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Over-reflection of magnetoacoustic ion-cyclotron plasma waves

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The over-reflection of magnetoacoustic ion-cyclotron waves in a warm Hall plasma is investigated. It is shown that the effect of the Hall term is to strongly support over-reflection, thereby destabilizing the flow. Its relevance to the current-vortex sheet of the magnetosheath resulting from the interaction of the solar wind and the Earth's magnetopause is pointed out.

1. Introduction

The problem of over-reflection of magnetoacoustic ion-cyclotron waves at a current-vortex sheet is investigated in this paper. This is essentially an extension of the classical problem of over-reflection of many types of waves propagating in a shear flow, as discussed comprehensively by many authors (e.g. McKenzie 1970; Eltayeb and McKenzie 1975; Acheson 1976), to include the effects of the Hall-current term.

Although the general dispersion equation that results from the usual one-fluid model of a Hall plasma has been derived by many authors (e.g. McKenzie 1979), its correct analysis and physical interpretations when applied to the problem of over-reflection at a current-vortex sheet have never been fully elucidated.

This can be attributed to many factors. First of all, the Hall-current term introduces considerable mathematical difficulties in the dispersion equation. Even in the simplest configuration of plane waves of prescribed sinusoidal dependence in the direction of the ambient magnetic field, the presence of the Hall-current term raises the order of the governing differential equation in the direction perpendicular to the magnetic field by two, and at the same time introduces coupling between the magnetoacoustic and Alfvén waves. The singularities of the governing equation are also changed in a radical manner from the usual ones of pure magnetoacoustic waves. In fact, these singularities are made to occur only where the phase speed of the waves matches the shear-flow speed. This point is particularly important, since the shear flow is essentially responsible for the energy changes required to account for major wave behaviours, such as over-reflection (by which a reflected wave at a vortex sheet extracts energy from the shear flow) and the critical-layer absorption mechanisms (Booker and Brethertom 1967; El Mekki and McKenzie 1977). From this point of view, the importance of including the Hall current in the

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equations of motion can hardly be exaggerated, even if it is very small. In fact, many authors working on the problem of over-reflection have stressed the fact that over-reflection arises only if either the incident or the refracted wave is a negative-energy wave, or, in more appropriate words, the Doppler-shifted wave frequencies (with respect to the shear flow) of the incident and transmitted waves have opposite signs (McKenzie 1972*a*, *b*). The boundary conditions must be as follows.

1. The total pressure (fluid + magnetic) must be continuous across the sheet (Acheson 1976), which provides one equation.

2 The displacement must be continuous across the sheet, which guarantees that the vortex sheet is a streamline common to both media (El Mekki 1985). This means that the displacement on either side of the vortex sheet must be equal to the amplitude of the distortion of the interface, which provides two more equations.

3. The interface must be a line of force common to both media. This also provides two further equations upon relating the components of the magnetic field normal to the discontinuity to the amplitude of the distortion of the interface.

Therefore in all we get five equations in the five unknowns that we have (namely the amplitude of the distortion of the interface and the two components of each of the displacement and magnetic field normal to the sheet), which ensures that the problem is physically consistent and well posed (McKenzie 1970).

The third difficulty that arises concerns the definitions of the energy density, the energy flux and momentum in the plasma. When the Hall-current term is present, the dispersion equation becomes so complicated that these quantities need both caution and patience to calculate. In calculating the wave energy flux, we must remember that it is made up of three parts:

(i) the fluid pressure only (not the total pressure), multiplied by the wave-velocity component;

(ii) the wave kinetic energy multiplied by the wave-velocity component;

(iii) the electromagnetic wave energy as given by the Poynting vector, which takes into account the magnetic pressure; the energy density may then be easily calculated by dividing the energy flux by the group velocity in any direction.

If all these considerations are taken into account then the resulting expressions for the physical quantities will be correct, and hence the physical conclusions will also be correct.

The expressions for energy flux and the reflection and refraction coefficients that we obtain coincide with those obtained by previous workers when the Hall term is neglected, and our modifications resulting from the inclusion of this term conform closely with the laws of conservation of energy and momentum, and therefore the physical conclusions that we draw are justified.

It is shown in this paper that the Hall-current term strongly supports the process of wave amplification or over-reflection. This conclusion is based on three facts. First, since the Hall term makes the only singularity of the

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dispersion equation occur where the Doppler-shifted frequency becomes zero, there is always a chance of this Doppler-shifted frequency becoming negative as well as positive. This means that the over-reflection mechanism, which takes place only if the Doppler-shifted frequencies of the incident and transmitted waves have different signs, is linked directly to the shear-flow strength. Hence the condition for over-reflection is always guaranteed if the shear flow is sufficiently strong. Secondly, it is shown that the energy flux converging on the vortex sheet with the incident wave equals the energy flux diverging from the vortex sheet with the reflected and transmitted waves, despite the fact that the reflection coefficient is greater than unity. This contradiction can only be removed if either the incident or the transmitted wave is a negative-energy wave, which again confirms the basic condition for over-reflection. Thirdly, as seen from the wavenumber curves, it is only the slow mode (with respect to the resultant wavenumber perpendicular to the magnetic field) that reaches the asymptote arising where the frequency is Doppler-shifted to zero. This requires that an incident fast (slow) wave couples with a transmitted slow (fast) wave if their wavenumbers normal to the vortex sheet are of opposite signs, whereas an incident wave couples with a refracted wave of the same type if their wavenumbers normal to the vortex sheet are of the same sign. This is also in conformity with the other basic condition for over-reflection, namely that the reflection coefficient must exceed unity.

