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Stochastic modeling of plasma mode forecasting in tokamak

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Abstract. The structure of magnetohydrodynamic (MHD) modes has always been an interesting study in tokamaks. The mode number of tokamak plasma is the most important parameter, which plays a vital role in MHD instabilities. If it could be predicted, then the time of exerting external fields, such as feedback fields and Resonance Helical Field, could be obtained. Autoregressive Integrated Moving Average (ARIMA) and Seasonal Autoregressive Integrated Moving Average are useful models to predict stochastic processes. In this paper, we suggest using ARIMA model to forecast mode number. The ARIMA model shows correct mode number ($m = 4$) about 0.5 ms in IR-T1 tokamak and equations of Mirnov coil fluctuations are obtained. It is found that the recursive estimates of the ARIMA model parameters change as the plasma mode changes. A discriminator function has been proposed to determine plasma mode based on the recursive estimates of model parameters.

1. Introduction

Structure of magnetohydrodynamic (MHD) mode has always been an interesting study in Tokamaks. It is important for the understanding of MHD behavior and its control. MHD instabilities and disruptions are prevalent and are important phenomena in tokamaks [1]. Evidence of magnetic islands in tokamak plasma is given by the external magnetic field fluctuations detected by Mirnov coils [2]. MHD modes produce fluctuating magnetic fields that can be measured by magnetic pick-up Mirnov coils [3]. Correlating the phases from on array of Mirnov coils yields useful information about the geometry of modes magnetic perturbations [4, 5]. It is well known that magnetic oscillations with poloidal and toroidal mode numbers m and n, respectively, are created by perturbation of current channel on rational magnetic surfaces [3–5]. The tokamak plasma is toroidally symmetric, so spectral analysis of the signals from two toroidally and poloidally separated Mirnov coils can identify toroidal and poloidal mode numbers.

The Mirnov signals are oscillating time series from which the overall structure is extracted. The mode number and structure are determined by obtaining phase angles from a set of Mirnov coils at a frequency of interest. This method is particularly useful for the analysis of Mirnov coils data with the aim of obtaining m and n numbers. This involves finding the optimal parameters, m, n, λ, and $\delta_\circ$, such that $\xi = m(\theta - \lambda \sin\theta) + n\varphi + \delta_\circ$ approximates the phase data, where $\lambda = (\beta_p + \frac{l_i}{2} + 1)$ is a free toroidal correction to the geometry of the mode and the off-center position of the mode with respect to the vessel center. β_p is the poloidal beta, l_i is the internal inductance, $\varepsilon = \frac{r}{R}$ is the inverse aspect ratio, $\delta_\circ$ is the phase constant, and θ and φ are the poloidal and toroidal angles of Mirnov coils [6].The next method is Singular Value Decomposition (SVD). SVD does not require any assumptions about the spatial structure of the mode, but determines its own basis vectors representing the fluctuation data in the directions of maximum coherence [5]. Since Mirnov signals are oscillating time series, these two methods could determine the mode numbers.

Artificial neural network, a prediction technique, has been used in DIII D [7], Asdex [8], and ITER [9] for plasma parameters and disruptions.

We present a statistical method to forecast mode number. If it could be predicted, then the time of external fields, such as feedback fields and Resonance Helical Field (RHF), could also be obtained. In forecasting, the goal is to predict future values of a time series based on the Mirnov coils data collected to the present. The forecasting technique used is a regression analysis or moving averages [10]. Since moving averages can work better with stationary data, the forecasting technique that should be considered is Autoregressive Integrated Moving Average (ARIMA) and seasonal ARIMA (SARIMA). ARIMA models have been already applied to forecast [11].

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2. ARIMA model

Autoregressive models are based on the idea that the present value of the time seriesx_t can be explained as a function of (p) past values, x_{t-1}, $x_{t-2}, \ldots, x_{t-p}$.

An autoregressive model (AR) of order p, AR(p), is $x_t = \sum \phi_j.\omega_{t-j}$, where ω_t is a sequence of uncorrelated random variables with mean zero and constant variance σ^2 [12].

