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Energy transfer from particles to low-frequency fields in a non-uniform magnetized plasma

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Energy transfer from particles to low-frequency turbulent fields in the non-uniform magnetized plasma is studied. Sufficient conditions for plasma stability are derived taking account of magnetic field shear.

1. Introduction

Experimental observations and numerical simulations of space and fusion plasmas reveal very complex behaviour due to low-frequency (with respect to the cyclotron frequency Ω) turbulence and anomalous transport. The dependence of the latter phenomena on the macroscopic parameters of the system are of great interest, as are the possibilities of controlling plasma behaviour in magnetic confinement devices by changing the macroscopic parameters.

The purpose of this paper is to study energy transfer from particles to low-frequency turbulent fields in a non-uniform magnetized plasma, and to derive sufficient conditions for plasma stability, taking account of magnetic field shear. Such conditions are obtained within the context of the kinetic theory from macroscopic adiabatic invariants of the system. The existence of the macroscopic invariants, related to the particle energy, has been revealed, and some of their implications for plasma stability have been shown (Sosenko and Decyk 1994*b*; Sosenko and Weiland 1995). These invariants can be obtained by means of the quasiparticle approach (Sosenko and Decyk 1993, 1994*a*; Sosenko 1995*a*, *b*), which is closely related to the general asymptotic methods of Krylov, Boholiubov and Mitropolskii (Kryloff and Bogoliuboff 1934*a*,*b*; Krylov and Bogoliubov 1936; Boholiubov 1950; Bogoliubov and Mitropolskii 1961; Mitropolskii 1971).

Section 2 presents basic quantities and equations. In Sec. 3, the macroscopic invariants are derived, taking account of the magnetic field shear. Section 4 deals with the sufficient conditions for plasma stability. Section 5 gives a summary.

2. Basic equations

Let us consider a system of charged particles, such as a plasma, with self-consistent interaction between them in the magnetic field $\mathbf{B}$. This system can be represented by

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the system of quasiparticles† with the following equations of motion (Boholiubov and Zubarev 1955; Brahinskyj 1956):

$$\frac{d\mathbf{r}}{dt} = \mathbf{V}, \quad \frac{dv_\parallel}{dt} = \mathbf{\Pi}_\parallel, \quad \frac{d\mu}{dt} = 0, \tag{1}$$

where $\mathbf{r}$ is the guiding-centre position, $v_\parallel$ is the projection of the particle velocity on $\mathbf{B}$ at the guiding-centre position, $\mu \equiv mv_\perp^2/2B$ is the magnetic moment, $\mathbf{v}_\perp \equiv \mathbf{v} - v_\parallel \hat{\mathbf{h}}$, $\mathbf{v}$ is the particle velocity. The general expressions for the guiding-centre velocity $\mathbf{V}$ and the effective acceleration $\mathbf{\Pi}_\parallel$ in the non-uniform magnetic field are given in Brahinskyj (1956). They are presented below for the case of interest.

The particle acceleration $\mathbf{a}$ is produced by self-consistent electric and magnetic fields in addition to the stationary magnetic field $\mathbf{B}$. The latter is assumed to be inhomogeneous and to have shear:

$$\mathbf{B} \equiv B\hat{\mathbf{h}} = B_z(x)\hat{\mathbf{z}} + B_s(x)\hat{\mathbf{y}} \tag{2}$$

(in Cartesian coordinates $\mathbf{r} = (x, y, z)$), where the straight-field model is employed for simplicity.

In the spirit of the asymptotic theory (Boholiubov and Zubarev 1955), the acceleration $\mathbf{a} = -(q/m)\boldsymbol{\nabla}\Phi$, due to the electric field, and the inhomogeneity of the background magnetic field are treated as small perturbations in the equations of particle motion (q and m are the particle charge and mass).

