{N_t} X_{it}^{\theta} \left(Z_{it}^{\alpha} \bar{Z}_t^{1-\alpha}\right)^{1-\theta} di\right]^{1/\theta}, \tag{3}$$

where p_{it}^I and w_t represent the price of intermediate good i and the wage rate, respectively.

2.2. The Intermediate Goods Sector

Monopolistic competition prevails in the intermediate goods sector. Here, there exists a one-to-one correspondence between the number of product varieties and firms. Firm i exclusively produces its differentiated good with quality Z_{it}. The monopoly of each firm is permanently protected by perfect patent protection. Producing one unit of intermediate goods requires one unit of final goods. Firms improve upon the quality of their specific product through their in-house R&D.[9] The law of motion of the firm-specific quality is

$$\dot{Z}_{it} = R_{it}, \tag{4}$$

where R_{it} is the input for in-house R&D. Fixed operating costs, $\phi \bar{Z}_t$ ($\phi > 0$), are also required at each point in time. This assumption implies that operating costs on establishment are more expensive, the higher the technological level of intermediate goods produced. Importantly, this assumption ensures the existence of a balanced growth path (the steady-state equilibrium).[10] The profit flow of firm i is $\Pi_{it} = \left(p_{it}^I - 1\right) X_{it} - \phi \bar{Z}_t - R_{it}$. Without loss of generality, the amount of equity per firm is normalized to 1. The return on equity of firm i is given by

$$r_t = \frac{\Pi_{it}}{V_{it}} + \frac{\dot{V}_{it}}{V_{it}}. \tag{5}$$

Integrating (5) forward yields the value of firm i as $V_{it} = \int_t^{\infty} \Pi_{is} \left(e^{-\int_t^s r_v dv}\right) ds$.

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Throughout the subsequent analysis, we focus on a symmetric equilibrium in which $Z_{it} = Z_t$ by assuming that any entering firm starts with the same quality level as that of incumbents so that the subscript i can be dropped. In equilibrium, $Z_t = \bar{Z}_t$ holds. Each firm maximizes its value, subject to (2) and (4), given $\bar{Z}$. Solving the intertemporal optimization problem, we obtain the optimality conditions

$$p_t^I = \frac{1}{\theta}, \tag{6}$$

$$q_t = 1 \;\Leftrightarrow\; R_t > 0, \tag{7}$$

$$r_t = \alpha(1-\theta)\theta^{(1+\theta)/(1-\theta)} Z_t^{\alpha-1} \bar{Z}_t^{1-\alpha} \frac{L_t}{N_t} \frac{1}{q_t} + \frac{\dot{q}_t}{q_t}, \tag{8}$$

and the transversality condition is given by $\lim_{s\to\infty} q_s Z_s e^{-\int_t^s r_v dv} = 0$, where q_t is a costate variable (a shadow value of in-house R&D) corresponding to the current-value Hamiltonian for this optimization problem. Equation (6) represents the pricing rule with constant mark-up, and equation (8) represents the no-arbitrage relationship between the return on equity and the return on in-house R&D.

2.3. Firm Entry

Development of a new product requires βZ_t ($\beta > 1$) units of final goods. Equation (4) implies that achieving the quality level Z_t requires cumulative investment costs of Z_t units of final goods. As noted above, any new entrant firm is assumed to start with the same quality level as that of incumbents. Then, entry costs should be proportional to Z_t. $\beta > 1$ means that a new entrant firm needs to pay an additional cost more than the cumulative investment costs already paid by incumbents.[11] A new firm is set up by issuing equities. The free-entry condition yields

$$V_t = \beta Z_t \;\Leftrightarrow\; \dot{N}_t > 0, \tag{9}$$

which implies that entry is positive until the value of a firm is equal to the setup cost. As we will see below, the population is perpetually growing and so the aggregate demand for intermediate goods also continues to grow. Therefore, in equilibrium, the entry of a new firm occurs at each point in time.

2.4. Households

The specification of household behavior is based on that developed by Weil (1989). At each point in time, a new generation is born at a constant rate $\lambda > 0$. Since the probability of death is assumed to be zero, each generation lives forever. Furthermore, there is no intergenerational altruism. Without loss of generality, the total population at time 0 is normalized to 1 so that the total population at time t is $e^{\lambda t}$.

At the beginning, time 0, there is an initial generation endowed with initial wealth. Hereafter, we call this generation "generation 0^-." The initial wealth

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consists of a real asset (equity) and an intrinsically useless asset (fiat money). To fix terminology, in what follows, we call the intrinsically useless asset the "bubbly asset." Specifically, generation 0^- initially possesses the equity of firms (the initial number of firms is N_0), whose value is given by N_0V_0 in real terms and B_0 pieces of the bubbly asset in nominal terms. That is, the total value of the initial asset is equal to $N_0V_0 + B_0/p_0^M$ in real terms, where $1/p_0^M$ is the price of the bubbly asset measured in numeraire goods at time 0. In contrast, the subsequent generations do not have any initial wealth. Hereafter, we denote the generation born at time $t \geq 0$ by "generation t."

Each household supplies one inelastic unit of labor at each point in time. The representative household in generation $s(\leq t)$ maximizes the utility function:

$$U(s,t) = \int_t^\infty \log\ [c(s,x)]e^{-\rho(x-t)}dx, \quad \rho > 0, \tag{10}$$

subject to the budget constraint of generation s:

$$\frac{db(s,t)}{dt}\frac{1}{p_t^M} + \frac{dk(s,t)}{dt} = r_t k(s,t) + w_t - c(s,t),$$

where $b(s,t)$ represents the holdings of pieces of a bubbly asset (in nominal terms) of generation s at time t, $1/p_t^M$ is the price of the bubbly asset, $k(s,t)$ represents the holdings of a real asset, and $c(s,t)$ is consumption.

