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OPTIMAL R&D SUBSIDIES WITH HETEROGENEOUS FIRMS IN A DYNAMIC SETTING

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When observably heterogeneous firms engage in R&D and policy can be conditioned on the heterogeneity, what is the optimal policy? This paper starts with a static duopoly model of R&D competition with uncertainty and finds it welfare enhancing to subsidize the larger firms with no subsidies for the smaller firm, extending existing results. This result follows because the policymaker's goal is to minimize the average cost of production. Our paper demonstrates that these results are not robust to a dynamic setting. The optimal policy depends on the equilibrium type of competition that emerges without intervention—an insight that cannot be found in a static setting. In a dynamic setting, the degree of competition becomes an endogenous variable. Interestingly, although the optimal policy in some cases provides a slightly larger subsidy for the larger firm, it is the smaller firm that benefits most in terms of firm value.

Keywords: R&D Policy, Dynamic Setting, Industrial Organization, Duopoly

1. INTRODUCTION

What is the optimal allocation of R&D subsidies across firms that are observably heterogeneous in size? The empirical Industrial Organization literature provides substantial evidence that the social rates of return to R&D greatly exceed private rates of return [see Griliches (1998)]. The endogenous growth literature emphasizes the importance of the R&D process in driving growth and largely concurs that R&D subsidies from the government can be welfare enhancing. Restrictive assumptions regarding market structure and firm homogeneity allow these models to be tractable [e.g., Aghion and Howitt (1992); Peretto (1999)], but questions about the optimal allocation of R&D subsidies across heterogeneous firms cannot be addressed in these frameworks.[1] We characterize the optimal policy in terms of R&D subsidies and/or taxes for heterogeneous firms in a dynamic setting and find that the optimal policies differ, at times significantly from those results in a

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static setting. For example, optimal policy in some cases increases the value of the relatively smaller firm and decreases the value of the larger firm even when a greater R&D subsidy is provided to the larger firm.

It is not uncommon for policymakers to target certain firms in terms of R&D support based on the size of the firm in practice. Larger firms receive R&D support due to their resource advantage, market impact or political influence. But there are also examples throughout Europe and the United States where R&D policies are designed to favor small and midsize enterprises (SMEs).[2] This paper takes a step toward understanding how to analyze these policies.

Prior approaches to the question posed have been limited to static analyses. In a static, symmetric duopoly model with linear demand, Hinloopen (1997, 2000) finds that the optimal R&D subsidy is zero if there are no R&D spillovers, and positive subsidies if there are. Lahiri and Ono (1988) consider asymmetric firms. They show that exogenously reducing the cost of production for the relatively small firm harms welfare when the initial cost differential between firms is sufficiently large. Lahiri and Ono (1999) extend the result to consider cost-reducing R&D investments and find that the initially more efficient firm should be subsidized and the less efficient firm be taxed. In the static setting subsidizing the larger firm is welfare enhancing because the reductions in the marginal cost of the largest firm have larger net effects on consumer and producer surplus. Lahiri and Ono (1999) find the optimal R&D subsidy to be dependent on the shape of the demand curve.[3]

In this paper, we also begin with a static duopoly model in which R&D is a stochastic process and the firms compete in Cournot fashion. We extend the framework of Lahiri and Ono (1999), who consider deterministic R&D, by introducing stochastic R&D and show the results carry through in the static setting. In addition, if R&D subsidies cannot be conditioned on firm size, we find that it is welfare enhancing for the policymaker to tax *both* firms to reduce wasteful duplication.

It is not clear whether these results extend to a dynamic framework given the relatively scant literature. The closest paper to our work is Acemoglu and Akcigit (2012) who consider a dynamic Bertrand setting emphasizing intellectual property rights in a winner-take-all scenario. They find that the optimal policy is "state-dependent" in which the degree of patent protection *increases* with the technology gap between the lead firm and follower firms, a result that mirrors the static Cournot duopoly conclusions.

In a dynamic, monopolistic model of R&D, Hinloopen et al. (2013) account for several stylized facts of R&D including the possibility of a monopolist engaging in R&D despite short-term negative profits. The long-term perspective of the monopolist allows for a unifying model but limits the policy implications to this specific market setting and does not address the scenario in which multiple firms are engaged in R&D competition as in this paper. Kovac and Zigic (2016) also consider a dynamic, duopolistic R&D model in which the leading firm conducts pre-emptive R&D under certain market conditions with limited spillovers to force

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the following firm to exit the market. While this motivating factor for the leader is present in our model, we focus on the optimal policies and how they change with R&D competition.

In our dynamic model, based on Ericson and Pakes (1995), we consider Cournot competition, as in the static literature. The policymaker can condition the subsidies (taxes) on relative firm size. In this case, the policymaker considers the impact of subsidies, not only on the implications for static welfare but also on the long-run market equilibrium concentration which affects welfare. Our results demonstrate that the static results are not robust to a dynamic setting. The optimal policy depends on the equilibrium type of competition that emerges without intervention, an insight that cannot be found in a static setting. When the environment is characterized by very little R&D by both firms, subsidizing the larger of the two firms and taxing the smaller firm raises welfare by providing an incentive to lower costs even though the result is greater product market concentration. However, if the initial environment (without intervention) is dominated by one firm, the optimal policy changes. In this case, the policymaker increases welfare through subsidizing both firms. Specifically, the policy subsidizes the larger firm slightly more. Counterintutively, and in contrast to the results in static models, the policy leads to significant gains in value for the smaller firm and a decrease in value of the dominant firm. Finally, if the environment is characterized by intense R&D competition, the optimal policy is to tax both firms because much of the firms' private gains derive from socially wasteful business stealing.

The remainder of the paper proceeds as follows. Section 2 begins by laying out the static duopoly environment with results. Section 3 sets up the infinite horizon model. Section 4 illustrates the core results from the dynamic model and analyzes the robustness of the core results. Finally, Section 5 concludes the paper.

2. STATIC DUOPOLY MODEL

2.1. Static Model Setup

We begin with a static duopoly model found elsewhere in the literature for two reasons. First, it is useful to understand the driving force behind the optimal policy because similar effects are present in the dynamic setting. Second, our static model is a close, but not an identical match, to the prior literature. Specifically, we allow for R&D uncertainty, rather than deterministic R&D, and the possibility of a negative shock to a firms stock of innovations to bound the state space when moving to the dynamic framework. We verify that our model extends the previous results and ensures they are robust to the alterations introduced. Thus, the changes in optimal policy are derived entirely from the dynamic environment and not from minor changes made to the period game.

Consider two firms, $i \in [1, 2]$, producing a homogenous product. The firms differ only in their initial stock of innovations, w_i, and compete in two stages. In the first stage, firms choose R&D effort, x_i, in order to increase the probability of a

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successful innovation which increases its stock of innovations. With homogenous products, we focus on process innovations; thus, the stock of innovations maps directly into marginal costs. A greater stock of innovations means lower marginal costs of production. In the second stage, firms compete in the product market in Cournot fashion, choosing production quantities denoted by q_i.

A welfare maximizing policymaker observes the initial stock of innovations and gives firm-specific *ad valorem* R&D subsidies (or taxes), s_i, financed through a lump-sum tax on consumers. The policymaker grants R&D subsidies (or taxes) to maximize the total welfare (consumer surplus plus producer surplus net of taxes).[4]

In the second-stage Subgame Perfect Nash Equilibrium firm i maximizes profits with respect to quantity given the demand and R&D outcome while the rival firm faces a symmetric optimization problem, $\pi_i = (P(Q) - mc_i)\, q_i$, where $P(Q)$ is the general inverse demand, $Q = q_i + q_j$, and mc_i is the constant marginal cost of production. Each firm chooses q_i to maximize profits competing noncooperatively and taking first period R&D decisions as given.

Each firm begins with an initial stock of innovations, w_i. The marginal cost of production for firm i is expressed as $mc_i = mc_0 e^{-\eta(w_i+\nu-\Delta)}$, where ν is an indicator taking on a value of 1, if R&D is successful and 0 otherwise. Δ is the outcome of a negative shock which takes the value of one if the shock occurs and zero otherwise. The negative shock occurs with the exogenous probability of δ and is required to bound the state space in the dynamic setting. In the remainder of this section, we suppress the negative shock, Δ, to simplify the exposition, i.e., we set $\delta = 0$, but use a nonzero value in the numerical simulations. mc_0 is the marginal cost of production with zero innovations and η is a parameter that governs the rate at which each innovation reduces the marginal cost.

We assume that R&D outlays increase the probability of innovation at a diminishing rate and R&D productivity is constant and symmetric across firms, as in Pakes and McGuire (1994). We utilize the following functional form: $\mathrm{Pr}_i(x_i) = \frac{\bar{w}x_i}{1+\bar{w}x_i}$, where $\bar{w} > 0$ is the constant, symmetric R&D productivity parameter.

In the first stage, each firm chooses their level of R&D to maximize their expected value, V_1 and V_2, given the R&D subsidies. The maximization problem for firm i is as follows:

$$\max_{x_i} V_i = \mathrm{Pr}_i \mathrm{Pr}_j \pi^i_{SS} + \mathrm{Pr}_i \left(1 - \mathrm{Pr}_j\right) \pi^i_{SF} + (1 - \mathrm{Pr}_i)\, \mathrm{Pr}_j \pi^i_{FS} + (1 - \mathrm{Pr}_i) \left(1 - \mathrm{Pr}_j\right) \pi^i_{FF} - (1 - s_i)\, cx_i \tag{1}$$

where c is the constant and symmetric unit cost of R&D and x_i and x_j are the R&D outlays for firm i and firm j, respectively. π^i_{xx} are the reduced form profits from stage two for firm i given the R&D outcome. The subscripts refer to the R&D outcome of firm i and the second letter refers to the R&D outcome for the rival firm, either *S*ucceed or *F*ail. For example, $\pi_{i,FS}$ is the stage two profits for firm i if firm i fails (F) and firm j succeeds (S) in innovation. There are four possible outcomes from the R&D competition between firms: (1) both firms successfully innovate;

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(2) firm 1 is successful while firm 2 is unsuccessful; (3) firm 2 is successful while firm 1 is unsuccessful; or (4) neither firm successfully innovates:

$$x_i = \left(\frac{Pr_j \left(\pi_{i,SS} - \pi_{i,FS} \right) + \left(1 - Pr_j \right) \left(\pi_{i,SF} - \pi_{i,FF} \right)}{\bar{w} \left(1 - s_i \right) c} \right)^{1/2} - \frac{1}{\bar{w}}, \tag{2}$$

where x_i represents the optimized amount of R&D from maximizing (1). Note that the second term in the above equation is a constant and creates regions of corner solutions for the optimal choice of x_i, *coasting states* in the terminology of Ericson and Pakes (1995).

Subsidies increase R&D holding the rival's probability of success constant. The shape of the R&D reaction functions depends on the difference between the marginal gain in profits from successful innovation conditional on the R&D outcome of the rival firm. The marginal gain from successful innovation is larger if the rival firm fails in their R&D efforts. In other words, R&D is a strategic substitute. If this condition does not hold, then R&D is a strategic complement. In this case, in order to guarantee existence of an equilibrium, the slope of the reaction function must be strictly less than 1.

The policymaker maximizes welfare (the sum of consumer and producer surplus minus taxes) by choosing subsidies for each firm which are paid for in the form of a lump sum tax on consumers given by $T = c \left(s_i x_i^* + s_j x_j^* \right)$. The policymaker chooses the level of subsidies (or taxes) and can distinguish between the large and small firms.

2.2. Static Results

We first consider the case in which firms are identical except for the initial stock of innovations but the policymaker cannot favor one particular firm. Without loss of generality, we assume that firm 1 holds a higher or equivalent stock of initial innovations, i.e., $w_1 \geq w_2$. Thus, by assumption, firm 1 begins with a lower marginal cost and a larger initial market share. We solve for the optimal firm specific subsidies utilizing a numerical approach due to the probabilistic nature of our model to illustrate the welfare maximizing policies. The results are qualitatively similar to those expressed in Lahiri and Ono (1999) and Kitahara and Matsumura (2006). Table 1 lists the parameters used in the approach taken here.