The quasiparticle density G in the phase space $(\mathbf{r}, v_\parallel, \mu)$ satisfies the continuity equation

$$\partial_t G + \boldsymbol{\nabla} \cdot (\mathbf{V}G) + \partial_{v_\parallel}(\mathbf{\Pi}_\parallel G) = 0. \tag{3}$$

This equation can be used to analyse the balance of the particle kinetic energy:

$$K = \int d\mathbf{r}\, dv_\parallel\, d\mu (\mu B + \tfrac{1}{2} m v_\parallel^2) G,$$
$$\frac{dK}{dt} = \int d\mathbf{r}\, dv_\parallel\, d\mu\, m\mathbf{a} \cdot \mathbf{V}G. \tag{4}$$

In the case under consideration the general expressions (Brahinskyj 1956) for the guiding-centre velocity and the effective parallel acceleration take the form

$$\mathbf{V} = v_\parallel \hat{\mathbf{h}} + \mathbf{V}_E + \mathbf{V}_g, \quad \mathbf{\Pi}_\parallel = a_\parallel, \tag{5}$$

where $\mathbf{V}_E = \mathbf{a} \times \hat{\mathbf{h}}/\Omega$ is the electric drift velocity, $\Omega \equiv qB/mc$, $\mathbf{V}_g = \mathbf{a}_g \times \hat{\mathbf{h}}/\Omega$ is the gradient drift velocity, and $\mathbf{a}_g \equiv -\mu\boldsymbol{\nabla}B/m$.

3. Macroscopic invariants

As usual, one splits the quasiparticle density $G = G_0 + \delta G$, into the equilibrium density G_0 and the fluctuation δG:

$$\delta G \sim \lambda G_0, \quad \lambda \ll 1. \tag{6}$$

The equilibrium electric field is assumed to be zero, $\mathbf{E} = \delta\mathbf{E}$. The space dependence of perturbations along $\hat{\mathbf{z}}$ is taken to be very weak, and E_z is disregarded.

† These are usually called guiding centres; however, in our approach (Sosenko and Decyk 1993, 1994*a*; Sosenko 1995*a*,*b*) the term 'guiding centre' is introduced before the asymptotic averaging, while the term 'quasiparticle' is introduced after statistical averaging of the guiding-centre equations of motion.

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The equilibrium density satisfies the equation ,

$$(v_\parallel \hat{\mathbf{h}} + \mathbf{V}_g) \cdot \boldsymbol{\nabla} G_0 = 0.$$

Its solution is an arbitrary function of $x, v_\parallel$ and μ:

$$G_0 = G_0(x, v_\parallel, \mu). \tag{7}$$

The fluctuation δG satisfies the equation

$$\partial_t\, \delta G + \boldsymbol{\nabla} \cdot (\mathbf{V}\, \delta G) = -\boldsymbol{\nabla} \cdot (\mathbf{V}_E G_0) - \partial_{v_\parallel}(a_\parallel G_0). \tag{8}$$

This equation takes into account the parallel acceleration of the particle in the sheared magnetic field, besides the drift motion across the magnetic field.

After statistical averaging of (4), one gets an equation for the balance of the mean kinetic energy:

$$\frac{d\langle K\rangle}{dt} = -m \int d\mathbf{r}\, dv_\parallel\, d\mu \langle (\mathbf{a}_g \cdot \mathbf{V}_E - v_\parallel a_\parallel)\delta G\rangle. \tag{9}$$

For the large-scale perturbations (with their characteristic length scales comparable to the inhomogeneity length scales of the equilibrium quasiparticle density), the nonlinear term on the right-hand side of (9), $\boldsymbol{\nabla} \cdot (\mathbf{V}_E\, \delta G)$, can be neglected according to (6). For the small-scale perturbations, with

$$\boldsymbol{\nabla} \delta G \sim \boldsymbol{\nabla} G_0, \tag{10}$$

this nonlinear term is to be retained. Therefore the results obtained below allow for the nonlinear effects.

One takes into account that $\boldsymbol{\nabla} B = \hat{\mathbf{x}} B \kappa_B$, where, for any quantity Q,

$$\kappa_Q \equiv \frac{1}{Q}\frac{dQ}{dx}, \tag{11}$$

and that $\mathbf{V}_E \cdot \hat{\mathbf{x}} \approx cB_z E_y/B^2$ and $a_\parallel \approx qB_s E_y/mB$. Then, in the same manner as in Sosenko & Decyk (1994*b*) and Sosenko & Weiland (1995), the reduced balance relation for the kinetic energy follows from (8) and (9):

