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CORRIGENDUM

Elliptic functions, theta function and hypersurfaces satisfying a basic equality

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Case (x) in Theorem 6 is missing. The correct statement of Theorem 6 in the paper should read as follows.

THEOREM 6. *Let* $\mathbf{x}: M^n \to H^{n+1}(-1) \subset \mathbb{E}_1^{n+2}$ $(n > 2)$ *be an isometric immersion of a conformally flat* n*-manifold. Then it satisfies equality* (1·2) *if and only if one of the following ten cases occurs.*

(i) M^n *is an open portion of* $H^n(-1)$ *and the immersion* $\mathbf{x}: M^n \to H^{n+1}(-1)$ *is totally geodesic.*

(ii) M^n *is an open portion of* A_a^n *for some* $a > 1$ *and, up to rigid motions, the immersion* $\mathbf{x}: A_a^n \to H^{n+1}(-1) \subset \mathbb{E}_1^{n+2}$ *is given by*

$$\mathbf{x}(x, y_1, \ldots, y_n) = (a \cosh x, \sinh x, \sqrt{a^2 - 1}\, y_1 \cosh x, \ldots \\ \ldots, \sqrt{a^2 - 1}\, y_n \cosh x) \tag{1·10}$$

with $y_1^2 + y_2^2 + \cdots + y_n^2 = 1$.

(iii) M^n *is an open portion of* G^n *and, up to rigid motions, the immersion* $\mathbf{x}: G^n \to H^{n+1}(-1) \subset \mathbb{E}_1^{n+2}$ *is given by*

$$\mathbf{x}(x, u_2, \ldots, u_n) = \left(\left(1 + \frac{u_2^2 + \cdots + u_n^2}{2}\right)\cosh x, \right. \\ \left. \frac{u_2^2 + \cdots + u_n^2}{2} \cosh x, \sinh x, u_2 \cosh x, \ldots, u_n \cosh x\right). \tag{1·11}$$

(iv) M^n *is an open portion of* H_a^n *for some* $a > 0$, *and, up to rigid motions, the immersion* $\mathbf{x}: H_a^n \to H^{n+1}(-1) \subset \mathbb{E}_1^{n+2}$ *is given by*

$$\mathbf{x}(x, y_1, \ldots, y_n) = (\sqrt{a^2 + 1}\, y_1 \cosh x, \ldots, \sqrt{a^2 + 1}\, y_n \cosh x, \\ a \cosh x, \sinh x) \tag{1·12}$$

with $y_1^2 - y_2^2 + \cdots - y_n^2 = 1$.

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(v) *M^n is an open portion of Y_a^n for some a, $0 < a < 1$ and, up to rigid motions, the immersion $\mathbf{x}\colon Y_a^n \to H^{n+1}(-1) \subset \mathbb{E}_1^{n+2}$ is given by*

$$\mathbf{x}(x, y_1, \ldots, y_n) = (\cosh x, a \sinh x, \sqrt{1-a^2}\, y_1 \sinh x, \ldots \\ \ldots, \sqrt{1-a^2}\, y_n \sinh x) \tag{1·13}$$

with $y_1^2 + y_2^2 + \cdots + y_n^2 = 1$.

(vi) *$n = 3$, M^3 is an open portion of F^3 and, up to rigid motions, the immersion $\mathbf{x}\colon M^3 \to H^4(-1) \subset \mathbb{E}_1^5$ is given by*

$$\mathbf{x}(x, u, v) = (\sqrt{2}\cosh u \cosh v, \sqrt{2}\cosh u \sinh v, \sqrt{2}\sinh u, \cos x, \sin x). \tag{1·14}$$

(vii) *$n = 3$, M^3 is an open portion of L^3 and, up to rigid motions, the immersion $\mathbf{x}\colon L^3 \to H^4(-1) \subset \mathbb{E}_1^5$ is given by*

$$\mathbf{x}(x, u, v) = \operatorname{sech} x \left(x^2 + u^2 + v^2 + \cosh^2 x + \tfrac{1}{4}, \right. \\ \left. x^2 + u^2 + v^2 + \cosh^2 x - \tfrac{1}{4}, x, u, v\right). \tag{1·15}$$