The inverse demand curve is given by $P = A - B\left(Q\right)$, where $A, B > 0$ and $Q \leq A/B$. The welfare maximizing subsidies are $s_1 = s_2 = 0$. Table 2 summarizes the maximum welfare results with each firm's efficiency level set to 10. The firms optimally choose R&D such that they have a 39.9% chance of success. Their expected profits are 87.42 each and overall welfare is 350.78. The policymaker optimally chooses no subsidies as in Hinloopen (1997).[5]

Table 3 summarizes the results for two cases of asymmetric stocks of innovations. The first combination consists of the more efficient firm with a stock of knowledge of $w_1 = 10$, while the less-efficient firm has a stock of knowledge of

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TABLE 1. Parameters

A	30	Demand intercept
B	1	Demand slope
mc_0	5	Initial marginal cost of production
c	0.25	Marginal cost of R&D
η	0.1	Rate of marginal cost decrease
$\bar{w}$	0.3	Productivity of R&D
δ	0.5	Probability of a negative shock
Δ	1	Size of a negative shock

TABLE 2. Symmetric firms

	Firm 1	Firm 2
Initial innovations	10	10
Subsidy	0	0
Probability of innovation	0.399	0.399
R&D outlays	2.213	2.213
Welfare	350.780	
Consumer surplus	175.939	
Firm 1 E[value]	87.420	
Firm 2 E[value]	87.420	
Lump sum tax	0	

TABLE 3. Asymmetric firms and symmetric subsidies

	Firm 1	Firm 2	Firm 1	Firm 2
Initial innovations	10	2	10	6
Subsidy	−0.0125	−0.0125	0.0	0.0
Prob. of innovation	0.418	0.557	0.408	0.491
R&D outlays	2.392	4.198	2.300	3.218
Welfare	325.592		339.885	
Consumer surplus	162.371		170.426	
Firm 1 E[value]	101.831		93.079	
Firm 2 E[value]	61.370		76.380	
Lump sum tax	−0.021		0.0	

$w_2 = 2$. In the second combination, $w_1 = 10$, while $w_2 = 6$. The policymaker optimally chooses to tax both firms to discourage the wasteful R&D by the smaller firm. The intuition is that the smaller firm's R&D efforts result in business stealing, a transfer of surplus between firms rather than social gains. By discouraging the small firm, the expected gains for the larger firm increase since it will be subject to a lower probability of business stealing. The larger firm increases R&D

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TABLE 4. Asymmetric firms and asymmetric subsidies

	Firm 1	Firm 2	Firm 1	Firm 2
Initial innovations	10	2	10	6
Subsidy	0.050	−0.075	0.025	−0.025
Prob. of innovation	0.436	0.544	0.416	0.485
R&D outlays	2.577	3.975	2.372	3.137
Welfare	325.596		339.885	
Consumer surplus	162.385		170.423	
Firm 1 *E*[value]	101.907		93.105	
Firm 2 *E*[value]	61.288		76.352	
Lump sum tax	−0.042		−0.005	

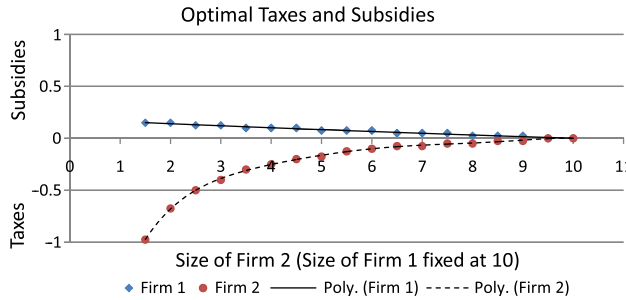


FIGURE 1. Optimal taxes and subsidies with asymmetric firms. The stock of innovations for firm 1 is fixed at 10. The *x*-axis displays the efficiency level of firm 2. The *y*-axis shows the optimal subsidy (tax) by firm.

despite the tax. Innovations by the larger firm lead to a lower average cost of production. Moreover, the policymaker does not have the power of a social planner who would produce all output at the firm with the lower marginal cost. Thus, the policymaker, through the tax, shifts production to the more efficient firm.

When firm 1 has a marginal cost advantage, the welfare maximizing subsidies are $s_1 > 0$ and $s_2 < 0$ if firms are taxed/subsidized asymmetrically. The larger the gap between firm sizes, the more the small firm is taxed and the larger firm is subsidized. Table 4 summarizes the results for two combinations of firm sizes allowing for asymmetric policy. Overall, the subsidy for the larger firm and the tax on the smaller firm both increase in the absolute value of the difference in efficiency levels, $|w_1 - w_2|$.

Figure 1 presents the optimal R&D subsidies and taxes when the firms differ with respect to efficiency and the policymaker is able to condition the subsidies on their size. Figure 1 shows the optimal subsidy (positive value) or tax (negative value) for a given stock of knowledge at firm 2.

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3. DYNAMIC MODEL

The static model yields results consistent with previous work when adding uncertainty to the outcome of the R&D process. The main question for the current paper pertains to the robustness of the optimal R&D policy results in a dynamic setting. Moving to a dynamic setting allows for several key features unavailable in the static context. First, and foremost, the analysis above by necessity needs to take the initial state, $s = [w_1, w_2]$, as exogenously given. By embedding the two-stage static game in a dynamic context, the equilibrium ergodic distribution of the states becomes endogenous. Thus, the policy choices will influence the distribution itself and, in turn, feedback into the optimal policy.[6]

Second, and crucially, the dynamic setting with forward looking firms means that firms will anticipate changes in the relative efficiency levels and their impact on firm values when making R&D policy decisions. When we analyze subsidies conditional on relative firm size, firms will account for possible gain/loss of subsidies or a switch to taxes in choosing R&D levels. For example, suppose that only the large firm is given a subsidy for R&D. In the static setting, since the game ends with product market sales even if the smaller firm becomes the larger firm after the R&D uncertainty is resolved, the change in relative size has no bearing on the R&D decisions. In the dynamic setting that is no longer the case as firms will anticipate potential changes to subsidies altering their incentives to engage in R&D.[7]

3.1. Dynamic Setting

In each period, firms choose R&D expenditures to maximize the expected present discounted value of their future stream of profits. The state of the industry is summarized by the vector of the efficiency levels of the firms, $s = [w_i, w_j]$. Denote $pr(s' \mid s)$ as a firms' perceptions of the probability that the industry state it faces will be s', conditional on the current industry state. The optimal R&D choice solves the following Bellman equation subject to a non-negativity constraint on R&D:

$$V_i(s) = \max_{x_i \geq 0} \{\pi_i(s) - (1 - s_i)cx_i + \beta E[V_i(s')]\} \tag{3}$$

where $\beta = 1/(1 + r)$, with r, standing for the interest rate, is the discount factor common to all firms (and consumers), and $\pi_i(s)$ denotes the profits of firm i in industry state s. c represents the unit cost of R&D. s_i again represents the percentage subsidy (tax) applied to firm i.

The first-order condition is as follows:

$$\frac{\partial V_i}{\partial x_i} = -\frac{(1 - s_i)c_x}{\beta} + \frac{\bar{w}}{(1 + \bar{w}x_i)^2}[C_S - C_F] \leq 0 \tag{4}$$

where, in the dynamic context, C_S represents the value of the firm conditional on successful innovation and C_F is the value conditional on failure to innovate.

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Rearranging, the optimal R&D level can be expressed as follows:

$$x_i^* = \left[\frac{\beta}{\bar{w}(1-s_i)c}(C_S - C_F)\right]^{1/2} - \frac{1}{\bar{w}}. \quad (5)$$

Similar to the two-stage game, R&D is positively related to the gains in value from innovation, $C_S - C_F$, the level of the subsidy, s_i, the discount factor, β, and negatively related to the cost of R&D, c.

As in the static environment, $1/\bar{w}$ is a constant and creates regions of corner solutions for the optimal choice of x_i, coasting states. These will apply whenever the expected marginal gain from innovating, $C_S - C_F$, is small. A small marginal gain occurs when one firm has high marginal costs relative to the other firm, but may apply to one or both firms. An inefficient firm may find that successful R&D produces little gain on its competitor and shuts down R&D conceding the market to its larger, more efficient rival. In addition, the more efficient firm may be a monopoly or near monopoly and thus find little gain from innovating and optimally choose to cease its R&D operations.

The evolution of the stock of innovations includes a negative shock which can be interpreted as obsolescence of innovations or exogenously increasing, though unmodeled, labor costs which increases all firms' marginal costs as suggested in Pakes and McGuire (1994). The negative shock helps ensure the state space is bounded. In each period, there is a random negative shock, Δ, to the stock of innovations for both firms, which occurs with probability δ. Thus, the stock of innovations for firm i evolves according to

$$w_i' = w_i + \nu_i - \Delta \quad (6)$$

where the new stock of innovations is the sum of the previous innovation level, a positive increment, ν_i, if firm i successfully reduces costs through R&D, and the outcome of the negative shock, Δ, which takes the value one if the shock occurs and zero otherwise.

The equilibrium here is Markov Perfect. The model satisfies the Unique Investment Choice criteria as described in Doraszelski and Satterthwaite (2010), so that an equilibrium exists. However, as is well known in this class of models, proving uniqueness remains elusive. The equilibrium is also a rational expectations equilibrium. The solution yields policy choices such that the probabilities of evolving from one state to another match the firms' perceptions of those probabilities.[8]

3.2. Computation

To analyze the dynamic model, we largely retain the same parameters as in Section 2. We do need to add a discount rate, β, which we assign as $\beta = 1/1.04$. In order to ensure that the equilibrium ergodic distribution lies within the state space, we initially set the maximum number of innovations to 100 for both firms and check *ex post* that the upper bounds do not have positive probability in the

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equilibrium ergodic distribution. We also need to reduce the demand intercept, set at $A = 30$ in Section 2, down to $A = 12$. The reason is that the firms' now look at the long-run value which greatly exceeds the one-period profits. The reduction from 30 to 12 approximately sets firm values in the dynamic model to the same level as the profits in the static model. Thus, firms are willing to conduct R&D with a much smaller demand. In fact, in the numerical example in the static setting when $A = 12$ firms optimally choose $x = 0$. However, in the dynamic setting with a large demand ($A = 30$) firms conduct significantly more R&D resulting in a high frequency of exceeding the state space, even as large as it is.

The solution method first computes the static profits for all possible states. The profits for each possible state are used in value function iteration which yields the equilibrium values and policy functions.[9] To compute the ergodic distributions under each R&D policy scenario, we construct the transition matrix from the optimal policy functions, denoted T. The equilibrium asymptotic ergodic distribution, T_e, can then be found as follows:

$$T_e = \lim_{t \to \infty} T^t. \tag{7}$$

We repeatedly square T until it converges, yielding the probability of being in each state. Our error tolerance for the iterations such that the probability of a state occurring was within 0.000001 margin of approximation error.

To find the optimal policy, we consider a range of subsidies (taxes) for each firm from +90% (subsidy) to −100% (tax) in steps of 10 percentage points which yields 400 possible combinations of subsidies and taxes for two firms. As in the previous section, the subsidies are conditional on relative firm size. In the event that the firms are of the same size, we use the policy for the larger firm. For every combination of subsidies and taxes, we evaluate total welfare and find the optimal policy via the maximum welfare value. Welfare in the dynamic context differs from the static model in several ways. First, since the firms are infinitely lived, we use values instead of one-period profits for producer surplus. Second, consumer welfare is the expected long-run value rather than the one period difference between the demand curve and the price. Thus, we use the probabilities for each possible price from the ergodic distribution to compute the weighted expected consumer surplus and find the long-run value, using the same discount factor, β, for consumers as applies to firms. Taxes and subsidies are computed in the same manner and are subtracted (added) to consumer surplus in each state prior to calculating the long-run expected value.