If we adopt the view that the wave amplification or over-reflection essentially promotes instability, we can assert that the Hall-current term clearly destabilizes the flow even if it is very small.

It remains to point out that the over-reflection mechanism requires that the wind shear be sufficiently strong that the Doppler-shifted wave frequency becomes negative (McKenzie 1970; El Mekki and McKenzie 1977). This condition is easily fulfilled in the case of the solar wind impinging on the Earth's magnetosphere and giving rise to the magnetosheath. Reliable data (see e.g. Clemmow and Dougherty 1969) show that the solar wind has a sonic Mach number of about eight and an Alfvénic Mach number of about ten, and thus the solar wind is both supersonic and super-Alfvénic. Hence over-reflection is a very relevant wave mechanism at the vortex sheet of the magnetosheath, and many instabilities can be explained in terms of it, for example, instabilities of the magnetopause and magnetic storms.

2. The dispersion equation

The equations of motion governing the motion of a collisionless unstratified, non-rotating and warm plasma are the equations of momentum and continuity for each of the two species of the plasma, together with Maxwell's equations. These equations, when linearized about some basic state, will apply to the motion of small-amplitude oscillations. We shall consider a basic state consisting of a quasi-uniform magnetic induction B and a quasi-uniform wind U, both in the x direction. The plasma will also be assumed to consist of equal numbers of protons and electrons, and we shall confine attention to oscillations of wavelength much greater than the width of the vortex sheet at the interface between two media, so that we may neglect both the electron inertia and the displacement current in Maxwell's equations (McKenzie 1971).

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The Fourier analysis of the linearized equations under these approximations readily yields the following consistent system of equations:

$$\begin{bmatrix} \rho\hat{\omega} & 0 & 0 & -k_x c^2 & 0 & 0 & 0 \\ 0 & \rho\hat{\omega} & 0 & -k_y c^2 & \frac{Bk_y}{\mu} & \frac{-Bk_x}{\mu} & 0 \\ 0 & 0 & \rho\hat{\omega} & -k_z c^2 & \frac{Bk_z}{\mu} & 0 & \frac{-Bk_x}{\mu} \\ \rho k_x & \rho k_y & \rho k_z & -\hat{\omega} & 0 & 0 & 0 \\ -ik_x B & 0 & 0 & 0 & i\hat{\omega} & \frac{V^2 k_x k_z}{\Omega_p} & \frac{-V^2 k_x k_z}{\Omega_p} \\ 0 & -ik_x B & 0 & 0 & \frac{-V^2 k_x k_z}{\Omega_p} & i\hat{\omega} & \frac{-V^2 k_x^2}{\Omega_p} \\ 0 & 0 & -ik_x B & 0 & \frac{-V^2 k_x k_y}{\Omega_p} & \frac{-V^2 k_x^2}{\Omega_p} & i\hat{\omega} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ \delta P \\ b_x \\ b_y \\ b_z \end{bmatrix} = 0, \quad (2.1)$$

where ρ is the basic state density, $\hat{\omega} = \omega - k_x U$ is the Doppler-shifted wave frequency, $(k_x, k_y, k_z) = \mathbf{k}$ is the wavenumber vector, c is the speed of sound in the plasma, V is the Alfvén-wave speed, Ω_p is the proton gyrofrequency, (u, v, w) is the perturbation-velocity vector and (b_x, b_y, b_z) is the perturbation-induction vector.

The consistency condition that the determinant of the coefficients in (2.1) should vanish readily yields the dispersion equation

$$\alpha k_\perp^4 - \beta k_\perp^2 + \gamma = 0, \quad (2.2)$$

in which

$$\alpha = \frac{\hat{\omega}^2 k_x^2 V^4 c^2}{\Omega_p^2}, \quad (2.3)$$

$$\beta = c^2\left[(\hat{\omega}^2 - k_x^2 V^2)^2 - \frac{2\hat{\omega}^2 k_x^4 V^4}{\Omega_p^2}\right] + \hat{\omega}^2 V^2\left[\hat{\omega}^2 - k_x^2 V^2\left(1 - \frac{\hat{\omega}^2}{\Omega_p^2}\right)\right], \quad (2.4)$$

$$\gamma = -(\hat{\omega}^2 - k_x^3 c^2)\left[(\hat{\omega}^2 - k_x^2 V^2)^2 - \frac{2\hat{\omega}^2 k_x^4 V^4}{\Omega_p^2}\right], \quad (2.5)$$