Since the bubbly asset has no intrinsic or fundamental value, households possess both the real asset and the bubbly asset only if the price appreciation of the bubbly asset is equal to the return on the real asset, that is,

$$-\frac{\dot{p}_t^M}{p_t^M} = r_t. \tag{11}$$

Let $m(s,t) \equiv b(s,t)/p_t^M$ and $a(s,t) \equiv k(s,t) + m(s,t)$. Using (11) and these definitions, the flow budget constraint of generation s can be rewritten as

$$\frac{da(s,t)}{dt} = r_t a(s,t) + w_t - c(s,t). \tag{12}$$

Maximizing (10) subject to (12) yields the following Euler equation of generation s:

$$\frac{dc(s,t)}{dt} = (r_t - \rho)\, c(s,t). \tag{13}$$

Furthermore, the no-Ponzi-game condition must be satisfied, that is, $\lim_{x\to\infty} a(s,x)e^{\left(-\int_t^x r_v dv\right)} = 0$.

In what follows, we aggregate the consumption and wealth of each generation. To do so, we sequentially define aggregate consumption, aggregate wealth, and

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aggregate wage income:

$$C_t \equiv c(0^-, t) + \int_0^t c(s,t)\lambda e^{\lambda s} ds,$$

$$A_t \equiv a(0^-, t) + \int_0^t a(s,t)\lambda e^{\lambda s} ds,$$

$$W_t \equiv w_t + \int_0^t w_t \lambda e^{\lambda s} ds,$$

where $\lambda e^{\lambda s}$ represents the population of generation s. In Appendix A, we show that the dynamic paths of aggregate consumption and aggregate wealth are, respectively, given by

$$\dot{C}_t = (r_t - \rho + \lambda) C_t - \rho\lambda A_t, \tag{14}$$

$$\dot{A}_t = r_t A_t + W_t - C_t. \tag{15}$$

3. MARKET EQUILIBRIUM AND DYNAMICS

In this section, we derive the dynamical system of the market equilibrium based on the model in the preceding section. The market equilibrium condition of final goods is given by $Y_t = C_t + N_t (X_t + \phi Z_t + R_t) + \beta Z_t \dot{N}_t$. The market equilibrium condition of assets is given by $A_t = M_t + N_t V_t$, where M_t is defined as the aggregate value of the bubbly asset that is B_0/p_t^M. This condition means that in equilibrium the aggregate wealth must be equal to the aggregate value of the bubbly asset and the aggregate value of equity ($N_t V_t$). Here, we remark that the only real asset in the economy is the equity of firms. The market equilibrium condition of labor is given by $e^{\lambda t} = L_t$.

We define the number of firms per capita by $n_t \equiv N_t/e^{\lambda t}$. From (7) and the market equilibrium condition of labor, if $R_t > 0$, (8) can be rewritten as

$$r_t = \alpha\theta(1-\theta)\frac{\Omega}{n_t}, \tag{16}$$

where $\Omega \equiv \theta^{\frac{2\theta}{1-\theta}}$. From (9) and the market equilibrium condition of labor, if $R_t > 0$, (5) can be rewritten as

$$r_t = \frac{1}{\beta}\left[\theta(1-\theta)\frac{\Omega}{n_t} - \phi\right] + \frac{\beta - 1}{\beta}\frac{\dot{Z}_t}{Z_t}. \tag{17}$$

From (16) and (17), the rate of quality growth is given by

$$\hat{z}_t(n_t) \equiv \frac{\dot{Z}_t}{Z_t} = \begin{cases} \dfrac{\alpha\beta - 1}{\beta - 1}\theta(1-\theta)\dfrac{\Omega}{n_t} + \dfrac{\phi}{\beta - 1}, & \text{if } \ n_t > \tilde{n}, \\ 0, \quad \text{if } \ \alpha\beta < 1 \ \text{ and } \ 0 < n_t \leq \tilde{n}, \end{cases} \tag{18}$$

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and the interest rate (the return on equity) is given by

$$r_t = \begin{cases} \alpha\theta(1-\theta)\dfrac{\Omega}{n_t}, & \text{if } \ n_t > \tilde{n}, \\ \dfrac{1}{\beta}\left[\theta(1-\theta)\dfrac{\Omega}{n_t} - \phi\right], & \text{if } \ \alpha\beta < 1 \ \text{ and } \ 0 < n_t \leq \tilde{n}, \end{cases} \tag{19}$$

where $\tilde{n} = \max\{0, \frac{(1-\alpha\beta)\theta(1-\theta)\Omega}{\phi}\}$. In equilibrium, the intensity of in-house R&D is determined at the point where the return on in-house R&D [the RHS of (16)] is equal to the return on equity [the RHS of (17)]. Equation (18) indicates that, if $\alpha\beta > 1$ (< 1), a lower number of firms per capita leads to a higher (lower) rate of quality growth. If α and β are sufficiently high (low), the return on in-house R&D is more (less) sensitive to variations in the number of firms per capita. Hence, a lower number of firms per capita raises the return on in-house R&D more than the return on equity, resulting in a higher (lower) rate of quality growth. On the other hand, equation (19) indicates that the interest rate is unambiguously decreasing in the number of firms per capita. The reason is that a lower number of firms per capita leads to a higher demand for intermediate goods at the level of an individual firm.