4. DYNAMIC MODEL RESULTS

4.1. Equilibrium Ergodic Distributions of States

Before looking at how taxes and subsidies affect welfare, we examine the ergodic distributions of industry states that emerge without any government intervention. There are four types of distributions that arise under varying parameterizations which we label: (1) *symmetric innovation*; (2) *bimodal innovation*; (3) *asymmetric*

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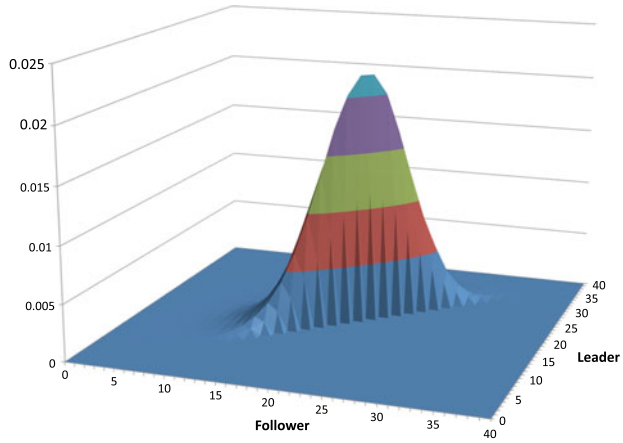


FIGURE 2. Symmetric innovation distribution: probability of occurrence for each state space combination. The efficiency level for the leader firm is on the x-axis while the efficiency level for the follower firm is on the y-axis.

innovation; and (4) *low innovation*. In general, welfare declines, respectively, with the types. In the symmetric innovation distribution, the innovation and product markets are intensely competitive with both firms heavily engaged in R&D. Figure 2 displays the equilibrium ergodic distribution where the efficiency level of the larger firm is on the x-axis and the efficiency level of the smaller firm is on the y-axis. The z-axis shows the probability of each industry state. Under symmetric innovation, both firms are efficient as indicated by the large peak in the center of the graph. These types of distributions typically exhibit low values of the C1 index ranging between 0.52 and 0.56, depending on the parameters, indicating that on average the larger firm holds between 2 and 6 percentage points more than half the market.[10] Moreover, the identity of the larger firm (firm 1 or firm 2) changes frequently.

As parameter changes weaken the market (e.g., lower demand or lower R&D productivity), the equilibrium shifts to a distribution as shown in Figure 3 which reveals a bimodal ergodic distribution. The symmetric innovation region remains but with lower probability. The new mode to the left is characterized by a near monopoly with the larger firm dominating the market and conducting significantly more R&D than the smaller firm. In essence, the larger firm conducts the R&D to maintain its large cost advantage over the small firm which is discouraged, but not entirely absent, from participating in R&D. As market conditions worsen, Figure 4 shows the asymmetric innovation distribution where the central mode disappears and the near monopoly mode that arose in Figure 3 now dominates.

The final distribution type in Figure 5 shows the low innovation distribution. Here the product market is nearly always monopolized by the larger firm, although the identity of the larger firm does change over time. The efficiency levels for both firms are always close to zero indicating that little innovation occurs. If the market conditions worsen further, the distribution collapses to an industry with no R&D, i.e., industry state $s = [0, 0]$ becomes an absorbing state.

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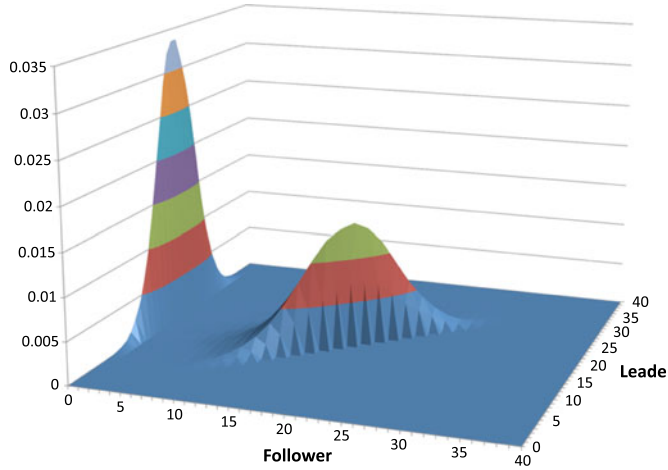


FIGURE 3. Bimodal innovation ergodic distribution: probability of occurrence for each state space combination. The efficiency level for the leader firm is on the *x*-axis, while the efficiency level for the follower firm is on the *y*-axis.

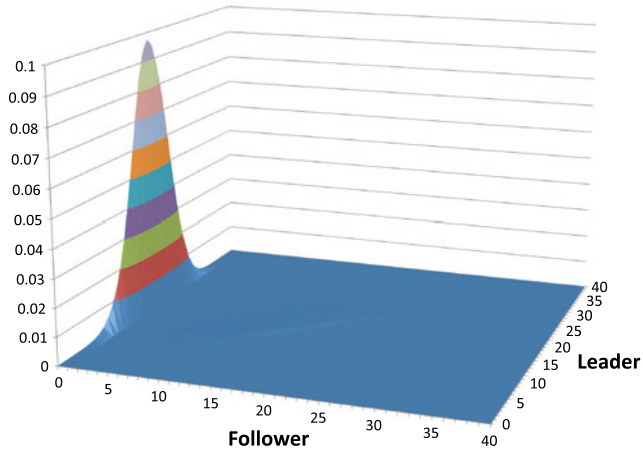


FIGURE 4. Asymmetric innovation ergodic distribution: probability of occurrence for each state space combination. The efficiency level for the leader firm is on the *x*-axis, while the efficiency level for the follower firm is on the *y*-axis.

4.2. Optimal Policies

The optimal policy achieves the highest welfare through modifying the innovation incentives at the lowest social cost in terms of the taxes levied on consumers and investment costs borne by the firms. Should the initial equilibrium be characterized by symmetric innovation as in Figure 1, the optimal policy is typically a *tax* on both firms as the marginal welfare gains from additional innovation are

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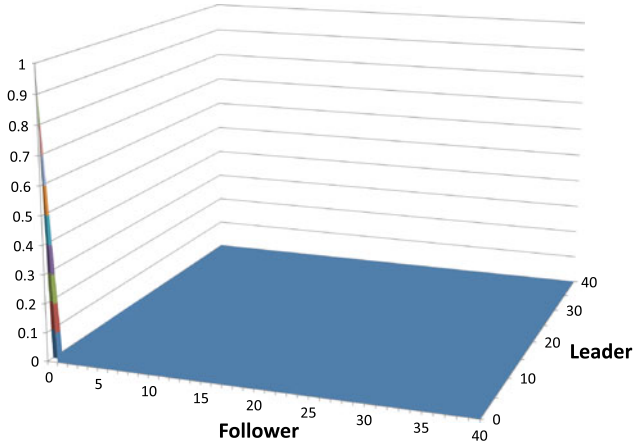


FIGURE 5. Low innovation ergodic distribution: probability of occurrence for each state space combination. The efficiency level for the leader firm is on the *x*-axis, while the efficiency level for the follower firm is on the *y*-axis.

outweighed by the losses through business stealing effects. In contrast, in the three other distribution types, the optimal policy involves subsidies to at least one firm and, in some cases, both.

To illustrate how the optimal policies depend on the ergodic distribution, we modify a single parameter, $\bar{w}$, which governs the productivity of R&D, while holding the other parameters fixed. By varying $\bar{w}$ over a range from 0.125 to 0.4, the model generates the different equilibrium distribution types described above. To get a sense of how much of a difference this range of $\bar{w}$ makes, consider the difference in R&D expenditures required to attain a probability of 0.5 of success. Rearranging the following equation and using $Pr = 0.5$, we have

$$x = \left(\frac{\Pr}{1-\Pr}\right)\frac{1}{\bar{w}} = \frac{1}{\bar{w}}. \tag{8}$$

Thus, at the high end, $\bar{w} = 0.4$ it requires 2.5 units of R&D expenditure to achieve a 50% probability of success. At the low end, $\bar{w} = 0.125$, the requirement more than triples to 8.0 units of R&D expenditure.

Four values for $\bar{w}$ that generate distribution types as described above are 0.125, 0.20, 0.275, and 0.35. The corresponding C1 indices are as follows: 0.5, 0.84, 0.57, and 0.52. At low values of $\bar{w}$, firms find it too costly to engage in R&D. As $\bar{w}$ increases, the equilibrium distribution proceeds through the types described above from low to asymmetric to bimodal to symmetric. The initial C1 value of 0.5 reflects a duopoly under which no innovation takes place and the distribution is degenerate. Once there is sufficient incentive to conduct at least a little R&D, the market moves from symmetric duopoly to monopoly. With further increases in $\bar{w}$, the equilibrium distribution moves from the asymmetric to the bimodal distribution and then to the symmetric distribution at high values of $\bar{w}$.

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TABLE 5. Results by environment

	No subsidy		Subsidy	
	Large	Small	Large	Small
$\bar{w} = 0.125$	**Low**		**Asymmetric**	
Subsidy	0	0	20%	−100%
R&D outlays	0	0	19.15	0.02
Welfare	565.2		904.1	
C1 index	0.50		0.82	
$\bar{w} = 0.200$	**Asymmetric**		**Bimodal**	
Subsidy	0	0	50%	40%
R&D outlays	12.06	0.04	11.54	11.51
Welfare	1088.9		1144.0	
C1 index	0.84		0.55	
$\bar{w} = 0.275$	**Bimodal**		**Symmetric**	
Subsidy	0	0	20%	10%
R&D outlays	8.19	8.11	8.28	9.23
Welfare	1295.4		1312.8	
C1 index	0.57		0.50	
$\bar{w} = 0.350$	**Symmetric**		**Symmetric**	
Subsidy	0	0	−10%	−20%
R&D outlays	6.33	7.48	6.48	7.12
Welfare	1392.0		1409.3	
C1 index	0.51		0.52	

Table 5 summarizes the results by initial distribution. For each initial environment without innovation, the table provides the optimal subsidy/tax depending on the firm's position. R&D outlays and the C1 index (largest firms' market share) are calculated by weighting the relevant values by the probability of each industry state in equilibrium. Welfare is calculated by first weighting firm values and consumer surplus by the probability of each industry state in equilibrium, applying the discount factor, β, to consumer surplus to find the long-run value and then adding it to the firm values. The optimal policy is the combination of taxes and subsidies that produces the highest level of welfare.

Figure 6 shows how the optimal R&D levels of the two firms change with varying levels of $\bar{w}$ both without any policy intervention and under the welfare maximizing policy. At low levels of R&D productivity $\bar{w}$, neither firm spends any resources on R&D. However with strong subsidies for the large firm (40%–50%) and high taxes on the small firm (90%–100%), the lead firm engages heavily in R&D which shifts the distribution to the asymmetric type. As the R&D

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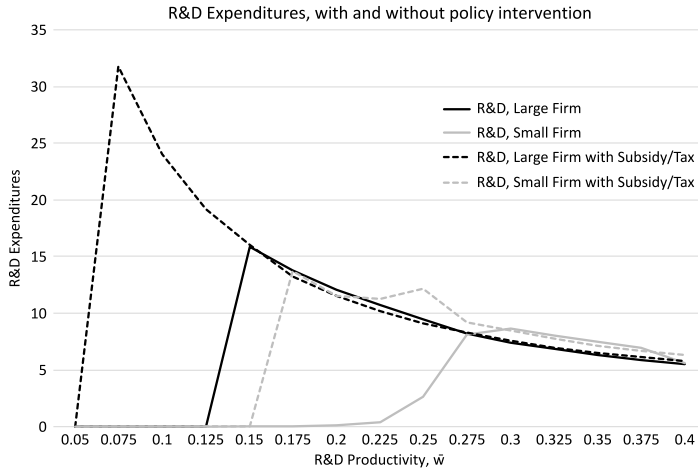


FIGURE 6. R&D expenditures.