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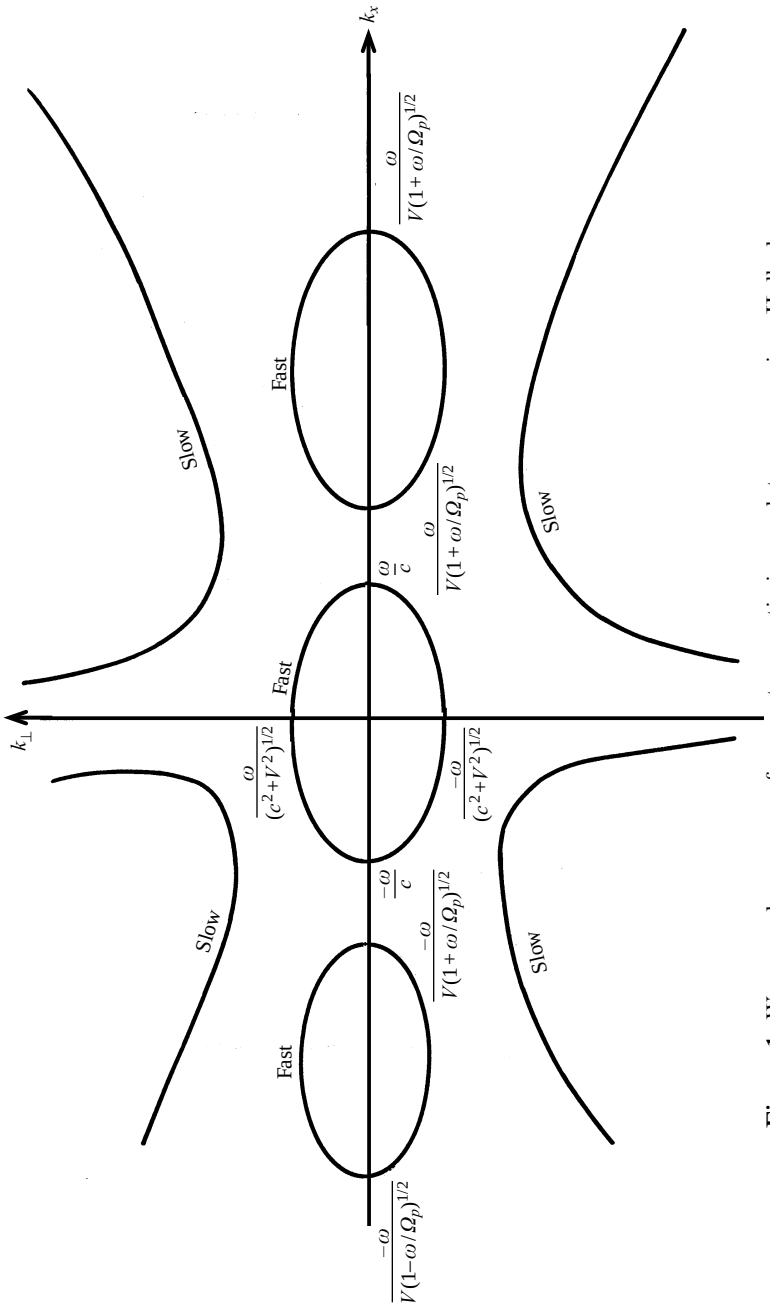


Figure 1. Wavenumber curves of magnetoacoustic ion-cyclotron waves in a Hall plasma in the $(k_x, k_\perp)$ plane for the case $U = 0$ and $\omega < \Omega_p$.

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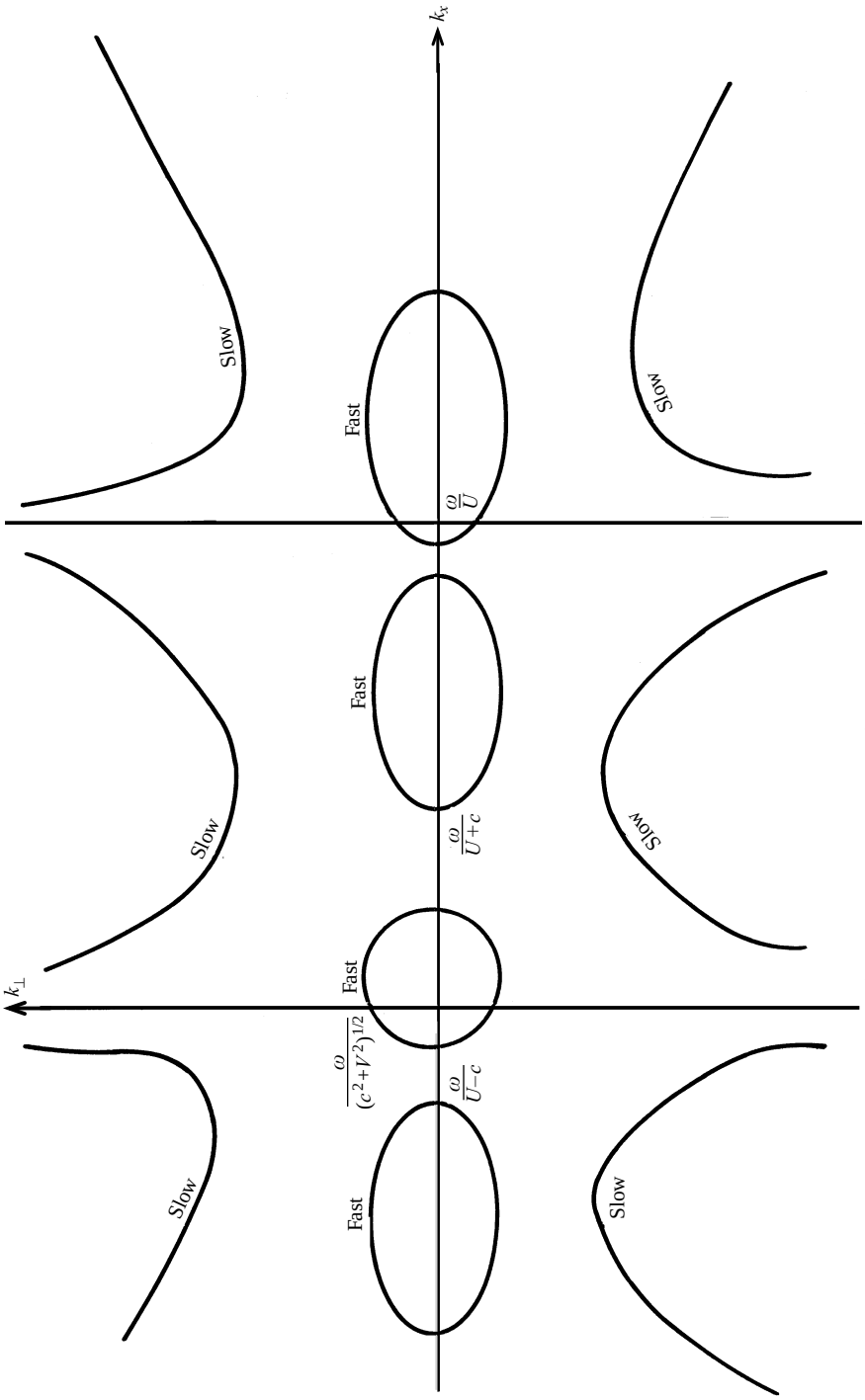