More intuitively, increasing the number of firms per capita has the following two opposite effects on quality growth. First, the higher interest rates associated with the lower number of firms per capita dampen incentives to in-house R&D. Hereinafter, we call this effect "the interest rate effect." Second, other things being equal, the lower number of firms per capita leads to a larger demand for intermediate goods at the level of an individual firm. This enlarges firm size (the scale of production at the level of an individual firm), which allows in-house R&D expenditure to be spread over more units of goods that the firm produces, which creates a positive incentive to perform in-house R&D. Hereafter, we call this effect "the cost spreading effect." Empirical evidence supports the existence of the cost-spreading effect [e.g., Cohen and Klepper (1996), Pagano and Schvardi (2003), and Laincz and Peretto (2006)]. If $\alpha\beta > 1$, the positive cost spreading effect outweighs the negative interest rate effect. The same logic applies to the case where $\alpha\beta < 1$.

The perfect distribution in the final goods sector implies $N_t X_t = \theta^2 Y_t$ and $w_t L_t = (1-\theta)Y_t$. Applying these relationships to (1), we obtain the following reduced-form aggregate production function of final goods:

$$Y_t = \Omega e^{\lambda t} Z_t. \tag{20}$$

From (20), we have the following lemma.

LEMMA 1. *Taking logarithms and differentiating with respect to time on both sides of (20) yields:*

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- *The growth rate of final outputs is given by*

$$\frac{\dot{Y}_t}{Y_t} = \hat{z}_t(n_t) + \lambda. \tag{21}$$

- *The growth rate of final outputs per capita ($y_t \equiv Y_t/e^{\lambda t}$) is given by*

$$\frac{\dot{y}_t}{y_t} = \hat{z}_t(n_t). \tag{22}$$

Equations (20)–(22) show that there is no productivity gain resulting from product proliferation in final goods production. This is a reasonable approximation mirroring the real economy, although it is somewhat extreme. Empirical evidence shows that the bulk of total factor productivity growth is accounted for by improvements of incumbent firms' productivity, rather than the entry of new firms [e.g., Foster et al. (1998) and Bartelsman and Doms (2000)].[12] Furthermore, the condition of no productivity gains resulting from product proliferation is not crucial for our main conclusion. In Section 5.1, we will discuss the case in which the reduced-form of the aggregate production function of final goods has nonzero social returns resulting from product proliferation.

Let us define the ratio of aggregate consumption to final outputs by $c_t \equiv C_t/Y_t$ and the ratio of aggregate value of the bubbly asset to final outputs by $m_t \equiv M_t/Y_t$. In Appendix B, we show that the dynamical system of the economy is constituted by the following three equations:

$$\dot{n}_t = \frac{\Omega}{\beta}\left(1-\theta^2-c_t\right) - \frac{n_t}{\beta}\left[\phi + \hat{z}_t(n_t) + \beta\lambda\right], \tag{23}$$

$$\dot{c}_t = \left[\varphi(n_t) - \rho\right] c_t - \lambda\rho\left(m_t + \beta\frac{n_t}{\Omega}\right), \tag{24}$$

$$\dot{m}_t = \left[\varphi(n_t) - \lambda\right] m_t, \tag{25}$$

where the difference between the interest rate and the rate of quality growth is given by

$$\varphi(n_t) \equiv r_t - \hat{z}_t = \begin{cases} \dfrac{(1-\alpha)\theta(1-\theta)}{\beta-1}\dfrac{\Omega}{n_t} - \dfrac{\phi}{\beta-1}, & \text{if } \; n_t > \tilde{n}, \\ \dfrac{1}{\beta}\left[\theta(1-\theta)\dfrac{\Omega}{n_t} - \phi\right], & \text{if } \; \alpha\beta < 1 \; \text{ and } \; 0 < n_t \le \tilde{n}. \end{cases} \tag{26}$$

4. THE STEADY-STATE EQUILIBRIUM

In this section, we first characterize the steady-state equilibrium in which $\dot{n}_t = 0$, $\dot{c}_t = 0$, and $\dot{m}_t = 0$. Then, we prove that there may exist multiple steady-state equilibria: the steady-state equilibrium without asset bubbles and

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steady-state equilibria with asset bubbles. We compare the properties of the steady-state equilibria and deduce our main implication. Last, we also examine the stability of the dynamical system. Hereafter, the time index, t, is omitted from variables that remain constant in the steady-state equilibrium.

From (23)–(26), we have the following lemma.

LEMMA 2. *In the steady-state equilibrium,*

- *The number of firms, N_t, grows with a constant population growth rate, λ. Thus, the number of firms per capita, n_t, remains constant over time.*
- *The final outputs Y_t, aggregate consumption, C_t, and aggregate value of the bubbly asset, M_t, grow by $\hat{z}(n)+\lambda$. Thus, the aggregate consumption ratio, C_t/Y_t, and the aggregate value of the bubbly asset to final outputs, M_t/Y_t, remain constant over time.*
- *The final output per capita, y_t, grows by $\hat{z}(n)$.*

Proof. The results are straightforwardly derived by using Lemma 1. ■

From (23), $(\dot{n}_t = 0)$-locus is given by

$$c = \begin{cases} (1-\theta)\left[1+\dfrac{\beta\theta(1-\alpha)}{\beta-1}\right] - \beta\dfrac{n}{\Omega}\left(\dfrac{\phi}{\beta-1}+\lambda\right), & \text{if } \ n > \tilde{n}, \\ (1-\theta^2) - \beta\dfrac{n}{\Omega}\left(\dfrac{\phi}{\beta}+\lambda\right), & \text{if } \ \alpha\beta < 1 \ \text{ and } \ 0 < n \le \tilde{n}. \end{cases} \tag{27}$$

This shows that the $(\dot{n}_t = 0)$-locus is independent of the values of m and c is negatively related to n on the locus. In Figures 2 and 3, the locus is depicted as a downward-sloping curve in (n, c) space. Figure 2 corresponds to the case of $\alpha\beta > 1$, whereas Figure 3 corresponds to the case of $\alpha\beta < 1$ and $\phi > \tilde{\phi}$, where $\tilde{\phi}$ is defined as $\tilde{\phi} \equiv \max\left\{0, \frac{(1-\alpha\beta)\lambda}{\alpha}\right\}$. The condition $\phi > \tilde{\phi}$ ensures a positive rate of steady-state quality growth. We restrict subsequent analyses to the parameter space in which $\phi > \tilde{\phi}$ is satisfied. This downward-sloping curve represents the resource constraint of the economy, which implies that a higher number of firms per capita absorb resources otherwise available for consumption by households.

