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Letter to the Editor

Nonlinear coupling between Alfvén and fast magnetosonic waves

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Abstract. Energy transformation between fast magnetosonic waves and Alfvén waves is considered. It is shown that the efficiency of this process can be significantly enhanced if the Alfvén wave propagation directions are appropriately chosen. The relevance of our investigation to the non-linear excitation of magnetohydrodynamic waves in space plasmas is discussed.

Large-amplitude magnetohydrodynamic (MHD) waves (Alfvén and magnetosonic waves) are frequently observed in the solar wind (Belcher and Davis 1971), in interplanetary space (Tsurutani and Ho 1999), in the terrestrial foreshock (e.g. Eastwood et al. 2002), as well as in other astrophysical fluids including stellar winds and the interstellar medium (Armstrong et al. 1995). The magnetohydrodynamic waves are typically generated by ion beams (Shevchenko et al. 2002) and ion temperature anisotropies (Cramer 2001). It is well known that finite-amplitude MHD waves can produce a number of non-linear effects including wave–wave (e.g. Sagdeev and Galeev 1969; Lashmore-Davies and Ong 1974; Hasegawa and Chen 1976) and wave–particle interactions (Shukla and Dawson 1984; Shukla and Stenflo 1999). Of particular interest are the decay and modulational interactions that provide the possibility of energy transfer within the MHD spectra and the localization of MHD wave energies. The linear and non-linear aspects of MHD waves have also been recently described by Cramer (2001).

Recently, Zaqarashvili and Roberts (2002) have shown that the energy of non-electromagnetic forces, which are able to support magnetosonic oscillations, may be transmitted into the energy of purely magnetic oscillations. Possible important astrophysical applications were discussed. In this letter, we show that the coupling can be even more efficient if one considers an alternative process where the polarization directions of the Alfvén waves are *not* necessarily perpendicular to the wavevector of the magnetosonic wave. The results are of interest for the origin of the fast magnetosonic waves that are observed (Eastwood et al. 2002) by the Cluster spacecraft in the terrestrial foreshock.

The interaction between various kinds of magnetosonic and Alfvén waves in a uniform plasma has been studied previously (Brodin and Stenflo 1988, 1990). In particular, considering the resonant interaction between two Alfvén waves $(\omega_1, \mathbf{k}_1)$ and $(\omega_3, \mathbf{k}_3)$ and one fast magnetosonic wave $(\omega_2, \mathbf{k}_2)$, assuming the resonance

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conditions $\omega_3 = \omega_1 + \omega_2$ and $\mathbf{k}_3 = \mathbf{k}_1 + \mathbf{k}_2$ to be satisfied, the governing equations (equation 29 of Brodin and Stenflo 1988) for the wave amplitudes were derived. In the present paper, we limit ourselves to the particular case where $\mathbf{k}_2 = k_{2x}\widehat{\mathbf{x}}$ is perpendicular to the external magnetic field $B_0\widehat{\mathbf{z}}$. Equations (29a,b,c) of Brodin and Stenflo (1988) then reduce to the remarkably simple form

$$d_t A_1^* = i\omega_1 C A_2 A_3^*, \tag{1}$$

$$d_t A_2^* = i\omega_2 \frac{c_A^2}{c_A^2 + c_s^2} C A_1 A_3^* \tag{2}$$

and

$$d_t A_3 = -i\omega_3 C A_2 A_1, \tag{3}$$

where $A_{1,3} = B_{1,3}/B_0$ is the magnitude of the magnetic field perturbation due to the Alfvén wave 1, 3, normalized by B_0, $A_2 = \rho_2/\rho_0$ is the density perturbation due to the magnetosonic wave 2, normalized by the unperturbed density ρ_0, and c_A and c_s are the Alfvén and ion-sound speeds, respectively. The asterisks denote the complex conjugate, and the coupling coefficient C is

$$C = \frac{1}{k_{1\perp}k_{3\perp}}\left(k_{1x}k_{3x} - 3k_y^2\right), \tag{4}$$

where $k_{1,3\perp} = \sqrt{k_{1,3x}^2 + k_y^2}$, with $k_{1y} = k_{3y}$ being denoted by k_y.

Next we regard wave 2 as a pump wave, which means that we now consider A_2 as a constant. From (1) and (3) it is then obvious that the remaining waves grow like $\exp(\Gamma t)$, where the squared growth rate is $\Gamma^2 = -\omega_1\omega_3 C^2|A_2|^2$. As in the present case, due to the resonance conditions, $\omega_3 = -\omega_1 = \omega_2/2$, we can rewrite the growth rate as

$$\Gamma = \frac{\omega_2|C|}{2}|A_2|. \tag{5}$$

The growth rate derived by Zaqarashvili and Roberts (2002) is $\Gamma_{ZR} = \omega_2|A_2|/2$. This agrees with our result, because in Zaqarashvili and Roberts (2002) it was assumed that $k_y = 0$, which in this special case means that $C = 1$. However, by inspecting (4), we note that in general C can be regarded as a function of the two parameters k_{1x} and k_y (as $k_{3x} = k_{1x} + k_{2x}$, where k_{2x} is fixed). The variation of C is shown in Fig. 1. We note that for $k_y \ll k_{1x}$ we have $C \approx 1$. However, the maximum growth rate occurs in the opposite limit $k_{1,3x} \ll k_y$. In this case we have $C \approx -3$, that is $|C| \approx 3$. *The maximum growth rate is thus three times as large as that of Zaqarashvili and Roberts (2002)*. We also note that for some intermediate parameter values $k_{1x} \sim k_y$ it occurs that $C \approx 0$.

A difference between the present three-wave interaction process and the swing wave–wave interaction of Zaqarashvili and Roberts (2002) is that our pump wave is propagating, whereas Zaqarashvili and Roberts use a standing pump. This means that at least one of our Alfvén waves propagates at an angle to $\widehat{\mathbf{z}}$, whereas in Zaqarashvili and Roberts (2002) both Alfvén waves can propagate strictly along $\widehat{\mathbf{z}}$. The three waves considered in the present letter satisfy both the resonance conditions and the wave dispersion relations. Our Alfvén waves propagate consequently almost perpendicular to $\widehat{\mathbf{z}}$ in the limit when both $k_y \gg k_{1,3x}$ and $c_A \gg c_s$. In the more interesting case, namely a high-β plasma ($c_s > c_A$), there is also fast growth ($|C| \approx 3$) at intermediate angles, however. In conclusion, we stress that the present

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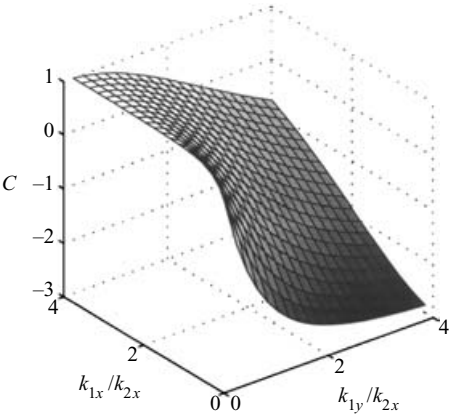


Figure 1. The coupling coefficient C as a function of k_{1x}/k_{2x} and k_y/k_{2x}.

results offer a plausible mechanism for exciting fast magnetosonic waves by Alfvén waves, which are indeed observed (Eastwood et al. 2002) by the Cluster spacecraft in the terrestrial foreshock.

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