Figure 2. Wavenumber curves for the case $U < \min(V, c)$ and $\omega < \Omega_p$.

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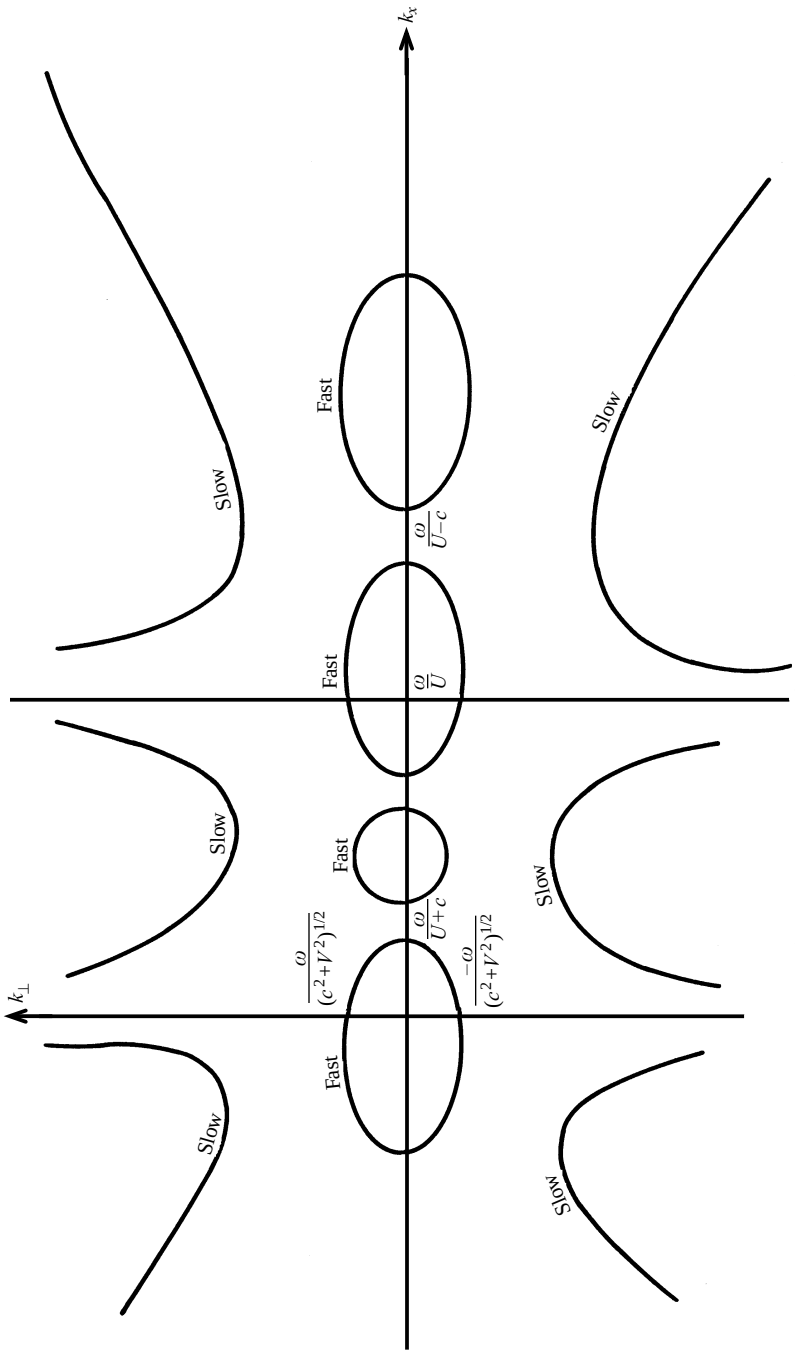


Figure 3. Wavenumber curves for the case $U > \max(V, c)$ and $\Omega < \Omega_p$.

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and

$$k_\perp = (k_y^2 + k_z^2)^{1/2} \tag{2.6}$$

is the wavenumber vector perpendicular to the magnetic field.

In the limit as $\Omega_p \to \infty$ (i.e. neglecting the Hall current) this dispersion equation reduces at once to the usual magnetoacoustic waves studied by McKenzie (1970) and also by Eltayeb (1977) in connection with linear wave motions through magnetic-velocity shears.