This is called the infinite moving average representation of x_t. The autoregressive moving average (ARMA) model of order q, MA(q), is defined as

$$x_t = \omega_t + \theta_1\omega_{t-1} + \theta_2\omega_{t-2} + \cdots + \theta_q\omega_{t-q}, \qquad (1)$$

where there are q lags in the moving average and $\theta_1, \theta_2, \ldots, \theta_q (\theta_q \neq 0)$ are parameters [12]. The noise ω_t is assumed to be the Gaussian white noise.

In (1) without loss of generality we impose the normalization restriction that $\theta_\circ = 1$ [10, 12]. The system is as infinite moving average define as the linear process (2) than $\theta_\circ = 1$ and $\theta_\circ = 0$ for other values [12]. The MA(q) is written in the form $x_t = \theta(B)\omega_t$, where the moving average operator is $\theta(B) = 1 + \theta_1 B + \theta_2 B^2 + \ldots + \theta_q B^q$. The AR and moving average (MA) polynomials are respectively defined as

$$\begin{aligned} \phi(z) &= 1 - \phi_1 z + \phi_2 z^2 + \ldots - \phi_p z^p \quad \phi_p \neq 0, \\ \theta(z) &= 1 + \theta_1 z + \theta_2 z^2 + \ldots + \theta_q z^q \quad \theta_q \neq 0, \end{aligned} \qquad (2)$$

where z is a complex number and θ and ϕ are the model parameters. The ARMA (p, q) model is shown as

$$\phi(B)x_t = \theta(B)\omega_t. \qquad (3)$$

Finally, suppose that there is a variable that is integrated of order d,

$$\nabla^d x_t = (1 - B)^d x_t, \qquad (4)$$

where ∇ is the backward difference operator. In general, we will write the model as

$$\phi(B)(1 - B)^d x_t = \theta(B)\omega_t. \qquad (5)$$

Such a process is said to be integrated ARMA process or an ARIMA (p, d, q). The Box–Jenkins modeling parameters (p, d, q) are used only in statistical modeling and do not have any physical meaning. The ARIMA (p, d, q) can describe both stationary ($d = 0$) and non-stationary ($d > 0$) time series [12, 13]. The multiplicative seasonal autoregressive integrated moving average or SARIMA model [10] is denoted as ARIMA $(p, d, q) \times$ (P, D, Q) [9, 10].

3. Identification of ARIMA model structure

Determine whether the series is stationary or not by considering the graph of Auto-Correlation Function (ACF). A model $\{Y_t\}$ is co-variance stationary when, the mean ($E(Y_t)$) and variance ($Var(Y_t)$) are constant for all times and correlation structure of the series $Corr(Y_t, Y_{t-h})$ is a function of h for all times and do not change. This function describes the strength of the relationships between different points in the series. For a covariance stationary time series model $\{Y_t\}$, ACF is defined as

$$\rho(h) = \mathrm{Corr}(Y_t, Y_{t-h}).$$

For $\{y_1, \ldots, y_n\}$ the sample estimate of the ACF is [13],

$$\rho(h) = r(h) = \frac{c(h)}{c(0)} \quad \text{where} \quad c(h) = \frac{1}{n}\sum_{i=h+1}^{n}(y_i - \tilde{y})(y_{i-h} - \tilde{y})$$

If a graph of ACF of the time series values either cuts off fairly quickly or dies down fairly quickly, then the time series values should be considered stationary. If a graph of ACF dies down extremely slowly, then the time series values should be considered non-stationary [12].

Partial Auto-Correlation Function (PACF) is simply the correlation between the two set of residuals obtained from regressing the element y_t and y_{t-h} on the set of intervening values $\{y_1, \ldots, y_{t-h+1}\}$ [12]. If the series is not stationary, it can often be converted into a stationary series by differencing, that is, the original series is replaced by a series of differences. An ARMA model is then specified for the differenced series. Differencing is done until a plot of the data indicates that the series varies about a fixed level, and the graph of ACF either cuts off fairly quickly or dies down fairly quickly [13].

Model for a non-seasonal series is called an autoregressive integrated moving average model denoted by ARIMA (p, d, q). Here $p$indicates the order of the autoregressive part, d indicates the amount of differencing, and q indicates the order of the moving average part. If the original series is stationary, $d = 0$ and the ARIMA model reduces to the ARMA models.