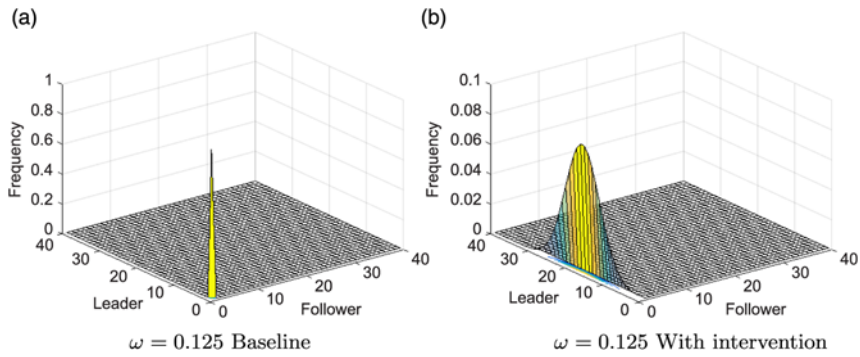


FIGURE 7. Low innovation policy: 20% subsidy to leader and 100% tax on follower.

productivity parameter increases, the amount of R&D necessary to induce the firm's desired success probability declines. Once the R&D productivity parameter is sufficiently large, $\bar{w} = 0.15$, at least one firm engages in R&D without any subsidies. The small firm also engages in a very small amount of R&D, but only when the larger firm's efficiency falls to near zero and the smaller firm has the opportunity to overtake the larger firm. At $\bar{w} = 0.175$, the optimal policy changes to subsidizing both firms leading to high values of R&D for both firms with slightly higher R&D for the smaller firm on average. At even higher levels of $\bar{w}$, the baseline distribution moves to the fully symmetric and an optimal policy of taxing both firms follows.

When $\bar{w} = 0.125$, we have the low innovation environment. Figure 7 shows the change in ergodic distribution under the optimal policy: the left-hand panel shows the (degenerate) ergodic distribution of industry states with no policy intervention; the right-hand side panel shows the distribution with the welfare maximizing

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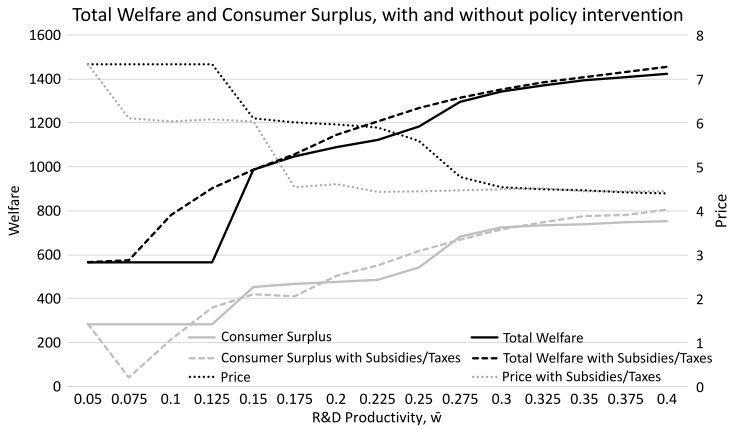


FIGURE 8. Total welfare, consumer surplus, and price.

R&D subsidy in place. The optimal policy subsidizes 20% of the large firm's R&D expenditures and taxes the small firm's expenditures at 100%. The larger firm's value rises by 265% and consumer surplus also increases by 26% while the smaller firm's value falls by 78%. The policy provides strong incentives to innovate because the welfare gains are large, even though it creates a monopoly. The tax on the follower (smaller) firm helps ensure a profitable position and reward for successfully innovating. There remains a positive probability that through a series of negative shocks and failures to innovate, the market returns to duopoly when both firms engage in R&D and vie to become the monopoly. Thus, the long-run market share of the lead firm is not 100% but 82%.

Figure 8 shows total welfare and consumer surplus over the range of $\bar{w}$ both with and without the policy intervention, while Figure 9 shows the two firms' values and total firm value. In addition, Figure 8 also displays the ergodic distribution weighted price on the secondary y-axis. Policy intervention makes the largest welfare difference at low levels and intermediate levels. Those correspond to shifts from the low innovation distribution policy to the asymmetric distribution and then from the asymmetric distribution to the bimodal or symmetric. They are also the two cases where the price drops the most under the optimal policy explaining the increase in consumer surplus. When the policy can induce a significant shift in the type of competition, the gains are at their largest. Also, noticeable in Figures 8 and 9 is that the welfare gains at low levels of $\bar{w}$ come from an increase in the large firm value at the expense of consumer surplus. Intuitively that makes sense since the market went from a duopoly with little or no R&D to a monopoly with R&D, i.e. the asymmetric distribution. In contrast, in the middle range (around $\bar{w} = 0.2$) where policy raises welfare, both consumer surplus and the value of the smaller firm increase at the expense of the larger firm. Counterintuitively, the policy in these cases typically has a slightly larger subsidy for the larger firm than the smaller firm, but it is the smaller firm that benefits, not the larger firm.[11] That is

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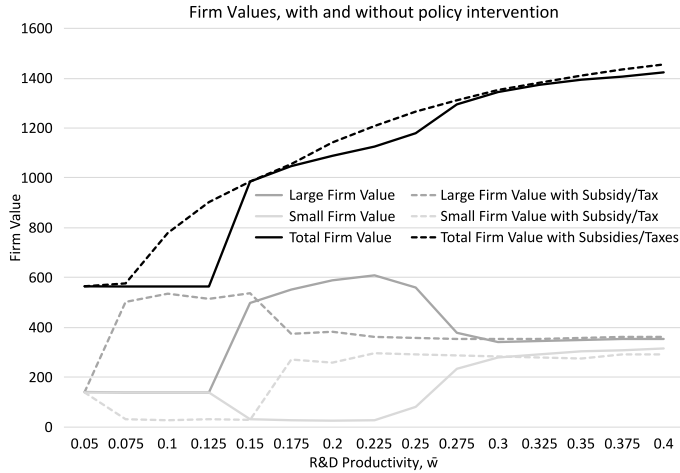


FIGURE 9. Firm values.

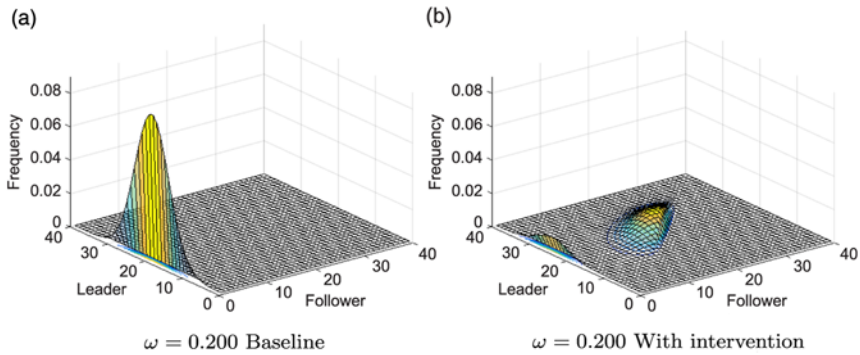


FIGURE 10. Asymmetric innovation policy: 0% subsidy to large and 20% to small.

not the case in static models where the main beneficiary corresponds directly to the firm receiving the larger subsidy.

When the initial environment is asymmetric (Figure 10), the policymaker can shift the ergodic distribution to the bimodal distribution by subsidizing both firms but with a slightly bigger subsidy to the larger firm, providing an additional incentive to become the large firm. The two firms conduct nearly the same amount of R&D on average despite the subsidy difference. The welfare gains are much more modest at 5% as compared to the previous scenario. The product market becomes significantly more competitive as the C1 index falls from 0.84 to 0.55. Further reductions in the C1 index are possible with higher subsidies but result in lower welfare due to the larger tax requirement.

Starting from the bimodal innovation distribution (Figure 11), the symmetric environment becomes attainable and generates a welfare improvement. Again,

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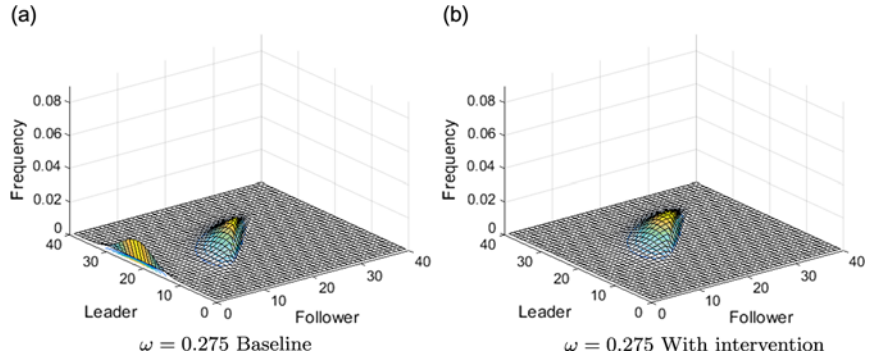


FIGURE 11. Bimodal innovation policy: 20% subsidy to large and 10% subsidy to small.

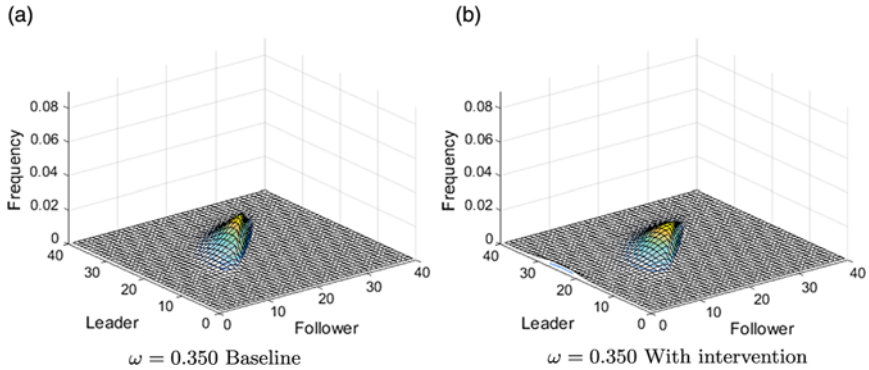


FIGURE 12. Symmetric innovation policy: 20% tax on both firms.

both firms are subsidized here. The welfare gain is comparatively small, at about 1%, and the C1 index falls from 0.57 to 0.50 reflecting the symmetric duopoly. When the distribution is initially symmetric (Figure 12), taxes become the optimal policy. For our particular parameterized case, the best policy is a 10% tax on the larger firm and a 20% tax on the other firm. Because of the intense competition, the marginal gains from further innovation are small and thus the R&D expenditures become socially wasteful. Note that the R&D outlays hardly change with the taxes applied. In fact, the average R&D for the larger firm rises, despite the tax, while that of the smaller firm falls.

The results indicate that the static policy recommendations are not robust to a dynamic environment. First, even though there are no spillovers, a zero subsidy policy is not optimal. The two environments closest to the symmetric static case are the no innovation case (low $\bar{w}$) and the symmetric innovation case at the other end with high $\bar{w}$. In the former case, the optimal policy calls for subsidies for the large firm and taxes on the other firm. In the latter case, the policy calls for taxes on both firms. Second, the optimal policy depends on the equilibrium distribution that emerges without intervention. That insight cannot be found in static models.

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TABLE 6. Parameter variation used for robustness

Demand:		Low	Asymmetric	Bimodal	Symmetric
Intercept	A	8.0	10.0	12.0	14.0
Slope	B	0.5	1.0	1.5	2.0
R&D related:		Low	Asymmetric	Bimodal	Symmetric
R&D unit cost	c	0.45	0.35	0.25	0.15
Neg. shock rate	δ	0.80	0.75	0.70	0.65
Rate of marg. cost decline	η	0.04	0.08	0.12	0.16

Third, the welfare implications differ. The larger increase in firm value does not necessarily correspond to the more favorable subsidy policy. Furthermore, which agents (large firm, small firm, or consumers) benefit from policy also depends on the initial distribution type as illustrated by Figures 7 and 8.

4.3. Robustness of Results

The results presented above are indicative of how the optimal policy changes in a dynamic setting. They were obtained by varying a single parameter, $\bar{w}$. To investigate the robustness and demonstrate more fully how the optimal policy is linked to the initial distribution type, we vary the other key parameters in the model. Specifically, we vary the demand parameters A and B and the R&D parameters c, δ, and η. Table 6 contains the parameter values used for the robustness check. Table 7 shows the numerical results and Figures 13–21 present the equilibrium distributions while the parallel sets of Figures 14–22 show how the welfare components and R&D expenditures change under the optimal policy. $\bar{w}$ is set to 0.25 at the middle of the range used above.