When the Hall current is taken into account, numerous dispersive features are introduced into the system. Solving (2.2) for $k_\perp^2$, we have

$$k_\perp^2 = \frac{\beta \pm (\beta^2 - 4\alpha\gamma)^{1/2}}{2\alpha}, \tag{2.7}$$

showing that, with respect to propagation perpendicular to the magnetic field, there are two modes, corresponding to the + sign (the slow mode) and the − sign (the fast mode). In the absence of the Hall term, this slow-and-fast classification is with respect to propagation parallel to the magnetic field.

As $\alpha \to 0$ (either $k_x \to 0$ or $\hat{\omega} \to 0$), $k_\perp^2$ tends to ∞ for the slow mode, whereas it tends to $\omega^2/(c^2 + V^2)$ as $k_x \to 0$ and to ω^2/U^2 as $\hat{\omega} \to 0$ in the case of the fast mode.

In Figs 1–3 we have depicted the wavenumber curves for three cases $U = 0$, $\omega < \Omega_p$ (Fig. 1), $U < \min(c, V)$, $\omega < \Omega_p$ (Fig. 2) and $U > \max(c, V)$, $\omega < \Omega_p$ (Fig. 3). The slow modes are the open branches that reach to the asymptotes, whereas the fast modes are the closed loops. The choice of parameters is intended to show the evolution of the wavenumber curves as the shear flow strength is increased, and this is important since the presence of the Hall term in the dispersion equation makes the main feature of the wavenumber curves (the asymptote at $\hat{\omega} = 0$) dependent on the shear-flow strength. In fact, the two asymptotes that occur at $\hat{\omega}^2 - k_x^2 V^2 = 0$ in the absence of the Hall term are recombined into a single asymptote at $\hat{\omega} = 0$.

However, as far as the over-reflection mechanism is concerned, we can see qualitatively and from the wavenumber curves that the Hall-current term closely relates the flow properties to the shear-flow strength, thereby strongly supporting over-reflection, which is known to depend on the shear-flow strength. In Secs 3 and 4 below we give a detailed mathematical analysis of the over-reflection mechanism.

3. Over-reflection at a current-vortex sheet

We assume the current-vortex sheet to coincide with the plane $y = 0$ parallel to both the magnetic field and the shear flow in medium 1 ($y < 0$) and medium 2 ($y > 0$). A wave incident on the current-vortex sheet will give rise to a reflected wave, a transmitted wave and a wave-like distortion of the interface (El Mekki 1985). The velocity component in the y direction may be written in the form

$$v_1 = e^{-ik_{y1}y} + Re^{ik_{y1}y} \quad (y < 0, \text{ medium 1}), \tag{3.1}$$

$$v_2 = Te^{-ik_{y2}y} \quad (y > 0, \text{ medium 2}), \tag{3.2}$$

where R and T are the reflection and transmission coefficients respectively.

The kinematic boundary condition guaranteeing that the vortex sheet is a

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streamline common to both media is that the displacement must be continuous across the vortex sheet (see e.g. Acheson 1976). This condition yields

$$\frac{v_1}{\hat{\omega}_1} = \frac{v_2}{\hat{\omega}_2} \quad \text{at} \quad y = 0, \tag{3.3}$$

which gives, on using (3.1) and (3.2),

$$\frac{1+R}{\hat{\omega}_1} = \frac{T}{\hat{\omega}_2}. \tag{3.4}$$

The dynamical boundary condition to be satisfied across the interface or current-vortex sheet is that the total pressure (fluid + magnetic)

$$\pi = \frac{Lv}{\hat{\omega} k_y M + iN} \tag{3.5}$$

be conserved in both media. Here

$$L = \rho\left[(\hat{\omega}^2 - k_c^2 V^2)^2 - \frac{\hat{\omega}^2 |\mathbf{k}|^2 k_x^2 V^4}{\Omega_p^2}\right], \tag{3.6}$$

$$M = \hat{\omega}^2 - k_x^2 V^2 - \frac{|\mathbf{k}|^2 k_x^2 V^4}{\Omega_p^2}, \tag{3.7}$$

$$N = \frac{|\mathbf{k}|^2 V^4 k_x^2}{\Omega_p}. \tag{3.8}$$

This dynamical boundary condition may be written as

$$\frac{L_1}{\hat{\omega}_1 k y_1 M_1 + iN_1} + \frac{RL_1}{-\hat{\omega}_1 k y_1 M_1 + iN_1} = \frac{T}{\hat{\omega}_2 k y_2 M_2 + iN_2}. \tag{3.9}$$

Solving (3.4) and (3.9), we obtain

$$R = \frac{X + iY}{Z - iY}, \tag{3.10}$$

$$T = \frac{\hat{\omega}_2}{\hat{\omega}_1} \frac{X + Z}{Z - iY}, \tag{3.11}$$

where

$$X(Z) = \hat{\omega}_1^2 k_{y1} L_1 M_1 (\hat{\omega}_2^2 k_{y2}^2 M_2^2 + N_2^2) - (+)\, \hat{\omega}_2^2 k_{y2} L_2 M_2 (\hat{\omega}_1^2 k_{y1}^2 M_1^2 + N_1^2), \tag{3.12}$$

$$Y = \hat{\omega}_1 L_2 N_2 (\hat{\omega}_2^2 k_{y1}^2 M_1^2 + N_1^2) - \hat{\omega}_1^2 L_1 N_1 (\hat{\omega}_2^2 k_{y2}^2 M_2^2 + N_2^2). \tag{3.13}$$

For over-reflection to arise, we must have $|R| > 1$, or $X^2 > Z^2$, a condition that readily yields

$$k_{y1} k_{y2} L_1 L_2 M_2 M_2 < 0. \tag{3.14}$$

In the absence of the Hall term, (3.14) reduces to

$$k_{y1} k_{y2} (\hat{\omega}_1^2 - k_x^2 V_1^2)(\hat{\omega}_2^2 - k_x^2 V_2^2) < 0, \tag{3.15}$$

as derived by McKenzie (1971).