Finally, model must be checked for adequacy by considering the properties of residuals, i.e., the residuals from an ARIMA model must have normal distribution and should be random.

The Mirnov coils spaced along the poloidal direction collect magnetic signals as time series. The plasma current within a rational surface $r_{m,n}$, which is usually associated with the development of resistive MHD modes, can be expressed in the form [4]

$$\widehat{j} = j_{m,n}\cos(\xi)\delta(r - r_{m,n}),$$

where $\zeta = m\theta^* + n\phi + \delta_0 - \omega t$ is the Merezhkin phase correction, and $\theta^* = \theta - \frac{r_{n,m}}{R_{n,m}}[\beta_p + \frac{1}{2}l_i + 1]\sin\theta$. In this study, we used the data for Mirnov coils IR-T1, a small tokamak with air core. Parameters are given in Table 1.

Table 1. Parameters of IR-T1.

Parameters	Value
Major radius	45.0 cm
Minor radius	12.5 cm
Toroidal field	<1.0 T
Plasma current	<30 kA
Discharge time	<35 ms
Electron density	$(0.7 - 1.5) \times 10^{13}$ cm^{-3}

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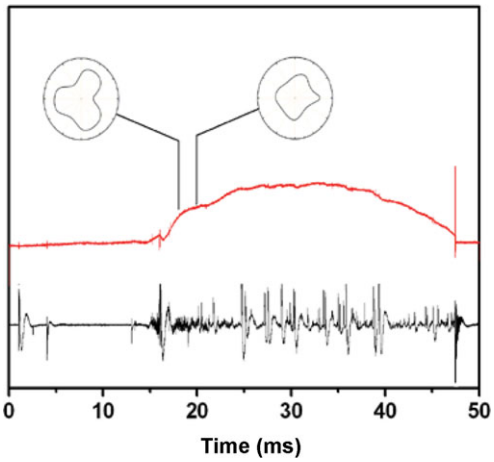


Figure 1. (Colour online) (a) Plasma current; (b) poloidal magnetic field fluctuations.

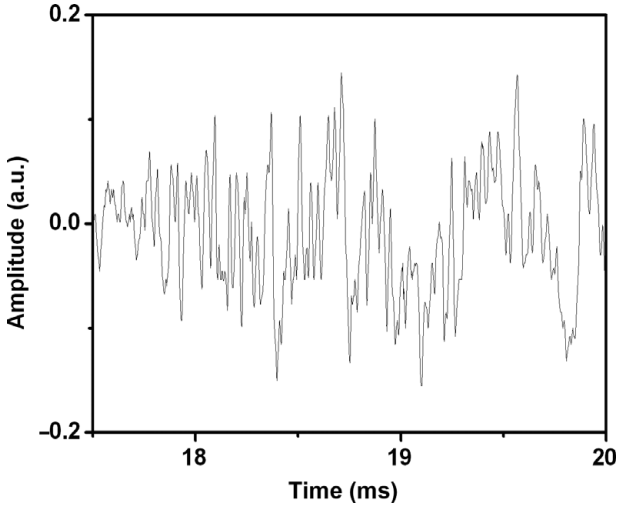


Figure 2. Mirnov coil fluctuations from 17.5 to 20.0 ms.

The coils are located at 160–170 mm from the vacuum vessel center. The poloidal array formed by coils 1–12 can detect by m number. SVD and Fast Fourier transform (FFT) are the techniques used for analyzing data from these signals [14].

The poloidal magnetic field detected by fluctuations of Mirnov coil (shot no. 8804251300) and plasma current are shown in Fig. 1.

The Mirnov coil fluctuations are analyzed from 17.5 to 20.0 ms. Figure 2 shows a plot of the data (y_t).

The sample ACF of the data is displayed in Fig. 3. We note slow decay in the ACF, indicating non-stationary behavior.

Differencing such a process will lead to a stationary process. Figure 4 shows the first difference of the data ∇y_t, and the trend has been removed [13, 15]. The sample ACF and partial auto-correlation function (PACF) of the Mirnov coil fluctuations are plotted in Fig. 4.

It is appropriate to introduce autoregressive and moving average polynomials that identify with seasonal lags.