Demand intercept, A: Starting with the demand intercept, A, the pattern of the optimal policies and resulting distributions (see Figure 13) mirrors the initial case when varying $\bar{w}$. The optimal policy shifts the distribution toward higher levels of innovation by incentivizing the firms. The graphs in Figure 14 show the changes in the welfare components and optimal R&D choices. At the lowest level where $A = 8$, the optimal policy calls for favoring the large firm. However, as demand increases to $A = 11$ and the baseline distribution becomes asymmetric, the optimal policy becomes one of subsidizing both firms at 30% which shifts the distribution to bimodal. Slightly higher demand at $A = 12$ generates a bimodal distribution with the optimal policy calling for subsidies on both firms, while slightly favoring the larger of the two. The result is the symmetric distribution with high levels of R&D by both firms. The final panel in Figure 13 shows that with high demand at $A = 14$ the symmetric distribution occurs without intervention while the welfare maximizing policy is to tax both firms. Under the optimal policy a small

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TABLE 7. Robustness results for A, B, C, δ, and η

		Base	Policy	Base	Policy	Base	Policy	Base	Policy
		$A = 8.0$		$A = 10.0$		$A = 12.0$		$A = 14.0$	
		Low	**Asym.**	**Asym.**	**Asym.**	**Bimod.**	**Sym.**	**Sym.**	**Sym.**
Subsidy	Lg.	0	0.8	0	−0.2	0	0.3	0	−0.1
	Sm.	0	−1	0	−1	0	0.2	0	−0.2
R&D	Lg.	0.00	9.69	9.65	9.62	9.48	9.13	8.86	9.06
	Sm.	0.00	0.00	0.01	0.00	2.65	10.15	10.47	10.13
Welfare		103.6	372.1	746.4	747.4	1181.1	1266.4	1881.6	1913.5
C1 index		0.50	1.00	0.94	0.92	0.77	0.52	0.52	0.53
		$B = 2.00$		$B = 1.50$		$B = 1.00$		$B = 0.50$	
		Low	**Asym.**	**Asym.**	**Asym.**	**Bimod.**	**Sym.**	**Sym.**	**Bimod.**
Subsidy	Lg.	0	0.2	0	0.3	0	0.3	0	−0.5
	Sm.	0	−1	0	0.2	0	0.2	0	−0.7
R&D	Lg.	0.00	9.58	7.51	7.35	9.48	9.13	8.84	9.14
	Sm.	0.00	0.01	2.08	8.08	2.65	10.15	10.52	9.95
Welfare		282.1	450.9	766.6	805.5	1181.1	1266.4	2946.6	3036.3
C1 index		0.50	0.82	0.77	0.52	0.77	0.52	0.51	0.53
		$C = 0.45$		$C = 0.35$		$C = 0.25$		$C = 0.15$	
		Low	**Sym.**	**Asym.**	**Sym.**	**Bmod.**	**Sym.**	**Sym.**	**Bimod.**
Subsidy	Lg.	0	0.9	0	0.6	0	0.3	0	−0.7
	Sm.	0	0.9	0	0.5	0	0.2	0	−0.5
R&D	Lg.	0.00	8.81	9.64	9.29	9.48	9.13	8.85	8.74
	Sm.	0.00	10.58	0.03	10.04	2.65	10.15	10.50	10.02
Welfare		565.2	1138.1	1053.0	1161.7	1181.1	1266.4	1433.0	1554.2
C1 index		0.50	0.50	0.83	0.52	0.77	0.52	0.51	0.55
		$\delta = 0.80$		$\delta = 0.75$		$\delta = 0.70$		$\delta = 0.65$	
		Low	**Asym.**	**Asym.**	**Sym.**	**Bimod.**	**Sym.**	**Sym.**	**Bimod.**
Subsidy	Lg.	0	0.9	0	0.5	0	0.3	0	0
	Sm.	0	−1	0	0.4	0	0.2	0	−0.1
R&D	Lg.	0.00	16.67	12.39	11.87	9.48	9.13	7.03	7.20
	Sm.	0.00	0.00	0.08	12.89	2.65	10.15	8.43	8.06
Welfare		565.2	932.6	1052.5	1095.0	1181.1	1266.4	1378.5	1393.4
C1 index		0.50	0.89	0.83	0.53	0.77	0.52	0.52	0.52
		$\eta = 0.05$		$\eta = 0.08$		$\eta = 0.12$		$\eta = 0.16$	
		Low	**Sym.**	**Asym.**	**Bimod.**	**Bimod.**	**Bimod.**	**Sym.**	**Bimod.**
Subsidy	Lg.	0	0.9	0	0.4	0	0.1	0	−0.3
	Sm.	0	0.9	0	0.3	0	0	0	−0.3
R&D	Lg.	0.00	8.88	9.60	9.21	8.90	9.10	8.84	9.00
	Sm.	0.00	10.18	0.05	9.78	10.26	9.93	10.77	9.61
Welfare		565.2	1152.8	1107.5	1212.8	1318.7	1322.3	1369.7	1420.5
C1 index		0.50	0.50	0.70	0.53	0.53	0.54	0.52	0.56

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asymmetric mode emerges. The increase in consumer surplus derives from a tax rebate and largely comes at the expense of the smaller firm.

Demand slope, B: For the demand slope, B, we find the pattern is largely the same (see Figures 15 and 16). As B declines the slope becomes less steep implying larger demand and thus the firms increase R&D. At $B = 2.0$, the distribution is the low type concentrated on the origin with zero R&D in this case. The optimal policy is to subsidize the larger firm at 20% and heavily tax the smaller firm at 100%. Doing so generates the asymmetric distribution where one firm becomes a monopoly but with significantly lower costs than without intervention. Welfare rises substantially by 56%, but entirely through a 175% increase in joint firm value while consumer surplus declines by 63%. In the asymmetric case at $B = 1.5$, the optimal policy calls for a 10% subsidy but only to the smaller firm while the distribution type does not change given the relatively low level of the intervention. The policy increases the R&D of the small firm and its value while having the opposite effect on the larger firm. The bimodal distribution in the third row of Figure 15 shows that the welfare maximizing policy of subsidizing the firms at the rates of 30% and 20%, respectively, moves the ergodic distribution type to the symmetric mode. In the case of the initial symmetric mode at $B = 0.5$, heavy taxes on both firms listed in Table 6 maximize welfare.

Unit costs of R&D, c: Changing the unit costs of R&D is akin to looking at the baseline where our unit costs were set at 0.25 and then imposing symmetric taxes or subsidies (see Figures 17 and 18). High enough unit costs stop R&D altogether at around $c = 0.45$. Now, the optimal policy is to heavily subsidize both firms at 90% which brings the effective cost down to 0.045, low enough to encourage high levels of R&D by both firms and generate a symmetric distribution. Thus, it should be no surprise that if the optimal policy can move the distribution to the symmetric mode when costs are quite high, the same happens again at lower values of c. For $c = 0.35$, the ergodic distribution moves from asymmetric to symmetric with large but slightly unequal subsides to both firms. Lower subsidies create the same at $c = 0.25$, which is the equivalent of the baseline used for $\bar{w} = 0.25$. When the unit R&D costs are particularly low at $c = 0.15$ and the initial distribution is symmetric, the optimal policy is again to tax both firms quite heavily.

Negative shock rate, δ: The parameter δ governs the negative shock to the stock of innovations. An increase in that parameter makes it harder for a firm to advance its R&D stock and keep production costs low. An increase to $\delta = 0.8$ from the baseline value of 0.7 creates a degenerate distribution with no R&D by either firm (see Figures 19 and 20). The optimal policy hits the extremes at both ends. Tax the small firm 100% and subsidize the large firm at the rate of 90%. Note that the C1 index moves to 0.89, suggesting that there is still some turbulence. The high level of δ means that there is an 80% chance the large firm's stock of innovations will fail. Once it falls sufficiently far, the firms again split the market in that state: both

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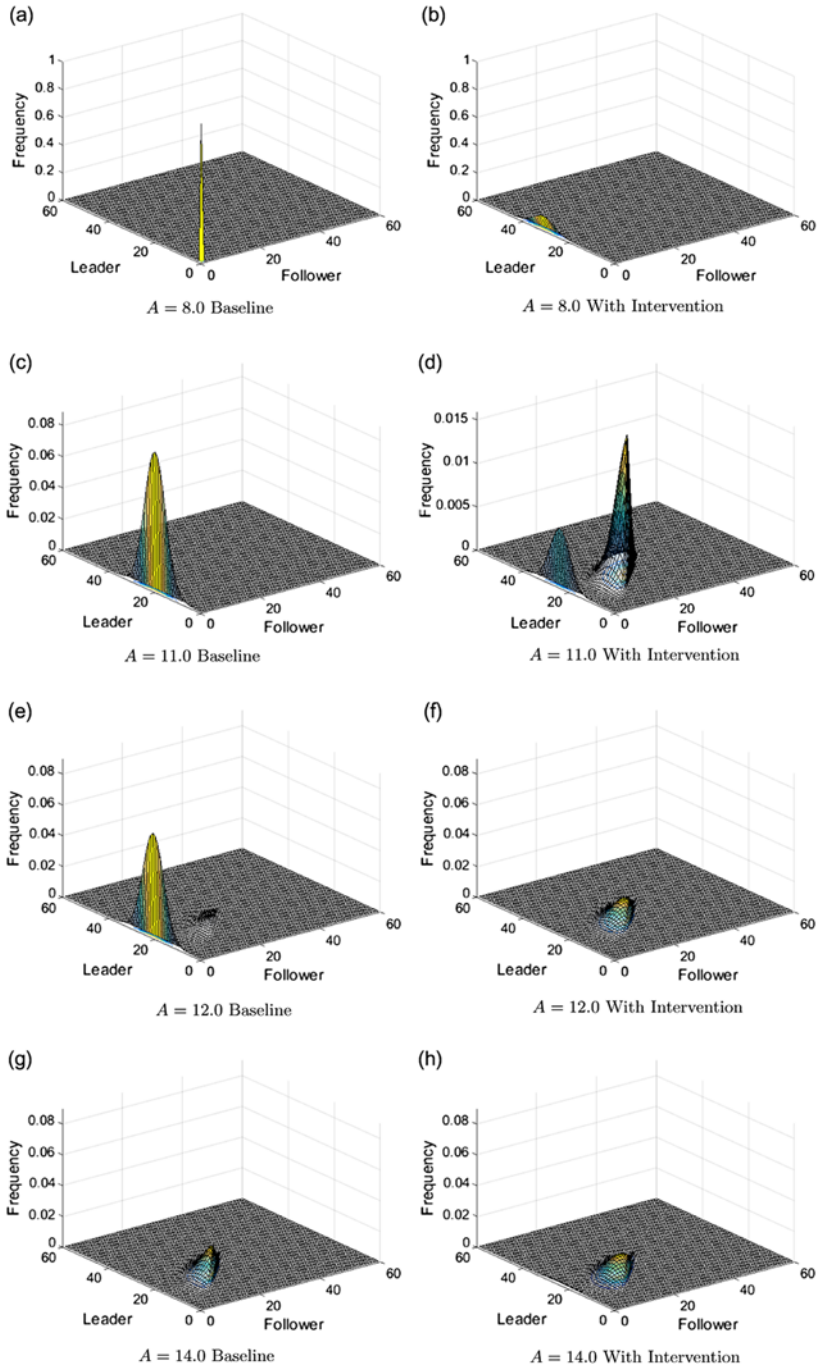


FIGURE 13. Variation in demand intercept, A.