$$\frac{d}{dt}\left[\langle K\rangle + \frac{1}{2}\int d\mathbf{r}\, dv_\parallel\, d\mu \langle \hat{f}^2\rangle \left(\frac{mv_\parallel \boldsymbol{\Omega}_s}{B} + \mu\kappa_B\right)\left(\partial_x + \boldsymbol{\Omega}_s \partial_{v_\parallel}\right)\frac{G_0}{B}\right] = 0, \tag{12}$$

where $\boldsymbol{\Omega}_s \equiv (qB_s/mc)B/B_z$, and $\hat{f}$ satisfies the equation

$$\partial_t \hat{f} + \boldsymbol{\nabla} \cdot (\mathbf{V}\hat{f}) = B\mathbf{V}_E \cdot \hat{\mathbf{x}}. \tag{13}$$

Therefore the quantity

$$S \equiv \langle K\rangle + \frac{1}{2}\int d\mathbf{r}\, dv_\parallel\, d\mu \langle \hat{f}^2\rangle \left(\frac{mv_\parallel \boldsymbol{\Omega}_s}{B} + \mu\kappa_B\right)(\partial_x + \boldsymbol{\Omega}_s \partial_{v_\parallel})\frac{G_0}{B}. \tag{14}$$

is a macroscopic invariant of the system.

4. Constraints on stable plasma states

The invariant (14) rules out the kinetic energy of particles as a source for driving fluctuations when the second term on the right-hand side, which is determined by fluctuations, is negative. This is the general sufficient stability condition.

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In what follows, only stability against perturbations with weak effects of thermal particle motion along the magnetic field is analysed in detail. In this case, the dependence of the fluctuation factor $\hat{f}$ on $v_\parallel$ can be disregarded (Sosenko and Weiland 1995), and the invariant simplifies after the integral over $v_\parallel$ has been calculated explicitly:

$$S \equiv \langle K \rangle + \frac{1}{2} \int d\mathbf{r}\, d\mu \langle \hat{f}^2 \rangle \left(\mu \kappa_B \partial_x - \frac{m \boldsymbol{\Omega}_s^2}{B} \right) \int dv_\parallel \frac{G_0}{B} \tag{15}$$

where particle flows along $\mathbf{B}$ are disregarded in equilibrium.

The second term on the right-hand side is negative when

$$\left(\mu \kappa_B \partial_x - \frac{m \boldsymbol{\Omega}_s^2}{B} \right) \int dv_\parallel \frac{G_0}{B} \leqslant 0. \tag{16}$$

This is the sufficient stability condition for the particular case under consideration, which demonstrates clearly the stabilizing effect of the magnetic field shear.

Let us consider the equilibrium quasiparticle distribution with separable dependence on $v_\parallel$ and μ, and take the normal distribution of perpendicular particle velocities as a particular example:

$$\int dv_\parallel\, G_0 = \frac{nB}{T_\perp} \exp\left(-\frac{B\mu}{T_\perp} \right), \tag{17}$$

where $T_\perp$ is the perpendicular temperature. Then the constraint (16) requires that

$$(\tilde{\kappa}_{T_\perp} - 1) X^2 + (\tilde{\kappa}_n - \tilde{\kappa}_{T_\perp}) X - \frac{\boldsymbol{\Omega}_s^2 m}{T_\perp \kappa_B^2} \leqslant 0, \tag{18}$$

where

$$X \equiv \frac{\mu B}{T_\perp}, \quad \tilde{\kappa}_Q \equiv \frac{\kappa_Q}{\kappa_B}.$$

This constraint can be satisfied only if $\tilde{\kappa}_{T_\perp} < 1$, which is the necessary stability condition. The inequality (18) is satisfied when either

$$\tilde{\kappa}_n < \tilde{\kappa}_{T_\perp} < 1 \tag{19}$$

or

$$(\tilde{\kappa}_n - \tilde{\kappa}_{T_\perp})^2 < 4(1 - \tilde{\kappa}_{T_\perp}) \frac{\boldsymbol{\Omega}_s^2 m}{T_\perp \kappa_B^2}, \tag{20}$$

which is the sufficient stability condition for the particular case of interest.