The condition (3.14), even in its simpler form (3.15), is rather complicated mathematically for one to draw physical conclusions from it. However, insight may be gained if we carry out a careful consideration of energy and momentum.

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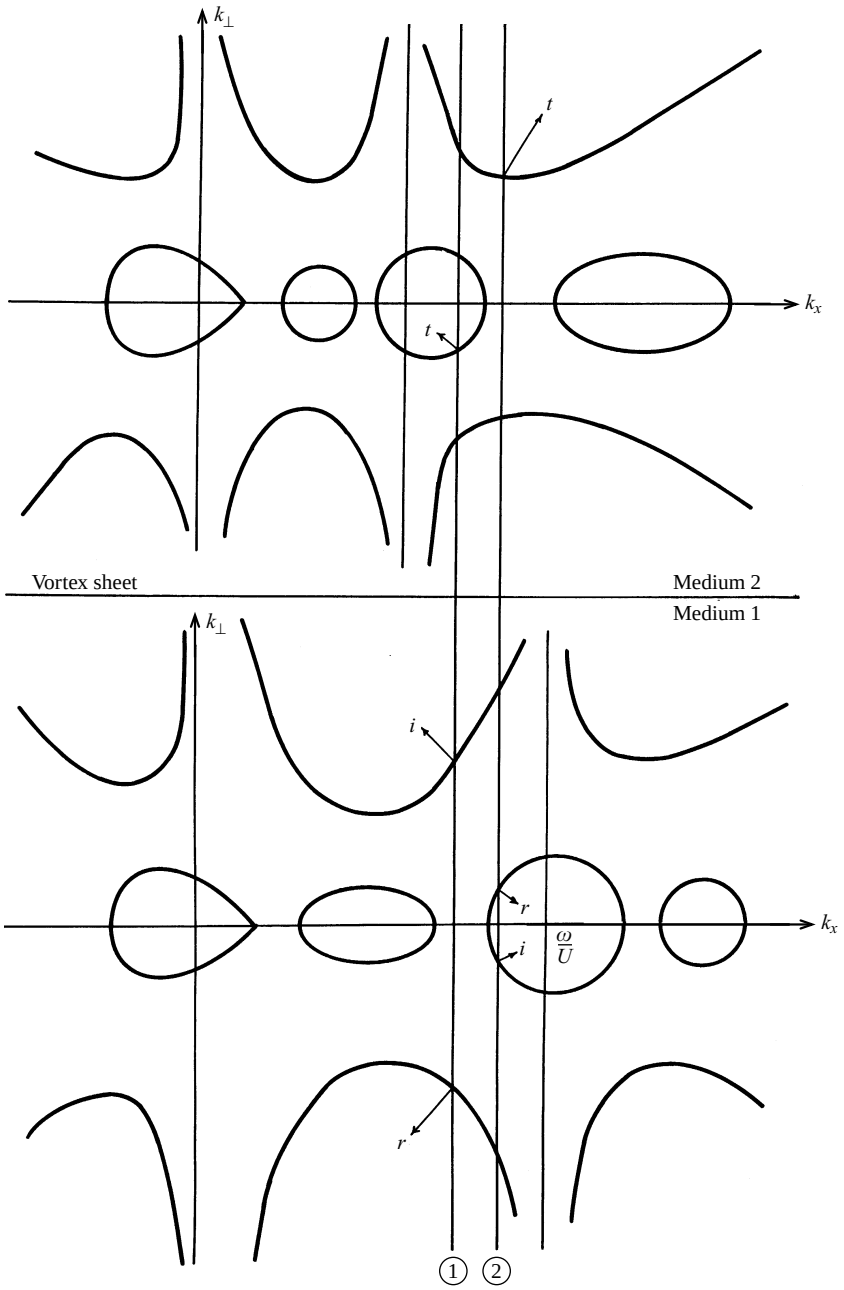


Figure 4. Over-reflection of magnetoacoustic ion-cyclotron plasma waves at a current-vortex sheet. For both rays $k_{y1}k_{y2} < 0$, and an incident fast (slow) wave couples with a slow (fast) transmitted wave.