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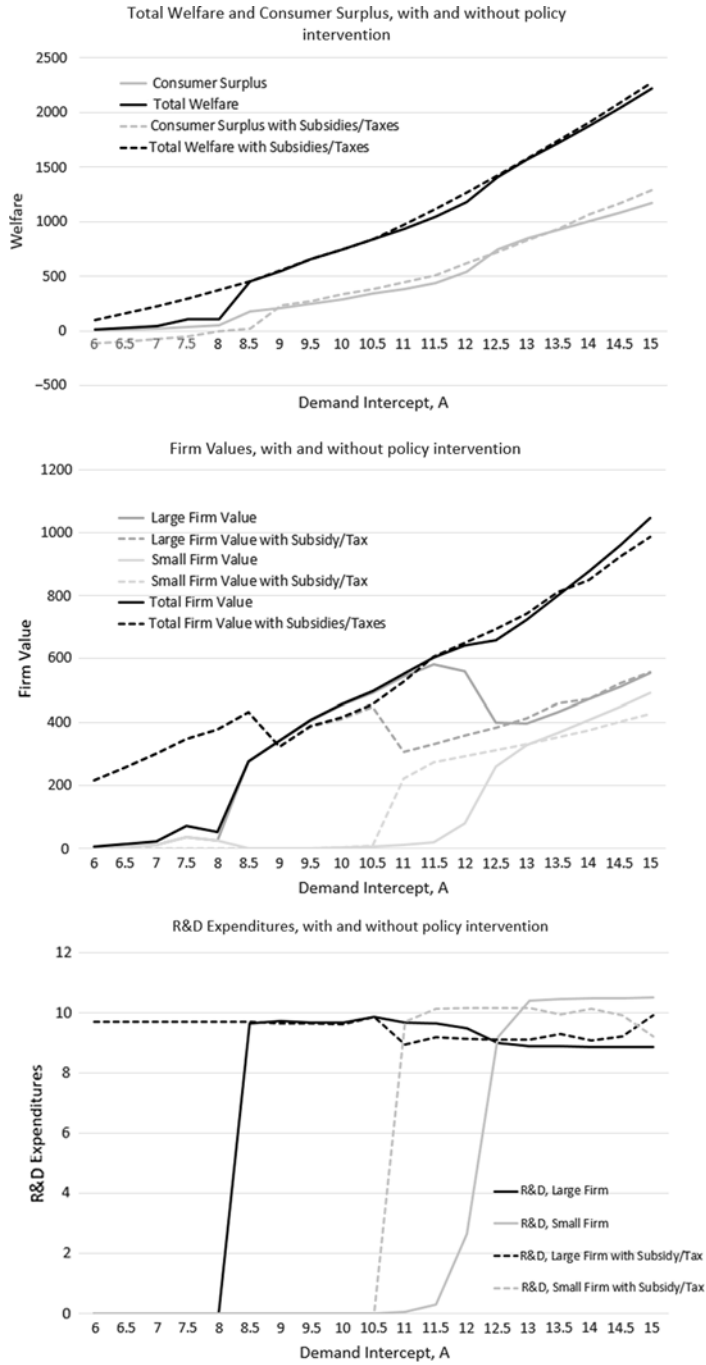


FIGURE 14. Variation in demand intercept, A: optimal policy and welfare vs. A.

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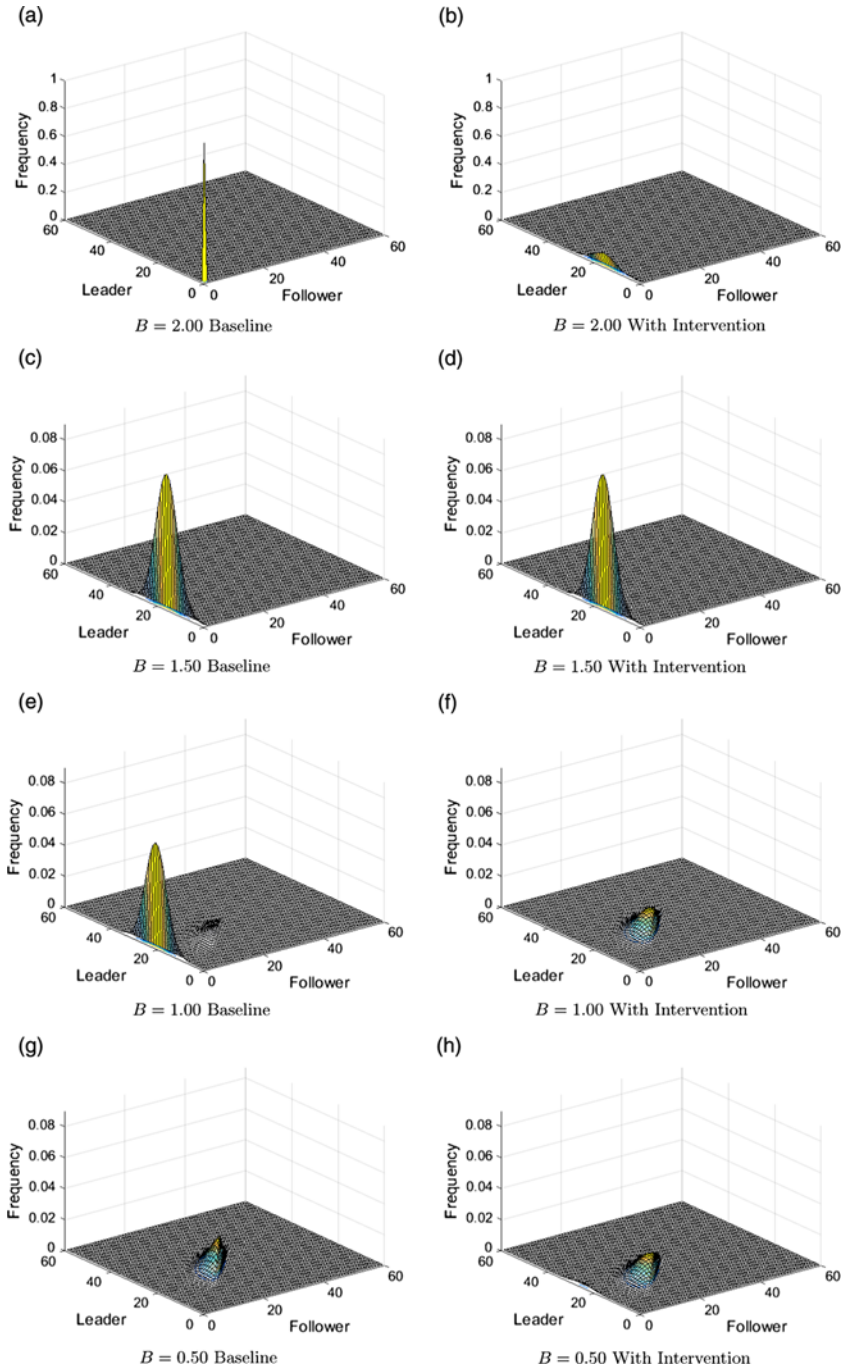


FIGURE 15. Variation in demand slope, B.

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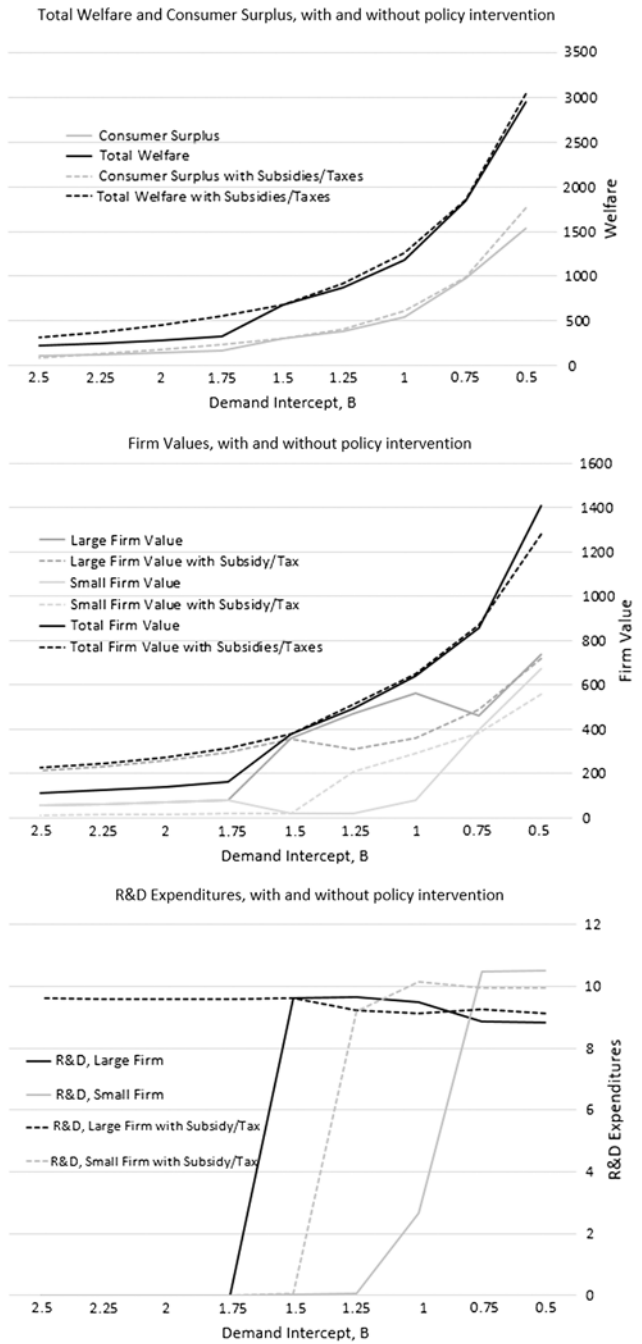


FIGURE 16. Variation in demand slope, *B*: optimal policy and welfare vs. *B*.

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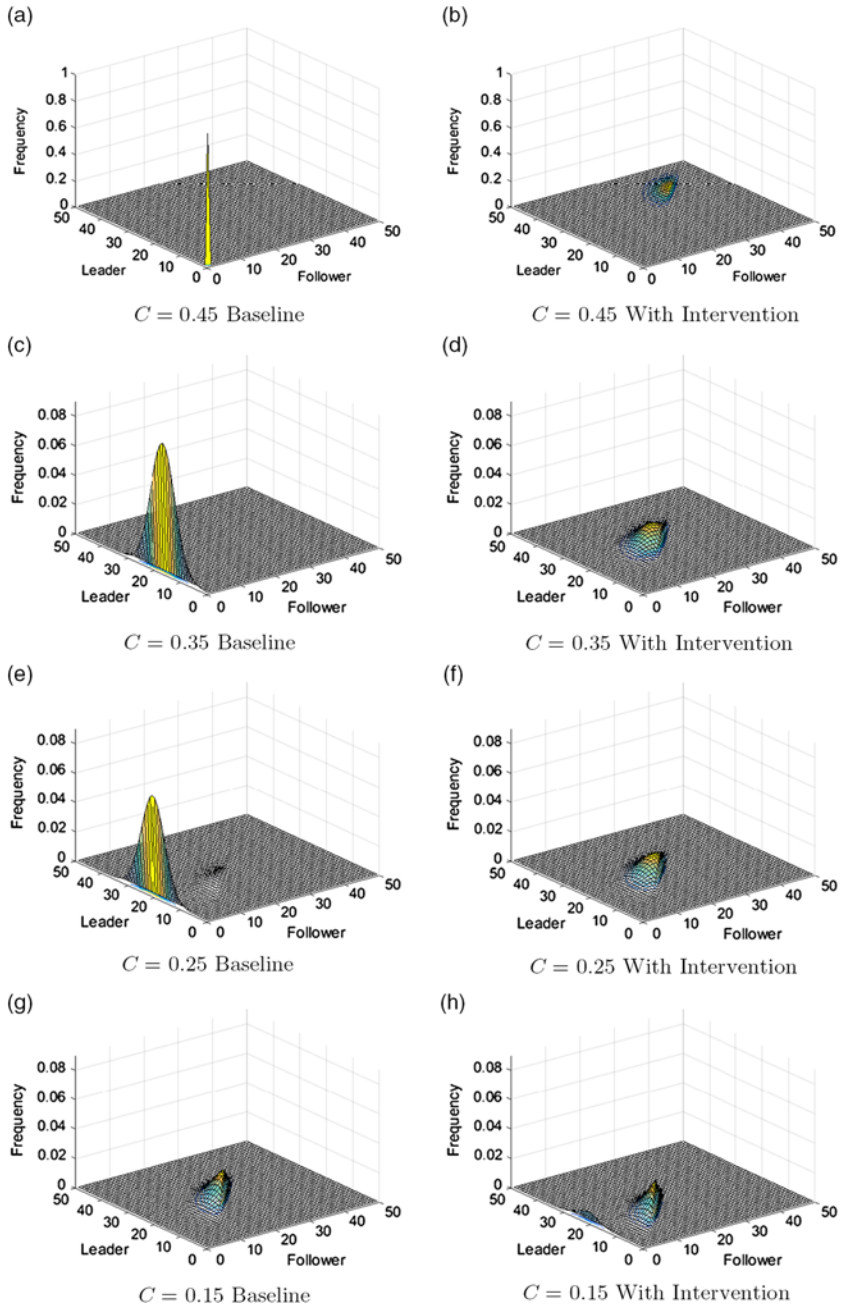


FIGURE 17. Variation in R&D unit cost, c.