It appears that the ACF and PACF tail off. We should consider a model with both $p > 0$ and $q > 0$ for non-seasonal components.

According to Fig. 4, the AR, MA, and seasonal lags can be identified [13, 15]. This result indicates ARIMA $(2, 1, 3) \times (1, 1, 1)$.

Figure 5 shows histogram of residuals, a normal Q–Q plots of residuals, and ACF and PACF of residuals. We see that residuals are somewhat close to normal except for a few extreme values in tails.

Similarly, we obtain ARIMA model for next 11 Mirnov coils that are presented in Table 2.

It is clear, for each coil one model is suggested that they are similar to each other, but attentively different at position of Mirnov coils, each of them shows different signals and has unique model.

4. Forecasting

The multiplicative SARIMA model of Box–Jenkins is given by

$$\Phi_P(B^s)\phi_p(B)(1-B)^d(1-B^s)^D w_t = \theta_q(B)\Theta_Q(B^s)a_t, \quad (6)$$

where Φ_P and ϕ_p are seasonal and non seasonal AR parameters (equ. 2), Θ_Q and θ_q are seasonal and non seasonal MA parameters (equ. 2), P, p, Q, p, D, d are SARIMA parameters (tab. 2) and w_t is the usual Gaussian white noise process [10, 13]. The ARIMA parameters in Table 2 are used in (6) and the forecasting Mirnov coil fluctuations are obtained.

The measured and forecasted data are shown in Fig. 6. It can be seen that there is fine accordance in the spectra.

The ARIMA model is used for IR-T1 Mirnov coils data. The obtained results have good correlation with experimental data. Hence, it is possible to predict Mirnov fluctuations at 20.0 to 20.5 ms.

It is expected that the ARIMA model equations can be useful for estimating the time of exerting external fields (RHF, feedback fields, etc.).

In order to do comparative studies between the experimental and our results, the mode number was calculated by the SVD and FFT methods. As can be seen in Fig. 7, there is well accordance between experimental and predicted data ($m = 4$). Thus, it can be believed that the ARIMA model is reliable.

Figure 1 shows the plasma mode is stable at 17.0 ms, however, after this recursive estimates begin to change in plasma mode and instability. At approximately 20.5 ms, the recursive estimates approach another stable state.

Hence, it is possible to use changes in the recursive estimates of ARIMA models to determine changes in plasma mode. The transition period can still be predicted based on recursive parameters (p, d, q).

In this interval, the recursive estimates of the ARIMA (2, 1, 2) model can be used to find out changes in plasma mode. For this purpose, three parameters need to be used as inputs of discriminator function. Let us consider

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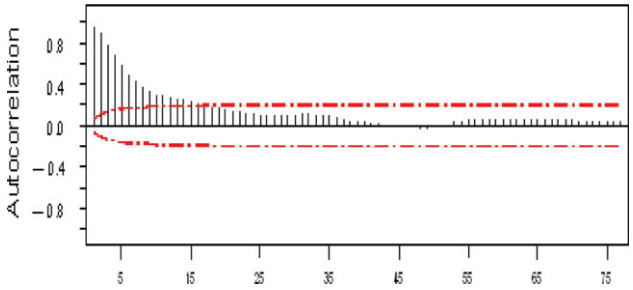


Figure 3. (Colour online) ACF of sample interval that shows trend in data.

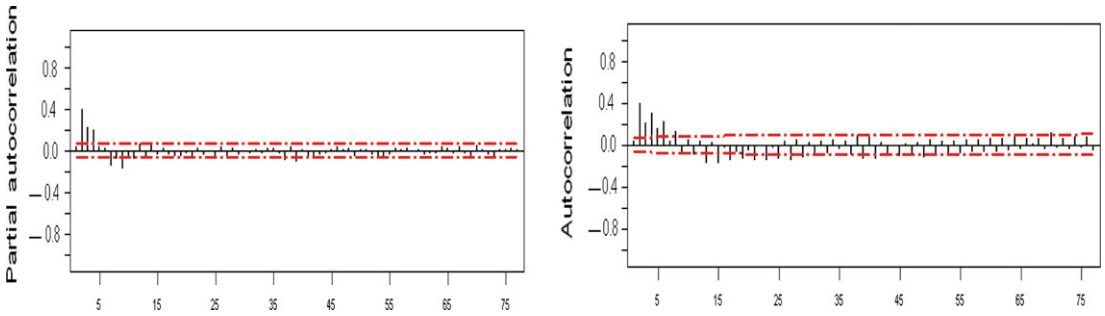


Figure 4. (Colour online) ACF and PACF of Mirnov coil fluctuations after first differencing. The dashed line is confidence limit.