4. Energy and momentum considerations

The energy flux in the y direction as measured in the laboratory frame, relative to which the fluid moves with speed U, is given by

$$F_y = \overline{\rho v} + \overline{\rho U u v} + \overline{\frac{1}{\mu}(\mathbf{E} \wedge \mathbf{B})_j}, \tag{4.1}$$

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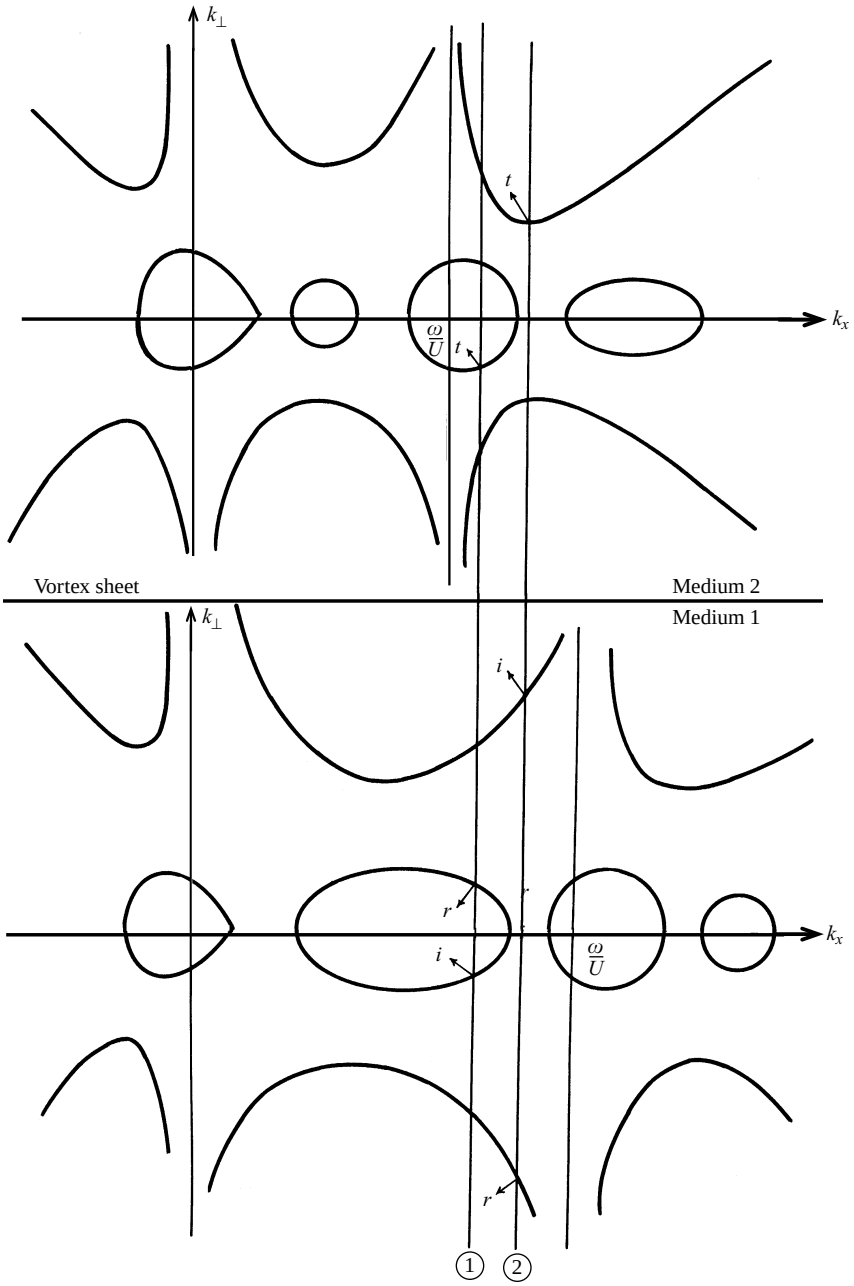


Figure 5. Over-reflection for two rays with $k_{y1}\,k_{y2} > 0$: an incident fast (slow) wave couples with a fast (slow) transmitted wave.

where p is the fluid pressure ($= c^2\,\delta\rho$) and the last term represents the Poynting vector, which includes the magnetic pressure ($= Bb_x/\mu$); the overbar denotes a time average. Using the linearized equations of 'frozen-in' lines of force ($\mathbf{E} = -\mathbf{u}\times\mathbf{b}$), we find

$$E_x = 0, \qquad E_2 = Bv - Ub_y \tag{4.2}$$

(McKenzie 1979).

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On the other hand, Maxwell's equation $\boldsymbol{\nabla}\times\mathbf{E}=\partial\mathbf{B}/\partial t$ gives

$$b_y=\frac{-k_x E_z}{\hat{\omega}}, \tag{4.3}$$

whereas the x component of the momentum equation gives $u=k_x p/\hat{\omega}$. Therefore

$$\begin{aligned} F_y &= \overline{\rho v}+\frac{k_x Up}{\hat{\omega}}+\frac{Bk_x v}{\mu}\left(1+\frac{Uk_x}{\hat{\omega}}\right) \\ &= \frac{w}{\hat{w}}\overline{\pi v}=\frac{\omega}{2\hat{\omega}}\mathrm{Re}(\pi v), \end{aligned} \tag{4.4}$$

where π, as before, is the total pressure (fluid + magnetic) and Re denotes the real part. Using (3.5), we have

$$F_y=\frac{\omega k_y LM}{\hat{\omega}^2 k_y^2 M^2+N^2}|v|^2. \tag{4.5}$$

We now show that the energy flux converging on the vortex sheet with the incident wave, $F_y^{(\mathrm{in})}$, is equal to the energy flux diverging from the vortex sheet with the refracted and transmitted waves, $F_y^{(\mathrm{out})}$, thus demonstrating the conservation of energy. Using (4.6) together with (3.10) and (3.14), we have

$$\begin{aligned} \frac{F_y^{(\mathrm{out})}}{F_y^{(\mathrm{in})}} &= |R|^2+|T|^2\frac{k_{y2}L_2M_2\hat{\omega}_1^2k_{y1}^2L_1^2+N_1^2}{k_{y1}L_1M_1\hat{\omega}_2^2k_{y2}L_2^2+N_2^2} \\ &= |R|^2+|T|^2\frac{\hat{\omega}_1^2Z-X}{\hat{\omega}_2^2Z+X} \\ &= \frac{X^2+Y^2}{Z^2+Y^2}+\frac{(Z+X)^2}{Z^2+Y^2}\frac{Z-X}{Z+X}=1. \end{aligned} \tag{4.6}$$

This result is in conformity with the general principle that the energy in the whole system is conserved during the process of over-reflection and proves that the physical approach of the problem is correct. However, we must account for the energies carried by each of the three waves during over-reflection.