From (24), $(\dot{c}_t = 0)$-locus is given by

$$c = \begin{cases} \dfrac{\lambda\rho}{\left[\dfrac{(1-\alpha)\theta(1-\theta)}{\beta-1}\dfrac{\Omega}{n} - \dfrac{\phi}{\beta-1} - \rho\right]}\left(m+\beta\dfrac{n}{\Omega}\right), & \text{if } \ n > \tilde{n}, \\ \dfrac{\lambda\rho}{\left[\dfrac{\theta(1-\theta)}{\beta}\dfrac{\Omega}{n} - \dfrac{\phi}{\beta} - \rho\right]}\left(m+\beta\dfrac{n}{\Omega}\right), & \text{if } \ \alpha\beta < 1 \ \text{ and } \ 0 < n \le \tilde{n}. \end{cases} \tag{28}$$

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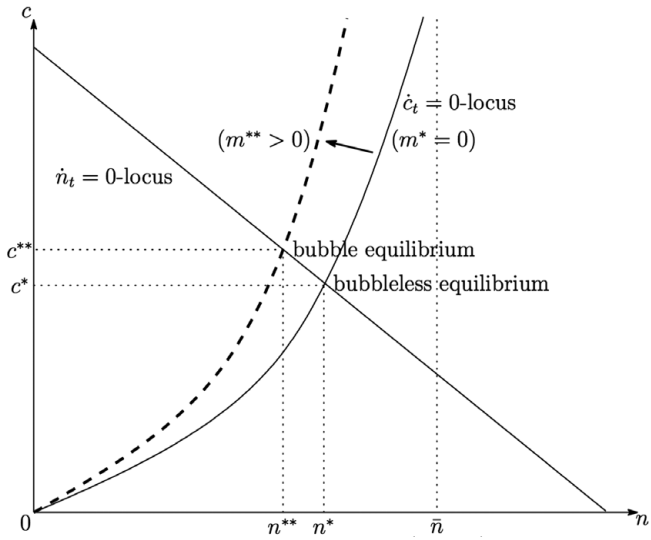


FIGURE 2. Steady-state equilibrium ($\alpha\beta > 1$).

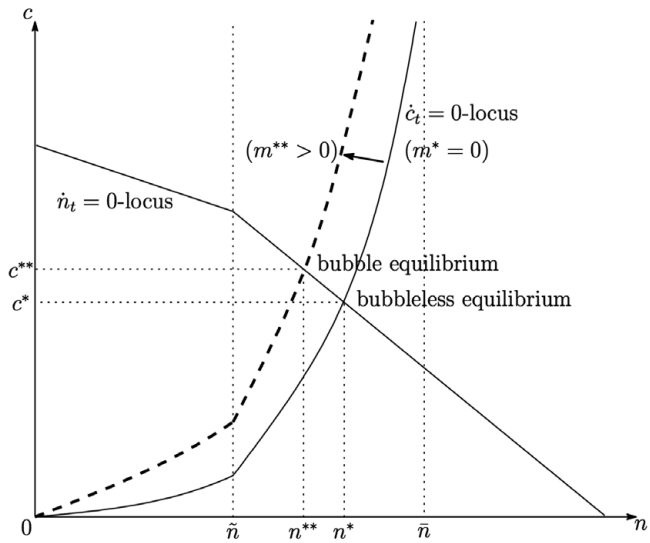


FIGURE 3. Steady-state equilibrium ($\alpha\beta < 1$ and $\phi > \tilde{\phi}$).

This shows that on the $\dot{c}_t = 0$-locus, for any nonnegative value of m, c is positively related to n starting from the origin in the region $0 < n < \bar{n}$, where $\bar{n} \equiv$ argsolve $\{\varphi(n) = \rho\}$.[13] In Figures 2 and 3, the locus is depicted as an upward-sloping curve. Here, the solid line corresponds to the case in which the steady-state value of m is equal to zero, whereas the dotted line corresponds to the case in

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which the stead-state equilibrium value of m is positive. This upward-sloping curve is derived from the aggregation of the Euler equation of households, which implies that a higher number of firms per capita leads to a lower interest rate, and thus households want to consume more.[14]

Before turning to the characterization of the multiple steady-state equilibria, it is useful to have the following definitions.

DEFINITION 1. *The bubbleless equilibrium is the steady-state equilibrium without asset bubbles, which is characterized by* $\{n^*, c^*, m^*\}$ *where* $m^* = 0$. *A bubbly equilibrium is a steady-state equilibrium with asset bubbles, which is characterized by* $\{n^{**}, c^{**}, m^{**}\}$ *where* $m^{**} > 0$.

First, we examine the bubbleless equilibrium. Recall that the $(\dot{n}_t = 0)$-locus is downward-sloping and the $(\dot{c}_t = 0)$-locus with $m^* = 0$ is upward-sloping (solid line) in Figures 2 and 3. Therefore, $\{n^*, c^*\}$ is uniquely determined by the point at which the lines intersect.