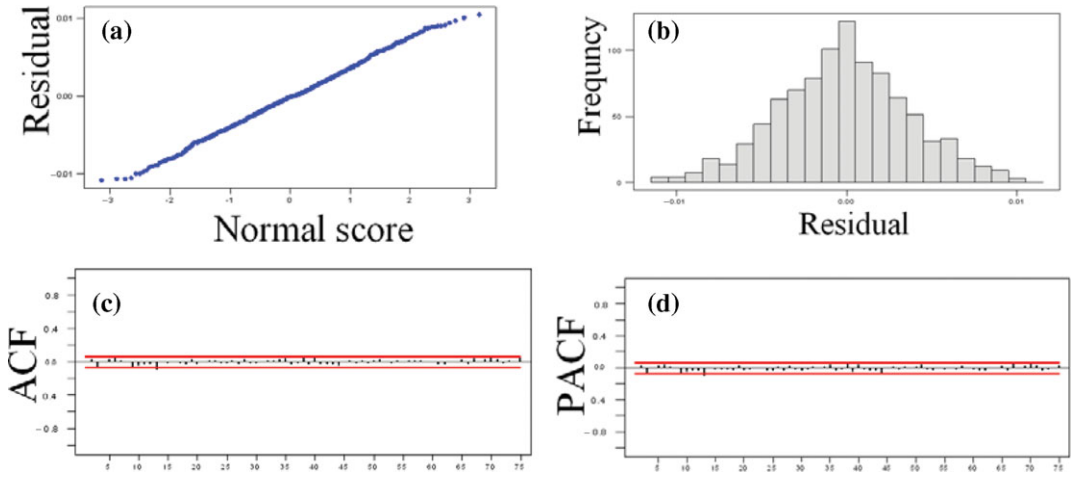


Figure 5. (Colour online) (a) Normal plot; (b) histogram; (c) ACF and (d) PACF of residuals for MP(0).

the Mirnov coil oscillations as a system with a white noise as the input and the plasma magnetic fluctuations as the output. Denote the transfer function as $H(z)$, then

$$H(z) = \frac{1 - \theta_1 z^{-1} - \theta_2 z^{-2}}{(1 - \theta_1 z^{-1} - \theta_2 z^{-2})(1 - z^{-1})^d}. \quad (7)$$

The frequency response of the system is [16, 17]

$$H(e^{iwt}) = \frac{1 - \theta_1 e^{-iwt} - \theta_2 e^{-2iwt}}{(1 - \theta_1 e^{-iwt} - \theta_2 e^{-2iwt})(1 - e^{-iwt})^d}, \quad (8)$$

where w is the frequency and t is the sampling period. The recursive estimates of the parameters can be used to determine system's frequency response at actual frequencies.

The state that produces magnetic fluctuation from the noise has higher frequency than the stable state. The discriminator function is proposed as [15]

$$f = \frac{|H(e^{iw_h t})|}{|H(e^{iw_l t})|}, \quad (9)$$

where w_h is the selected high frequency and w_l is the low frequency. The discriminator function results are shown in Fig. 8. As can be seen in Fig. 8, change in plasma mode at 19.70 ms can be determined from the discriminator function. The discriminator function decreases after 19.70 ms.

The discriminator function determines exactly the time of change in plasma mode.

5. Conclusion

We have suggested and analyzed the SARIMA model for forecasting mode number. We have obtained correct

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Table 2. Value of ARIMA parameters for 12 Mirnov coils data.