The energy flux in the y direction as measured in a frame of reference moving with the fluid is given by

$$F'_y=\frac{\hat{\omega}}{\omega}F_y=\frac{\hat{\omega}k_y LM}{\hat{\omega}^2k_y^2M^2+N^2}. \tag{4.7}$$

Therefore, since both incident and transmitted waves have their group velocities in the same direction, it follows that $F'_{y1}F'_{y2}>0$ or

$$\omega_1\omega_2k_{y1}k_{y2}L_1L_2M_1M_2>0. \tag{4.8}$$

Combining this with (3.14), we immediately obtain

$$\hat{\omega}_1\hat{\omega}_2<0. \tag{4.9}$$

Thus, for over-reflection to arise, this condition must be satisfied, which means that the Doppler-shifted frequencies of the incident and transmitted waves must have opposite signs. A wave for which $\hat{\omega}<0$ is termed a negative-energy wave (Eltayeb and McKenzie 1975, Acheson 1976; El Mekki and McKenzie 1977). Thus, for over-reflection to arise, either the incident or the transmitted wave must be a negative-energy wave.

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This condition will be fulfilled for prescribed ω and k_x provided that the shear-flow strength is sufficiently strong. However, we still have the condition (3.14) to fulfil. Now the quantity LM is positive for a slow wave (since $k_\perp^2$ is large), whereas it is negative for a fast wave belonging to the closed loops (since $k_\perp^2$ is small). This can be seen readily from the wavenumber curves in Fig. 3, in which the shear-flow strength is sufficiently high to ensure $\hat{\omega} < 0$.

It therefore follows from (3.14) that if $k_{y1}\, k_{y2} < 0$ then over-reflection will arise if a fast (slow) incident wave in medium 1 couples with a slow (fast) transmitted wave in medium 2. Similarly, if $k_{y1}\, k_{y2} > 0$ then over-reflection will arise provided that a slow (fast) incident wave in medium 1 couples with a slow (fast) transmitted wave in medium 2. This is depicted in Figs 4 and 5 respectively. In both cases it is assumed that $U_2 > U_1$ and $\omega/U_2 < k_x < \omega/U_1$ such that all transmitted waves are negative-energy waves.

5. Conclusions

The inclusion of the Hall-current term in the dispersion equation for magnetoacoustic waves, even under all permissable approximations, results in many features. First, it transpires that the dispersion equation splits into two modes: slow and fast. However, this classification is with respect to the resultant wave phase speed perpendicular to the direction of the magnetic field (and shear flow), rather than parallel to it as is the case with hydromagnetic wave in non-ionized fluids (McKenzie 1972*a*, *b*). Secondly, the two asymptotes, which gives rise to critical levels characteristic of magnetoacoustic waves, are found to recombine into one such asymptote, which occurs where the phase speed parallel to the shear flow matches the shear-flow strength. In this respect, the Hall current counteracts the effect of the ambient magnetic field in a similar manner to magnetic fields aligned with the direction of flow (Schwartz and Bell 1984; Eltayeb and Hassan 1986).

The changes in the wavenumber curves due to the Hall term provide two features that favour over-reflection. The first is that the slow mode exists for all k_x, which allows more chances of over-reflection. This can be seen from Figs 1–3. The second is that the asymptote at $\hat{\omega} = 0$ always ensures the existence of positive- and negative-energy waves, and hence the basic condition is readily fulfilled (see Figs 2 and 3, in which $U \neq 0$). In fact, Figs 4 and 5 demonstrate the ease with which over-reflection can take place, in contrast to the case in which the Hall term is absent (McKenzie 1970).

Thus we conclude that the effect of the Hall current is clearly to favour over-reflection, and if we adopt the view that over-reflection is a destabilizing mechanism of the flow then we can conclude with certainty that the Hall effect is a destabilizing factor for the plasma.

The basic condition for over-reflection to arise, namely that either the incident or transmitted wave should be a negative-energy wave, requires that the shear-flow strength be sufficiently strong. This is particularly suited to the case of the solar wind impinging on the Earth's magnetosphere, which has a speed much in excess of both the sound speed and the Alfvén-wave speed (McKenzie 1970).

Therefore the results of this paper are most appropriate to the vortex sheet of the magnetosheath of the magnetosphere, and it can be asserted that the

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instability of any wave motion in the magnetosphere or solar wind can be attributed to over-reflection of magnetoacoustic ion-cyclotron waves at the magnetosheath. Such instabilities may also include phenomena reaching the Earth's atmosphere, such as magnetic storms. This is also strongly supported by the fact that other wave behaviours that could lead to instabilities (e.g. critical-level emission; see El Mekki 1982) are not favoured.

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