Second, we consider the possibility of the existence of a bubbly equilibrium. From (25), if $m^{**} > 0$, $\dot{m}_t = 0$ implies $\varphi(n^{**}) = \lambda$. Therefore, from (18) and (26), we obtain

$$n^{**} = \frac{(1-\alpha)\theta(1-\theta)\Omega}{(\beta-1)\lambda+\phi}, \quad \text{if} \quad \alpha\beta > 1 \quad \text{or} \quad \alpha\beta < 1 \text{ and } \phi > \tilde{\phi}, \tag{29}$$

and

$$\hat{z}(n^{**}) = \frac{(\alpha\beta-1)\lambda+\alpha\phi}{1-\alpha}, \quad \text{if} \quad \alpha\beta > 1 \quad \text{or} \quad \alpha\beta < 1 \text{ and } \phi > \tilde{\phi}. \tag{30}$$

From (27), (29), and (30), c^{**} is given by

$$c^{**} = (1-\theta), \tag{31}$$

and from (28), (29), and (31), m^{**} is given by

$$m^{**} = \frac{(1-\theta)}{\lambda\rho}\left[\lambda - \rho - \frac{(1-\alpha)\beta\theta\lambda\rho}{(\beta-1)\lambda+\phi}\right], \quad \text{if} \quad \alpha\beta > 1 \quad \text{or} \quad \alpha\beta < 1 \text{ and } \phi > \tilde{\phi}. \tag{32}$$

Thus, we have the following lemma.

LEMMA 3. *Suppose that* $\alpha\beta > 1$ *or that* $\alpha\beta < 1$ *and* $\phi > \tilde{\phi}$. *Then,* $m^{**} > 0$ *is established if*

$$\lambda - \rho > \frac{(1-\alpha)\beta\theta\lambda\rho}{(\beta-1)\lambda+\phi} > 0. \tag{33}$$

If (33) holds, we find that, in the bubbleless equilibrium, the steady-state growth rate of final outputs exceeds the interest rate[15] and we have the following main proposition.

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PROPOSITION 1. *Suppose that* $\alpha\beta > 1$ *or* $\alpha\beta < 1$ *and* $\phi > \tilde{\phi}$ *and that (33) holds. Then, there exist two steady-state equilibria: a bubbleless equilibrium* $\{n^*, c^*, m^*\}$ *and a bubbly equilibrium* $\{n^{**}, c^{**}, m^{**}\}$. *Comparing the bubbleless equilibrium and the bubbly equilibrium yields the following results.*

- *The number of firms per capita in the bubbly equilibrium,* n^{**}, *is strictly lower than that in the bubbleless equilibrium,* n^*.
- *The consumption ratio in bubbly equilibrium,* c^{**}, *is strictly higher than that in the bubbleless equilibrium,* c^*.
- *If* $\alpha\beta > 1 (< 1)$, *the growth rate of outputs per capita in the bubbly equilibrium is higher (lower) than that in the bubbleless equilibrium.*

Proof. If $m^{**} > 0$, in Figures 2 and 3, the $(\dot{c}_t = 0)$-locus moves up in counterclockwise direction as compared to the case of $m^* = 0$. We plot the $(\dot{c}_t = 0)$-locus with $m^{**} > 0$ as a dotted line. These figures show that n^{**} is strictly lower than n^* and c^{**} is strictly higher than c^*. Moreover, applying these results to (18) and (22) leads to the last part of this proposition. ■

The key mechanism is as follows. If asset bubbles emerge, households think that they are wealthier, and thus want to consume more. This leads to a higher reservation interest rate for households in the asset market. In order to satisfy the higher reservation interest rate, the return on equity must increase. This requires the number of firms per capita to be lower for any given market size of the economy because the return on equity is positively related to demand for intermediate goods at the level of an individual firm. As a result, the presence of asset bubbles unambiguously gives rise to larger firms.

Consequently, if the cost spreading effect outweighs the interest rate effect (if $\alpha\beta > 1$), as previously noted, the presence of asset bubbles encourages in-house R&D by firms and promotes economic growth. The same logic applies to the case in which the interest rate effect outweighs the cost spreading effect (if $\alpha\beta < 1$). Besides, lower product proliferation makes available economic resources for a higher household consumption, regardless of the effect on economic growth.

The next proposition examines the stability of the corresponding dynamical system. Here, n is the state variable, and c and m are jump variables.

PROPOSITION 2. *Suppose that* $\alpha\beta > 1$ *or* $\alpha\beta < 1$ *and* $\phi > \tilde{\phi}$ *and that (33) holds. The equilibrium path toward bubbleless equilibrium is locally indeterminate, whereas the equilibrium path toward bubbly equilibrium is locally determinate (saddle-point stable). Therefore, for any given initial value of* $n (< \bar{n})$, *a global indeterminacy arises.*

Proof. See Appendix C. ■

Proposition 3 indicates that sunspot equilibria arise, which is consistent with previous studies, such as Grossman and Yanagawa (1993) and Futagami and Shibata (2000).

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5. DISCUSSION

In this section, we first extend the foregoing analysis to the case of a more general final goods production function that involves positive productivity gains resulting from product proliferation. Second, we also discuss how our main conclusion holds in alternative model setups proposed in the literature of R&D-based growth theory so that we can further clarify an important theoretical channel through which the presence of asset bubbles can promote economic growth.