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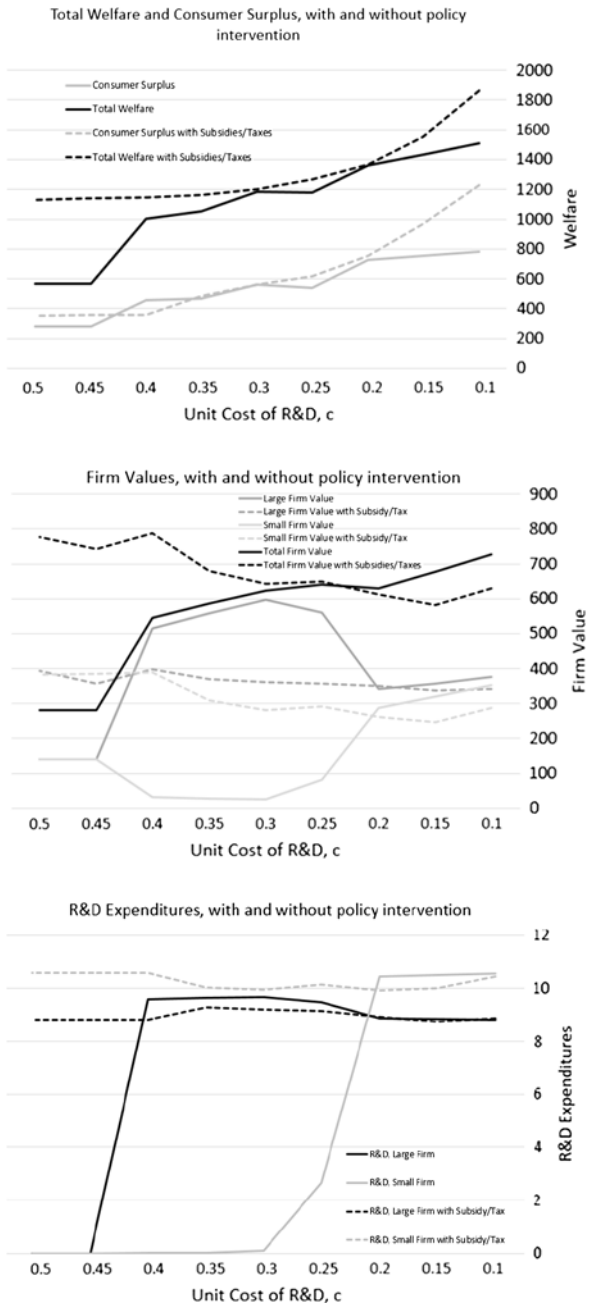


FIGURE 18. Variation in R&D unit cost, *c*: optimal policy and welfare vs. *c*.

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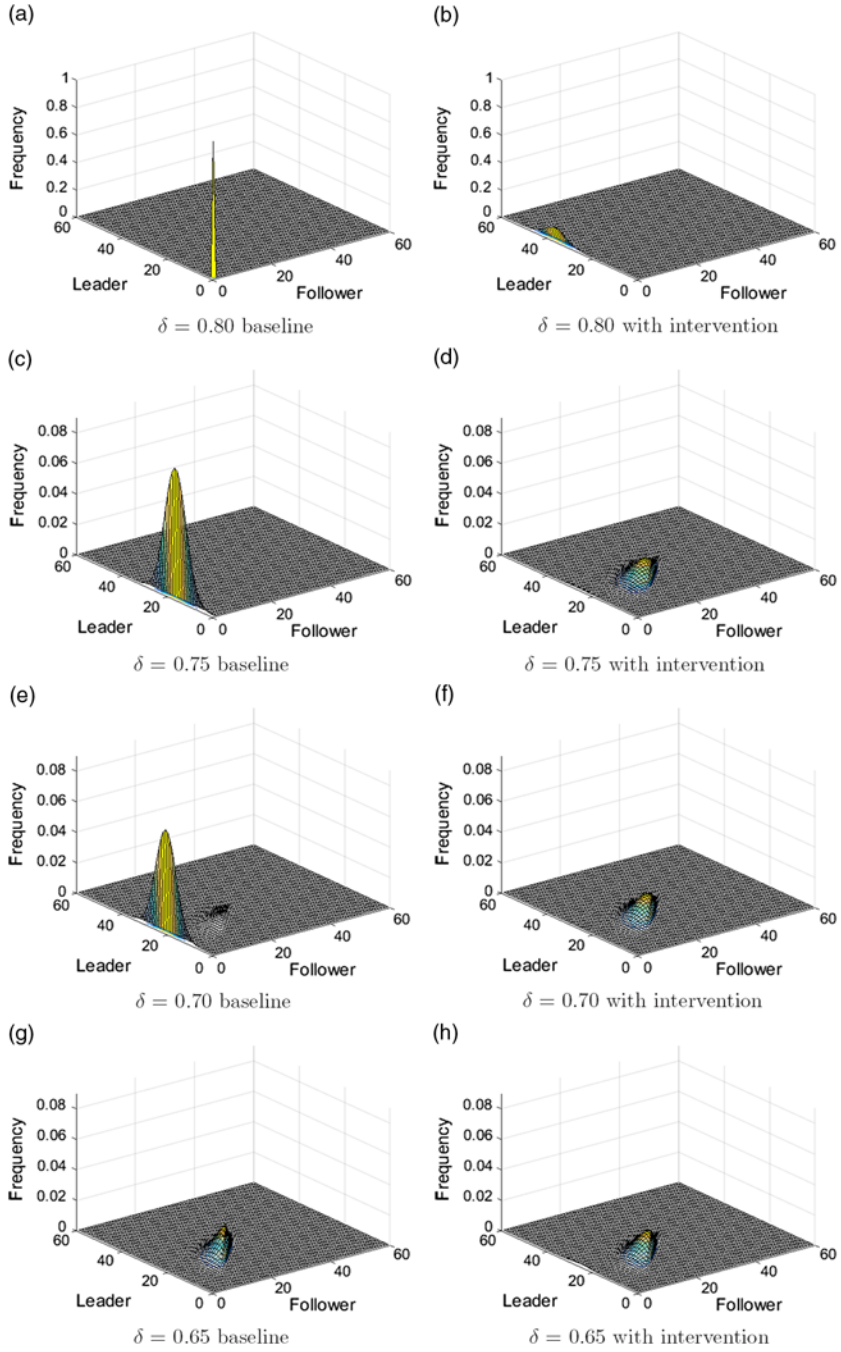


FIGURE 19. Variation in negative shock, δ.

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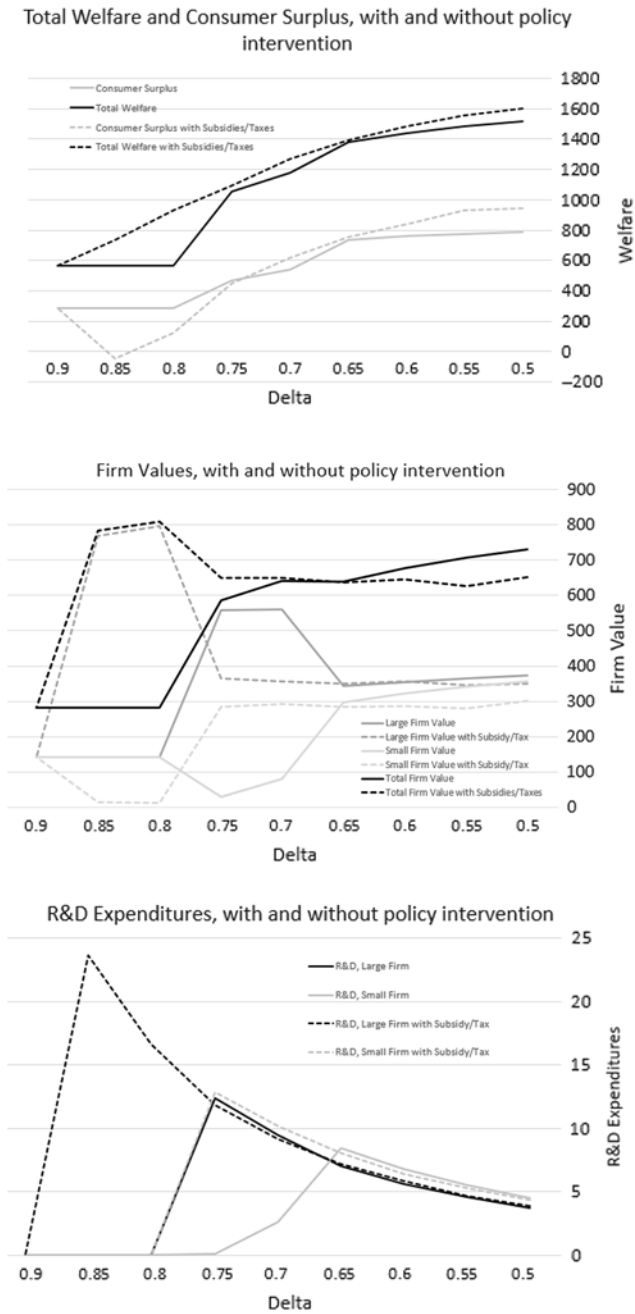


FIGURE 20. Variation in negative shock, δ: optimal policy and welfare vs. δ.