Coils (Deg)	p	d	q	P	D	Q
0	2	1	2	1	1	1
30	1	1	1	1	0	0
60	2	1	2	1	1	0
90	2	1	1	1	0	1
120	1	1	3	0	0	1
150	3	1	2	1	1	1
180	2	1	2	1	1	1
210	2	1	2	1	2	2
240	1	1	2	1	1	0
270	2	1	3	1	0	1
300	2	1	3	1	1	1
330	2	1	2	1	1	1

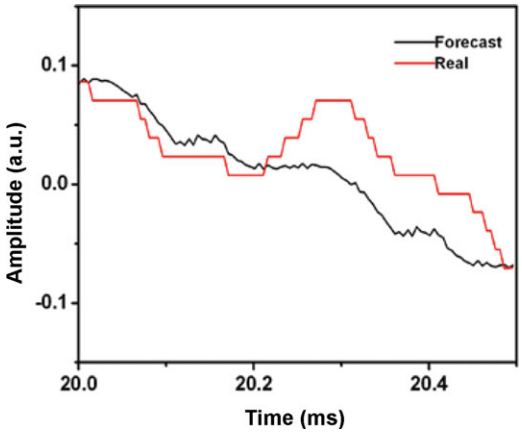


Figure 6. (Colour online) A Mirnov coil fluctuation data at 20.0 to 20.5 ms and its forecast.

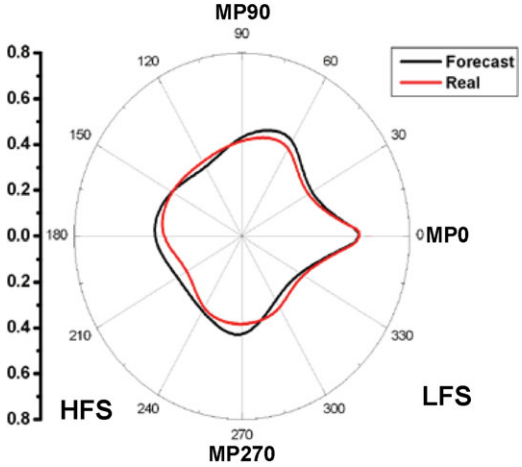


Figure 7. (Colour online) Plasma cross-section at 20.5 ms.

mode number ($m = 4$) and fluctuation's equation ((6) and Table 2).

Seasonal parameters (p, d, q) in the SARIMA model made very small fluctuations. These small peaks (Fig. 6) are not affected on mode number prediction. It is found that seasonal fluctuations and there parameters (P, D, Q) which are used in the SARIMA model for mode detection are unnecessary parameters (Fig. 6), thus for this purpose we can use ARIMA model.

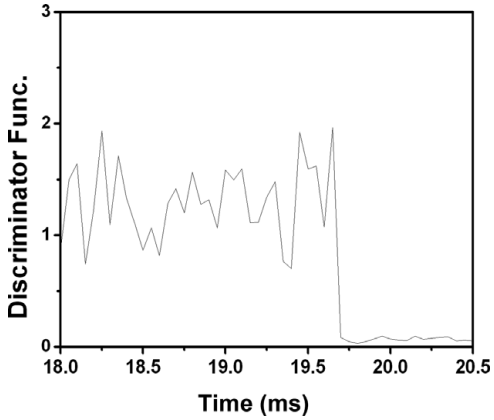


Figure 8. Frequency response and discriminator function curves determined using recursive estimates of ARIMA (2, 1, 2) parameters at 18.8–20.5 ms.

Parameters of the ARIMA model were estimated recursively. It was found that the recursive estimates could converge if the plasma mode remained unchanged. If the mode changes, the recursive estimates of the parameters and the parameters of the ARIMA change accordingly. The ARIMA model can be considered a transfer function. The ARIMA system produces magnetic fluctuation as output from a noise input. The frequency response of the ARIMA system has different characteristics in different modes. According to the characteristics in the frequency response, a discriminator function can be formed to simplify decision-making in detecting and can also predict change in plasma mode.

It is expected that the ARIMA model equations can be useful for estimating the time of applying biasing and external fields (RHF, feedback fields, etc.).

The instabilities have duration less than time of our forecast, be controlled by external fields. It should be pointed out that this study is focused on the development of the methodology. In order to have general Mirnov coil fluctuation equations and obtain ARIMA parameters to estimate the duration of entire tokamak shut, we should use professional statistical software and special computer.

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