5.1. Productivity Gains Resulting from Product Proliferation

Following Aghion and Howitt (1998), Peretto (2015) and Chu and Ji (2016), we consider the case in which the production function of final goods is specified as

$$Y_t = \int_0^{N_t} X_{it}^{\theta} \left[Z_{it}^{\alpha} \bar{Z}_t^{1-\alpha} \left(\frac{L_t}{N_t^{\sigma}} \right) \right]^{1-\theta} di, \tag{34}$$

where $\sigma \in (0, 1]$ represents an inverse measure of the degree of productivity gains resulting from product proliferation. The other specifications are the same as those in the foregoing analysis. In equilibrium, the reduced-form of (34) is given by

$$Y_t = N_t^{1-\sigma} \Omega e^{\lambda t} Z_t. \tag{35}$$

This shows that the number of firms directly contributes to the efficiency of the final goods production. By redefining n_t as $n_t \equiv N_t^{\sigma}/e^{\lambda t}$, however, we have qualitatively the same properties of the steady-state equilibrium as in the foregoing analyses. The reason for this is that the growth rate of the number of firms is tied to the population growth rate in the steady-state equilibrium.

We can also prove that the stability of the corresponding dynamical system exhibits global indeterminacy if there exist multiple (bubbleless and bubbly) equilibria.[16]

5.2. Alternative Model Setups

Since the foregoing analysis is based on an R&D-based endogenous growth model, we should discuss whether our theoretical conclusions hold in alternative model setups proposed in the literature.

First, we note that our main conclusion does not hold in first-generation canonical R&D-based endogenous growth models, such as those of Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992). Since the R&D investment opportunity in these models is one dimensional (i.e., quality improvement *or* variety expansion), the presence of asset bubbles leads to only a crowding out of investment in the same way as the AK growth model.[17]

The key feature in our analysis is that investment opportunities are two dimensional (i.e., quality improvement *and* variety expansion), which causes the structure of the goods market to be endogenous. In particular, through this endogenous market structure, the presence of asset bubbles gives rise to larger firms, which

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has a positive cost-spreading effect on the intensity of R&D investment for quality improvement. However, alternative model setups in the class of second-generation R&D-based endogenous growth models do not necessarily capture this feature. For example, Howitt (1999) has also developed a model in which there exist such two-dimensional investment opportunities. However, if an analogous analysis to ours is conducted on the basis of the production economy developed by Howitt (1999), it will be found that the presence of asset bubbles will unambiguously impede economic growth.

This difference stems from the differing characteristics of our model framework and that of Howitt. First, in Howitt's model, quality growth is derived from the so-called creative destruction: That is, quality improvement is not brought about by current incumbents but is created by newborn firms that replace the current incumbents. Since both innovations are determined by free-entry conditions, returns to both vertical innovation and horizontal innovation depend on profits. In contrast, in our analysis, returns to vertical innovation depend on marginal increases in profit by in-house R&D, whereas returns to horizontal innovation depend on profits. Second, in Howitt's model, there is an absence of fixed costs for firms (i.e., fixed operating costs and in-house R&D costs).

Due to these differences, in Howitt's model, the intensity of horizontal innovation is tied to exogenous parameters such as R&D efficiency. The presence of asset bubbles affects only the intensity of vertical innovation directly and does not affect the intensity of horizontal innovation at all. In particular, there is no secondary effect on vertical innovation through variations in the number of firms. Only the usual interest rate effect, which has a negative effect on the vertical innovation, is present in Howitt's model.

6. CONCLUSION

In this paper, we have proposed a new theoretical channel through which the presence of asset bubbles can promote economic growth in concurrence with high consumption. The present analysis is based on a continuous-time overlapping generation model with in-house R&D investment and firm entry. The theoretical mechanism is simple and endogenous market structure plays a key role. The presence of asset bubbles leads to a higher equilibrium interest rate. This requires product proliferation to be impeded, which would result in an increase in the demand for differentiated goods, at the level of an individual firm. The resulting larger firm size can encourage incentives to in-house R&D through the cost-spreading effect and promote economic growth. In addition, the lower product proliferation provides economic resources for a higher consumption by households.

Some remarks on potential future work are in order. First, although we provided empirical support for a positive relationship between asset price and firm size in Section 1, the evidence is barely more than anecdotal. One potential avenue for future research is to investigate precise econometric tests for the hypothesis we proposed. Second, the present analysis focuses on an environment in which

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quality improvement is derived from only incumbents in order to simply capture the growth effect of asset bubbles under endogenous market structure. Recently, Acemoglu and Cao (2015) developed an R&D-based endogenous growth model with more general characterization in which incumbents engage in incremental quality improvement and entrants engage in radical quality improvement to replace incumbents. Since the model generates a nondegenerate equilibrium firm-size distribution, it will be interesting to replicate the present analysis using this model. We leave this for future research.

NOTES

1. An asset bubble is defined as the existence of a difference between the fundamental value of an asset and its market value. For example, if an intrinsically useless asset whose fundamental value is zero has a positive value, we say there exists an asset bubble.

2. These models belong to the second-generation class of R&D endogenous growth models. In Section 5, we will discuss whether the main implications of the present paper hold in alternative setups that have been developed in R&D-based growth theory.

3. If we employ the alternative setup that there exists a positive death rate, following Blanchard (1985), our main conclusion does not change.

4. As one notable exception, Olivier (2000) analyzes the case in which bubbles attach directly to real assets that have fundamental value, such as the equity of firms, and shows that bubbles on equity provide a channel to enhance economic growth.

5. Average employment per establishment is computed from total employment divided by the total number of establishments in all industries. Data on total employment and the number of establishments were obtained from County Business Patterns data published by the US Census Bureau (http://www.census.gov/econ/cbp/index.html). Here, an establishment is defined to be a single physical location at which business is conducted or services or industrial operations are performed. It is not necessarily identical to a company or enterprise, which may consist of one or more establishments. If firm size is measured as average employment per firm, we obtain a similar empirical result using data derived from Business Dynamics Statistics (BDS). Here, a firm is defined as a business organization consisting of one or more domestic establishments that are under common ownership or control. Data on household and nonprofit net worth for the United States were obtained from the Flow of Funds at the Federal Reserve (http://www.federalreserve.gov/releases/z1/Current/data.htm). Net worth is defined as the difference between the value of all assets, financial and nonfinancial, and all liabilities at a particular point in time.