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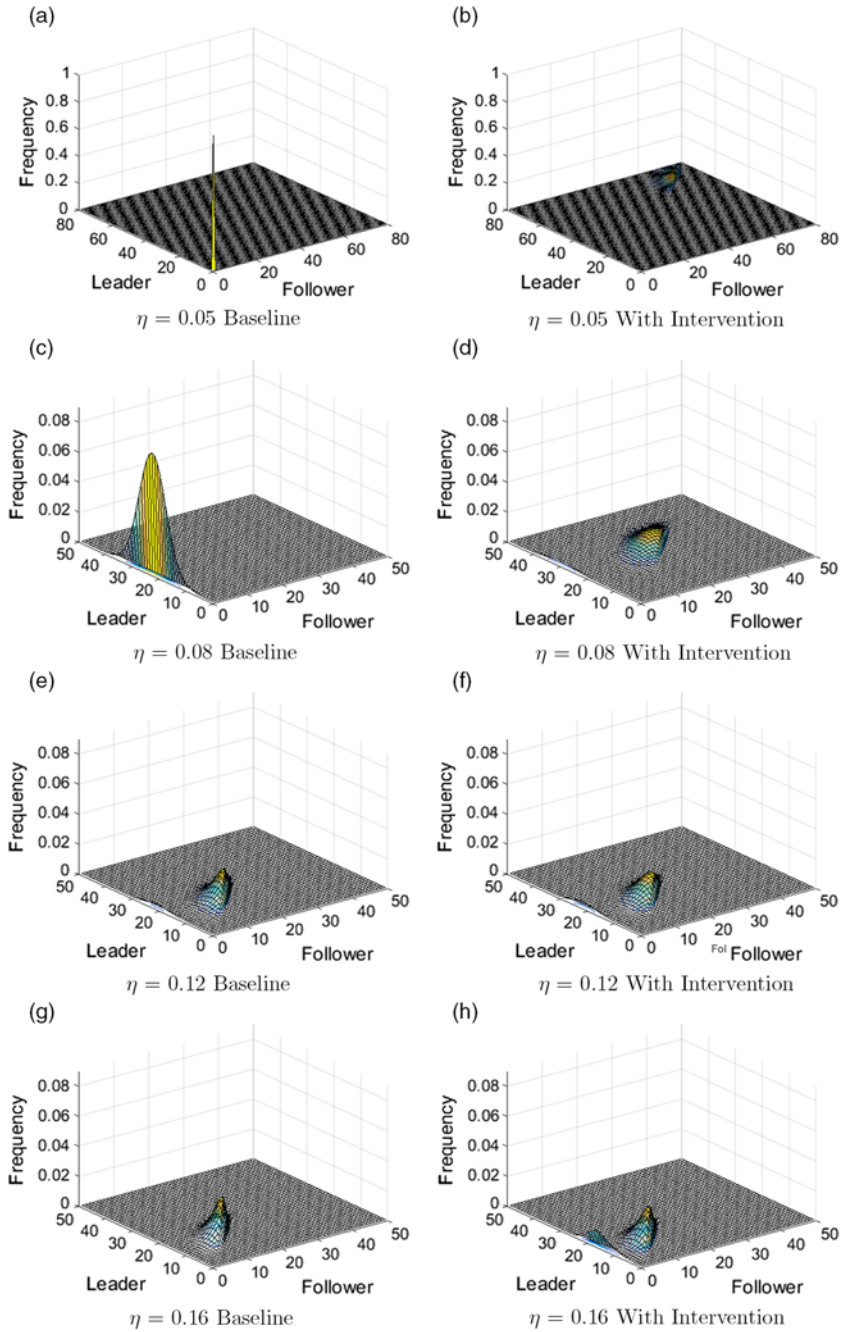


FIGURE 21. Variation in rate of marginal cost decline, η.

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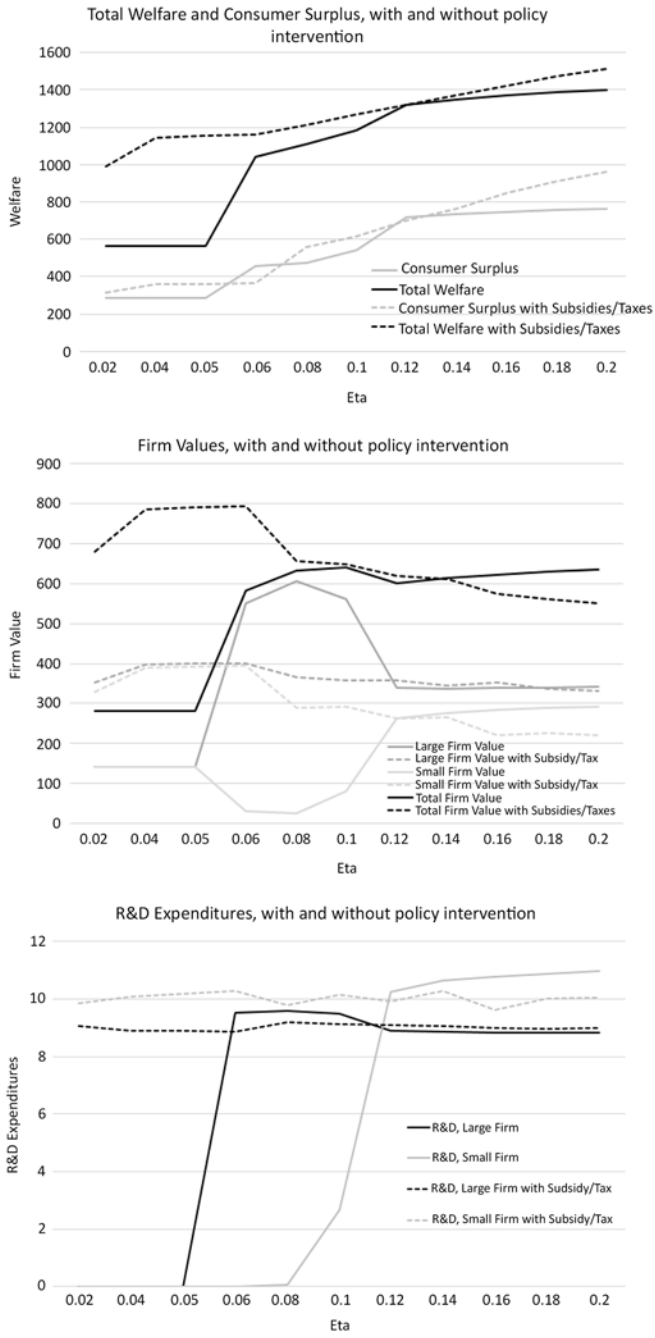


FIGURE 22. Variation in rate of marginal cost decline, η: optimal policy and welfare vs. η.

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conduct R&D in an effort to become the next large firm and sustain its monopoly for as long as it can while reaping large profits. As δ declines, the pattern of distributions and optimal policy is similar to the earlier results. At 0.75, from the asymmetric distribution the optimal policy is to subsidize both firms which generates a symmetric distribution. From the bimodal distribution ($\delta = 0.7$) again, the policy calls for subsidies, lower than in the asymmetric case but with the same result of a symmetric distribution. A further lowering of δ to 0.65 generates a policy of just taxing the smaller firm only but leaving the larger firm without a subsidy or tax. The welfare gain in this case is quite small at 1%. Lower values of δ lead to the symmetric innovation ergodic distribution (not shown) resulting in taxes on both firms.

Rate of marginal cost decline, η: The parameter η governs the rate at which marginal costs fall per innovation. Thus, a small value of η means each R&D success has a smaller impact on marginal costs (see Figures 21 and and 22). Starting from $\eta = 0.05$ where each innovation leads to a 5% reduction in marginal costs, large subsidies to both firms at 90% generate a large increase in welfare and a symmetric distribution. As η increases to 0.08, 0.12, and 0.16, respectively, the no-subsidy distributions move through the baseline types and the optimal policy follows the familiar pattern. In the asymmetric distribution, the optimal policy is to subsidize both firms, slightly favoring the larger firm. The optimal policy under the bimodal distribution in this case barely moves the distribution as it calls for a small (10%) subsidy to the larger firm and no subsidy or tax for the small firm. At $\eta = 0.16$, the baseline is a symmetric distribution which generates an optimal policy of a 30% tax on both firms.

5. CONCLUSIONS

We find that the static model, as in previous research, suggests that optimal government policy should target subsidies toward the more efficient firm and tax the less-efficient firm. Moving the same game to a dynamic setting shows that the results change. In general, the model suggests that in weak, low R&D environments, the policymaker should favor the large firm to encourage R&D. This policy takes the form of a subsidy to the large firm and a heavy tax on the small firm. As a result of this policy, welfare rises and results in a large increase in the market share of the larger firm. However, as the environment improves in terms of R&D and market demand, the policy shifts toward also subsidizing the small firm to promote more intense competition in both R&D and in the product market. In this case, the optimal policy benefits the smaller firm in terms of value at the expense of the larger firm. At the other end, when both demand and R&D are strong, the optimal policy is to tax both firms. The implications show the complexity faced by policymakers of how to subsidize or tax R&D when conditioning on the size of competing firms in a dynamic environment. The policy implication of our result

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is that using static size differential alone as a mechanism for allocating R&D subsidies is insufficient to determine the optimal policy. One must also understand the dynamic environment.

There is reason to believe that certain adjustments to the model, available in the dynamic context but not in the static context, may further extend and clarify these results. The dynamic context allows for the endogenous determination of the degree of competition that arises under various subsidies. In particular, allowing for entry and exit, and by extensions, a larger number of firms may well yield different effects since those alterations will almost certainly affect the nature of the competition and degree of pricing distortion in the equilibrium ergodic distributions. Related work on knowledge spillovers (see Laincz and Rodrigues 2016) also shows significant effects on the ergodic distribution which would also affect the optimal policies.

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NOTES

1. Segerstrom (2007) allows for incumbents and entrants to differ in research productivity, yet the firm heterogeneity is limited to monopolies or symmetric duopolies. Laincz (2005) allows for heterogeneity within industries by assuming Cournot competition and finds a uniform subsidy increases the growth rate as well as the degree of market concentration. Laincz (2005) also considers a general equilibrium dynamic environment with endogenous entry and exit; however, the R&D subsidy is assumed to be constant and symmetric for all firms.

2. Examples of R&D support targeted at SMEs including The Seventh Framework Program for Research and Technological Development - Research for the Benefit of SMEs (2007–2013) and SMEs in Health Research (2007–2013) in the European Union; Research and Development Relief for Corporation Tax (2008–present) in the United Kingdom; High Tech Strategy 2020 for Germany (2010–2020) in Germany; as well as Small Business Innovation Research (1982–present), Small Business Technology Transfer (1992–present), Small Business Investment Company (1958–present), and Research & Experimentation Tax Credit (1981–present) in the United States.

3. There has been extensive analysis of static two-stage oligopoly models [see, e.g., Brander and Spencer (1983), Spence (1984), D'Aspremont and Jacquemin (1988), Lahiri and Ono (1988, 1999), Hinloopen (1997, 2000), and Kitahara and Matsumura (2006)]. Others have looked at R&D competition among firms with varying environments [see Dasgupta and Stiglitz (1980), Gilbert and Newberry (1982), Levin and Reiss (1988), Cohen and Levinthal (1989), Budd et al. (1993), and Song, 2006, 2011]. Most of the research, however, focuses on symmetric oligopolies in a static environment.

4. We do not consider subsidies or taxes on output. Distortionary policy on output creates a significantly more complicated game. If the policymaker cannot observe costs (or innovations) firms have rent-seeking incentives to manipulate the level of output when the subsidies are conditioned on output. Similarly, we place the lump-sum taxes on the consumers, rather than the firms as in Brander and Spencer (1983) because the dynamic nature makes solving for lump-sum taxation levels significantly more complicated if placed on the firms.

5. We considered nonlinear demand $P = A - B\,(Q)^{\alpha}$, where $0 < \alpha < 1$ and $\alpha > 1$. These results are not included for brevity but they do not change the main results found in the paper.

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6. Although beyond the scope of this paper, an analysis of the trade off between concentration and growth becomes feasible. Growth in a static model is conditional upon the initial state and can only be assessed as a one period amount of growth and no inferences on the sustainability of that level can be obtained. Since the dynamic version of the model endogenizes the ergodic distribution of states, the growth rate itself is endogenous to the system. Therefore, the welfare analysis can account for the social gains that occur through long-run cost reductions.

7. Entry and exit could also be incorporated in the dynamic model. We suspect this feature may have a large effect on optimal policy. Without entry and exit, subsidies targeted to small firms may be inefficient since weak firms lack the incentive to exit. With exit as a choice, weak firms will leave the market and new firms will enter helping to maintain more intense competition. In addition, larger firms will possess incentives to drive competitors out of the market, a stronger form of Aghion et al.'s (2001) *escape competition* effect.

8. R&D spillovers are not considered here. Laincz and Rodrigues (2016) in a working paper address that topic in the Ericson and Pakes (1995) framework. Adding spillovers adds significant complication to the model because under spillovers the model will no longer satisfy the Unique Investment Criterion described in Doraszelski and Satterthwaite (2010) to ensure the existence of an equilibrium.

9. Here lies the technical reason for applying lump-sum taxes to consumers rather than firms. Since firms profits are calculated before the R&D levels are known, the net profits after taxes cannot be used in the value function iteration.

10. The C1 index, the market share of the largest firm, is calculated by weighting the market share of the largest firm by the probability of each industry state in the ergodic distribution.

11. This result has an interesting parallel with the results of Davis and Hashimoto (2015), who examine welfare effects from subsidy differences between large and small countries in a model of endogenous growth.

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