(viii) *$n = 3$, M^3 is an open portion of $\hat{C}_a^3$ for some $a > 1$ and, up to rigid motions, the immersion $\mathbf{x}\colon \hat{C}_a^3 \to H^4(-1) \subset \mathbb{E}_1^5$ is given by*

$$\mathbf{x}(x, y_1, y_2, y_3) = \\ \frac{1}{ak'}\left(\sqrt{a^2k'^2 + cn^2(ax)}\cosh\left(\frac{k'}{k}x - \tfrac{1}{2}\ln\frac{\Theta(ax-\gamma)}{\Theta(ax+\gamma)} - a\mathrm{Z}(\gamma)x\right),\right. \\ \sqrt{a^2k'^2 + cn^2(ax)}\sinh\left(\frac{k'}{k}x - \tfrac{1}{2}\ln\frac{\Theta(ax-\gamma)}{\Theta(ax+\gamma)} - a\mathrm{Z}(\gamma)x\right), \\ \left. y_1 cn(ax), y_2 cn(ax), y_3 cn(ax)\right), \quad y_1^2 + y_2^2 + y_3^2 = 1, \tag{1·16}$$

where $\gamma = sn^{-1}(1/(ak^2))$, $k = \sqrt{a^2+1}/(\sqrt{2}a)$ and $k' = \sqrt{a^2-1}/(\sqrt{2}a)$.

(ix) *$n = 3$, M^3 is an open portion of D_a^3 for some $0 < a < 1$ and, up to rigid motions, the immersion $\mathbf{x}\colon D_a^3 \to H^4(-1) \subset \mathbb{E}_1^5$ is given by*

$$\mathbf{x}(x, u, v) = \frac{1}{ak'}\left(k\, dn\left(\frac{a}{k}x\right)\cosh u \cosh v,\right. \\ k\, dn\left(\frac{a}{k}x\right)\cosh u \sinh v, k\, dn\left(\frac{a}{k}x\right)\sinh u, \\ \sqrt{k^2 dn^2\left(\frac{a}{k}x\right) - a^2k'^2}\cos\left(k'x - \frac{i}{2}\ln\frac{\Theta((a/k)x-\gamma)}{\Theta((a/k)x+\gamma)} - i\frac{a}{k}\mathrm{Z}(\gamma)x\right), \\ \left.\sqrt{k^2 dn^2\left(\frac{a}{k}x\right) - a^2k'^2}\sin\left(k'x - \frac{i}{2}\ln\frac{\Theta((a/k)x-\gamma)}{\Theta((a/k)x+\gamma)} - i\frac{a}{k}\mathrm{Z}(\gamma)x\right)\right), \tag{1·17}$$

where $\gamma = sn^{-1}(k/a)$, $k = \sqrt{2}a/\sqrt{1+a^2}$ and $k' = \sqrt{1-a^2}/\sqrt{1+a^2}$.

(x) *M^n is an open portion of the warped product $O^n =: \mathbb{R} \times_{e^x} S^{n-1}(1)$ and, up to rigid motions, the immersion $\mathbf{x}\colon O^n \to H^{n+1}(-1) \subset \mathbb{E}_1^{n+2}$ is given by*

$$\mathbf{x}(x, y_1, \ldots, y_n) = \left(e^x + \frac{e^{-x}}{2}, \frac{e^{-x}}{2}, e^x y_1, \ldots, e^x y_n\right), \tag{1·18}$$

where $y_1^2 + y_2^2 + \cdots + y_2^n = 1$.

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Case (x) occurs due to the fact that besides $f(x) = \alpha \sinh(x + b)$ and $f(x) = \alpha \cosh(x + b)$ the differential equation (8·6) (appeared on page 497) has yet another solution given by $h(x) = e^x$. The latter solution gives rise to case (x).

Also, we point out that α appearing in (6·15) and (6·16) on page 485 should read a.

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