6. If we employ the exact model specified by Peretto (1998, 2007a, 2007b) and Dinopoulos and Thompson (1998), our main conclusion does not qualitatively change but the analysis becomes more cumbersome.

7. If we consider an environment in which vertical dimension of technological change is driven from in-house R&D for the cost reduction of intermediate goods production, the conclusion of the present paper is not changed.

8. Peretto (2015) and Chu and Ji (2016) use the same specification. Peretto (2011) specifies the final goods production function in a different way. Specifically, in Peretto (2011), the final goods production function is given by $Y_t = \int_0^{N_t} X_{it}^{\theta} (Z_{it}^{\alpha} \bar{Z}_t^{1-\alpha} L_{it})^{1-\theta} di$ in which the term (L_t/N_t) is replaced by L_{it} that represents the input of labor that uses intermediate goods i. Since $\int_0^{N_t} L_{it}\ di = N_t L_{it} = L_t$ holds due to the assumption of a symmetric equilibrium, this difference does not change the subsequent analyses. However, the present specification is more clear-cut and useful for generalization. See Peretto (2015) for a more detailed discussion.

9. Since a perfect credit market prevails and there are no distortionary taxes, it does not matter whether firms finance in-house R&D investments by issuing equity or by using retained earnings.

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10. If, instead, the fixed operating costs are assumed to be proportional to production volumes, the model also works well without changing our main conclusion.

11. If the entry costs are assumed to be proportional to the average production volumes of the incumbents, our main conclusion does not change.

12. See Acemoglu and Cao (2015) for a precise review of the empirical literature.

13. If the steady-state number of firms per capita is higher than $\bar{n}$, the steady-state consumption ratio becomes negative.

14. In the present model framework, a higher number of firms per capita also yields a higher ratio of the aggregate value of equity to outputs, NV/Y, as NV/Y is given by $\beta n/\Omega$ in equilibrium. Hence, for any given interest rate, the higher wealth ratio induces households to consume more through the wealth effect. However, this only reinforces the upward-sloping gradient of this locus and is not crucial for our main implications.

15. See Appendix D for the proof.

16. Derivations are available upon request.

17. This is the case when asset bubbles attach to an intrinsically useless asset as with the foregoing analysis. As noted in footnote 5, if asset bubbles attach directly to equities, the presence of asset bubbles can enhance economic growth even in a model with only one-dimensional R&D investment, as shown by Olivier (2000).

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APPENDIX A

Integrating (12) yields

$$\lim_{x\to\infty} a(s,x)e^{\left(-\int_t^x r_v dv\right)} - a(s,t) + \int_t^\infty c(s,x)e^{\left(-\int_t^x r_v dv\right)} dx = \int_t^\infty (w_x)\, e^{\left(-\int_t^x r_v dv\right)} dx \equiv h_t.$$

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Applying (13) and the no-Ponzi-game condition to the above equation, we obtain $c(s,t) = \rho\,[a(s,t) + h_t]$. Aggregating this yields

$$C_t = \rho\left[a(0^-,t) + h_t\right] + \rho\lambda\int_0^t a(s,t)e^{\lambda s}ds + \rho\lambda\int_0^t h_t e^{\lambda s}ds,$$

$$\Leftrightarrow\ C_t = \rho\,(A_t + H_t)\,,\quad \text{where}\ \ H_t \equiv h_t + \lambda\int_0^t h_t e^{\lambda s}ds. \tag{A.1}$$

From (13) and (A.1), differentiating the aggregate consumption with respect to time yields

$$\dot{C}_t = (r_t - \rho)\,c(0^-,t) + c(t,t)\lambda e^{\lambda t} + (r_t - \rho)\,\lambda\int_0^t c(s,t)e^{\lambda s}ds,$$

$$\Leftrightarrow\ \dot{C}_t = \rho\lambda e^{\lambda t}\,[a(t,t) + h_t] + (r_t - \rho)\,C_t,$$

$$\Leftrightarrow\ \dot{C}_t = \rho\lambda H_t + (r_t - \rho)\,C_t,\quad (\because\ a(t,t) = 0)\,,$$

$$\Leftrightarrow\ \dot{C}_t = (r_t - \rho + \lambda)\,C_t - \rho\lambda A_t.$$

And aggregating (12) yields (15).

APPENDIX B

Let $a_t \equiv A_t/Y_t$. From (14) and (20),

$$\frac{\dot{c}_t}{c_t} = \frac{\dot{C}_t}{C_t} - \frac{\dot{Y}_t}{Y_t} = \frac{\dot{C}_t}{C_t} - \hat{z}_t - \lambda\ \Leftrightarrow\ \dot{c}_t = [\varphi(n_t) - \rho]\,c_t - \lambda\rho a_t. \tag{B.1}$$

From (20), the market equilibrium condition of assets can be rewritten as

$$a_t = m_t + \beta\frac{n_t}{\Omega}.$$

Then, substituting this condition into (B.1) yields (24).

From (20) and $N_t X_t = \theta^2 Y_t$, the market equilibrium condition of final goods can be rewritten as

$$1 = c_t + \theta^2 + \frac{n_t}{\Omega}\left(\phi + \hat{z}_t + \beta\frac{\dot{N}_t}{N_t}\right).$$

Then, from this condition and the definition of n_t, we obtain (23).