This allows plasma equilibria with $\tilde{\kappa}_{T_\perp} \leqslant 0$ and $\eta \equiv \tilde{\kappa}_{T_\perp}/\tilde{\kappa}_n \leqslant 1$, similar to the effective-gravity model (Sosenko and Decyk 1994*b*; Sosenko and Weiland 1995), as well as the states with $\tilde{\kappa}_n \geqslant 0$ and $\eta \geqslant 1$, or with $\tilde{\kappa}_n < 0$ and $0 < \tilde{\kappa}_{T_\perp} < 1$ (inverse density and temperature profiles, $\eta < 0$), which are not predicted by the latter model. The shear (inhomogeneity) of the magnetic field is responsible for the larger stability domain (20) ((19)). The bound in (19) yields $T_\perp/B = \text{const}$ and the adiabatic type law

$$p_\perp / n^\gamma = \text{const}, \tag{21}$$

where $p_\perp = nT_\perp$ is the perpendicular pressure and $\gamma = 2$.

The theory developed here can be compared with linear stability analyses for electron drift waves (Krall and Rosenbluth 1965; Pearlstein and Berk 1969; Antonsen 1978; Ross and Mahajan 1978; Tsang *et al.* 1978; Manheimer and Lashmore-Davies

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1989; and references therein). In this case, (15) can be associated with ions. For cold particles, it takes the form

$$S = \langle K \rangle - \frac{1}{2} \int d\mathbf{r}\, dv_\parallel\, d\mu \langle \hat{f}^2 \rangle \frac{m\Omega_s^2 G_0}{B^2}. \tag{22}$$

This latter shows the stabilizing role of a cold-particle subsystem in the presence of magnetic shear, in agreement with the results (Pearlstein and Berk 1969; Ross and Mahajan 1978) obtained in the absence of temperature gradients.

The constraints that follow from the adiabatic invariant (15) take into account the effects of finite Larmor radius. The relevant parameter (the Larmor radius over the wavelength) does not enter explicitly into such constraints. This means that such a parameter can be irrelevant for stability considerations, similarly to the analytical and numerical results (Antonsen 1978; Ross and Mahajan 1978; Tsang *et al.* 1978), and in contrast to the predictions from perturbation theory; see the discussion and references in Ross and Mahajan (1978) and Tsang *et al.* (1978).

For electrons, it is necessary to allow for their thermal motion along the magnetic field. Some understanding of the implications of the invariant (14) for electrons can be made after disregarding the magnetic field gradient, as in the low-pressure approximations (Ross and Mahajan 1978; Tsang *et al.* 1978), and taking into account that the parallel phase velocity of the electron drift waves is much smaller than the thermal velocity of electrons. Then the solution of (13) can be approximated by $\hat{f} = -(cB_z/v_\parallel B_s)\phi$, with ϕ the electric field potential, and (14) simplifies to

$$S = \langle K \rangle + \frac{e^2}{2m_e} \int d\mathbf{r}\, dv_\parallel\, d\mu \langle \phi^2 \rangle \frac{1}{v_\parallel} \partial_{v_\parallel} G_0. \tag{23}$$

Therefore the kinetic energy of electrons cannot be a source of instability when

$$\frac{1}{v_\parallel} \partial_{v_\parallel} G_0 \leqslant 0\ .$$

This constraint does not include gradients of density or temperature, or the shear length, and it is satisfied by the bi-Maxwellian electron distribution function. This is in agreement with the results Ross and Mahajan (1978) and Tsang *et al.* (1978).

However a detailed study of constraints on energy transfer, taking account of the thermal particle motion along the magnetic field, is beyond the scope of the present paper.

5. Summary

The basic results of this paper are the macroscopic invariant (14) (or (15)), together with the sufficient condition for stability (16) and its implications (19) and (20) for the particular equilibrium (17). These results are valid for both the large-scale (global) fluctuations and the small-scale ones. They take into account the kinetic effects, as well as the nonlinear effects. The latter can be important for the small-scale fluctuations.

The invariants are determined by the equilibrium quasiparticle density. This justifies the quasiparticle concept. Stable plasma states are determined according to the constraints on the macroscopic parameters (the density and the temperature in (17)) of the system of quasiparticles. The latter are equal to the corresponding

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macroscopic plasma parameters (the plasma density and temperature), when finite-Larmor-radius effects are assumed to be very weak in the equilibrium state.

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