From the market equilibrium condition of labor and $N_t w_t L_t = (1-\theta)Y_t$, (15) can be rewritten as

$$\frac{\dot{A}_t}{A_t} = r_t + (1-\theta)\frac{1}{a_t} - \frac{c_t}{a_t}.$$

Using the above equation and (20), we obtain

$$\dot{a}_t = [\varphi(n_t) - \lambda]\,a_t + (1-\theta) - c_t.$$

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Differentiating the market equilibrium condition of assets with respect to time and using (23) and the law of motion of the asset ratio, we obtain (25).

APPENDIX C

From (23), (24), and (25), the system of linearized differential equations around the steady state is given by

$$\begin{pmatrix} \dot{n}_t \\ \dot{c}_t \\ \dot{m}_t \end{pmatrix} = \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix} \begin{pmatrix} n_t - n^{st} \\ c_t - c^{st} \\ m_t - m^{st} \end{pmatrix}, \tag{C.1}$$

where $\{n^{st}, c^{st}, m^{st}\}$ represents $\{n^*, c^*, m^*\}$ or $\{n^{**}, c^{**}, m^{**}\}$ and

$$J_{11} = -\frac{1}{\beta}\left[\phi + \hat{z}(n^{st}) + \beta\lambda\right] - \frac{n^{st}}{\beta}\frac{\partial \hat{z}(n)}{\partial n}\bigg|_{n=n^{st}} < 0,$$

$$J_{12} = -\frac{\Omega}{\beta} < 0,$$

$$J_{13} = 0,$$

$$J_{21} = c^{st}\frac{\partial \varphi(n)}{\partial n}\bigg|_{n=n^{st}} - \lambda\rho\frac{\beta}{\Omega} < 0,$$

$$J_{22} = \begin{cases} \varphi(n^*) - \rho > 0 & \text{if } m^{st} = m^* \quad (\because\ n^* > n^{**}), \\ \lambda - \rho > 0 & \text{if } m^{st} = m^{**}, \end{cases}$$

$$J_{23} = -\lambda\rho < 0,$$

$$J_{31} = \begin{cases} 0 & \text{if } m^{st} = m^*, \\ m^{**}\dfrac{\partial \varphi(n)}{\partial n}\bigg|_{n=n^{**}} < 0 & \text{if } m^{st} = m^{**}, \end{cases}$$

$$J_{32} = 0,$$

$$J_{33} = \begin{cases} \varphi(n^*) - \lambda < 0 & \text{if } m^{st} = m^* \quad (\because\ n^* > n^{**}), \\ = 0 & \text{if } m^{st} = m^{**}. \end{cases}$$

Defining the matrix $J \equiv \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix}$, we obtain the characteristic equation of the coefficient matrix on the right-hand side of (C.1):

$$0 = |J - qI| = -q^3 + \mathrm{Tr}Jq^2 - BJq + \mathrm{Det}J \equiv f(q),$$

where I is the identity matrix, $\mathrm{Tr}J$ is the trace of J, $\mathrm{Det}J$ is the determinant of J, and $BJ \equiv \begin{vmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{vmatrix} + \begin{vmatrix} J_{22} & J_{23} \\ J_{32} & J_{33} \end{vmatrix} + \begin{vmatrix} J_{11} & J_{13} \\ J_{31} & J_{33} \end{vmatrix}$.

The dynamical system has one state variable, n_t, and two jump variables, c_t, and m_t. Hence, if the number of negative eigenvalues around the steady state is 1, there exists a

https://doi.org/10.1017/S1365100517000712 Published online by Cambridge University Press

unique equilibrium saddle path toward the steady state. If the number of negative eigenvalues around the steady state is more than 1, there exists a continuum of equilibrium paths toward the steady state.

Following Benhabib and Perli (1994), we now check the number of negative eigenvalues around the steady state by applying the Routh–Hurwitz theorem.

THEOREM. *The number of roots of the polynomial in (C.1) with positive real parts is equal to the number of variations of sign in the scheme*

$$-1 \quad \mathrm{Tr}J \quad -\mathrm{B}J + \frac{\mathrm{Det}J}{\mathrm{Tr}J} \quad \mathrm{Det}J. \tag{C.2}$$

Around the steady state without asset bubbles, we obtain $\mathrm{Det}J = J_{12}J_{23}J_{31} - J_{12}J_{21}J_{33} > 0$ and $\mathrm{Tr}J = J_{11} + J_{22} + J_{33} < 0$ ($\because \varphi(n^*) < \lambda$). Then, we can conclude that around the steady state without asset bubbles, there is one positive eigenvalue and two eigenvalues with negative real part because there is only one change of the sign in the scheme (C.2) regardless of the value of $\mathrm{B}J$. Therefore, there exists a continuum of equilibrium paths toward the steady state without asset bubbles.

Around the steady state with asset bubbles, we obtain $\mathrm{Det}J = J_{12}J_{23}J_{31} < 0$, $\mathrm{Tr}J = J_{11} + J_{22} < 0$, and $\mathrm{B}J = J_{11}J_{22} - J_{12}J_{21} < 0$. Hence, we also obtain $-BJ + \mathrm{Det}J/\mathrm{B}J > 0$. Then, we can conclude that around the steady state with asset bubbles there are two eigenvalues with positive real parts and one negative eigenvalue, because there are two changes of the sign in the scheme (C.2). Therefore, the equilibrium path toward the steady state with asset bubbles is locally determinate.

APPENDIX D

From (26), we find that the difference between the interest rate and the growth rate of final output, $\varphi(n) - \lambda$, is decreasing in n. In the steady-state equilibrium with asset bubbles, we obtain $\varphi(n^{**}) - \lambda = 0$. Since it has been shown that n^* is higher than n^{**}, we find that $\varphi(n^*) - \lambda < 0$.

https://doi.org/10.1017/S1365100517000712 Published online by Cambridge University Press