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Analytical models of water impact

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Mathematical models for the prediction of the hydrodynamic pressure distribution and the force on a body entering liquid are investigated. Particular attention is paid to analytical models which are based on the velocity potential given by the classical Wagner theory. Formal use of the Wagner theory provides the loads on an entering body, which are higher than the measured ones. To improve the predictions, the higher order terms in the Bernoulli equation are taken into account within the generalized Wagner model and the Logvinovich model. It is shown that the Logvinovich model corresponds better to the experimental data than the generalized Wagner model. A rational derivation of the Logvinovich model is given in the paper for the two-dimensional case. The analytical models are tested against both numerical and experimental results.

1 Introduction

Water impact problems have received considerable attention from both mathematical and engineering sciences, because of their practical importance in several technologies, ship hydrodynamics and ocean engineering. The basis of the applied theory of water impact was created by Wagner [14], who was concerned with the landing/alighting problem of flying boats. The concepts and ideas introduced by Wagner are still in use and have essentially not been changed up to now. Already at the time of Wagner, it was clear that water impact is a complex phenomenon, the analysis of which requires reasonable simplifications; one cannot expect to obtain all details of the fluid flow caused by the impact. There must be a compromise between accuracy of a model and the difficulties associated with obtaining its solution. Wagner developed the theory of impact, which is based on a flat-disc approximation and potential flow theory for ideal and incompressible liquid. Within this approximation the wetted part of the entering body, where the hydrodynamic pressures are very high during the impact, is substituted with a flat disc, the shape of which is unknown in advance and must be determined together with the solution; the boundary conditions and the Bernoulli equation are linearized and the boundary conditions are imposed on the initial position of the liquid boundary (see Howison *et al.* [2] for more details about the Wagner model). This theory can be used for blunt bodies during an initial stage of their entry, when the local deadrise angles in the contact region are small and substitution of the wetted part of the body surface with a flat disc can be justified. The shape of the disc is determined from the so called Wagner condition, which requires continuity of the liquid boundary at any time instant along the contact line between the actual positions of the body surface and the

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liquid free surface. Despite the linear field equations and the linear leading-order boundary conditions, the Wagner model is nonlinear due to the additional condition for the flat disc shape. The Wagner condition was reduced to a transcendental equation in both two-dimensional [4] and axisymmetric [6] cases, which greatly reduced the difficulties connected with analysis of the Wagner problem. Several exact solutions of the three-dimensional Wagner problem were derived by Scolan & Korobkin [11]. All of them are restricted to three-dimensional problems, with the equivalent flat disc being elliptical. Asymptotic analysis of the Wagner problem for almost axisymmetrical bodies was performed by Korobkin & Scolan [6]. The Wagner theory accounts for the important 'piled-up' effect, which is additional wetting of the entering body due to the elevation of the liquid free surface towards the moving body. This effect is pronounced in experiments and the wetting correction evaluated with the help of the Wagner condition is in fairly good agreement with the experimental data in both two-dimensional and axisymmetric cases. The Wagner theory has strong potential to be developed further and to be applied to impact problems with a porous or perforated body surface [9], to problems of impact on compressible liquid [4] and on liquid with an aerated upper layer, and to problems of elastic body impact [5]. The development of the Wagner theory is still in progress. More complex equations of flow and more complex boundary conditions can be analyzed by using advanced mathematical methods.

On the other hand, the classical Wagner model cannot describe some important details of the impact process. Different models of the impact have been proposed depending on the purpose of analysis. It might be expected that global characteristics of the impact, such as the hydrodynamic force on the entering body, can be accurately predicted by using rather simplified models, like that developed by Wagner. Unfortunately, this is not the case. It was clear from the very beginning that nonlinear effects associated with the geometry of the flow region (geometrical nonlinearity is well described by the Wagner condition), the boundary conditions and the Bernoulli equation are important. Wagner introduced the local nonlinear flow analysis close to the periphery of the contact region and provided some ideas about nonlinear effects coming from the Bernoulli equation. The solution of the original Wagner model predicts both unbounded velocities and unbounded pressure in a small vicinity of the contact area periphery. Correction of the Wagner solution taking account of nonlinear effects just in this vicinity was found to be very important. Some corrections were performed with the help of the method of matched asymptotic expansions by Armand & Cointe [1], Howison *et al.* [2] and Oliver [10]. Different asymptotic models, which much better predict the loads on the entering body compared with the original Wagner model, were derived.

Another group of models is based on the idea of improving the Wagner solution in a global way by adding some extra terms to the distribution of the velocity potential in the contact region to make the flow velocity at the periphery of this region bounded. In order to do this, results of local asymptotic analysis close to the contact region periphery are usually used. Among these models the model by Logvinovich [7] is the most famous one. This model predicts the hydrodynamic loads on an entering body, which are almost identical to the measured ones even for moderate penetration depth and for bodies with moderate deadrise angles. However, owing to several severe assumptions underlying the Logvinovich model, it is hard to believe that in its original form this model can be

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mathematically justified. In the present paper the Logvinovich model is rederived by using different reasonings and techniques, which are still not mathematically rigorous but have the potential of being justified in the future. The asymptotic models of impact are more promising from the mathematical point of view but up to now only the leading order uniform asymptotic solution is available. Attempts to derive the second-order asymptotic solution of the two-dimensional impact problem have been made several times in the past but the obtained results are still far from their final forms.

There is also an important group of semi-analytical impact models. These models are based on the so called generalized Wagner approach, where only the boundary conditions on the liquid free surface are simplified. This means that these boundary conditions are linearized and imposed on the horizontal line at the splash-up height, which is calculated by using the Wagner condition. The real shape of the body, the original body boundary conditions and the nonlinear Bernoulli equation are employed. These models received considerable attention after the paper by Zhao *et al.* [17] was published. In this approach the flow domain is known at any time instant if the splash-up height and the penetration depth are prescribed. Zhao *et al.* [17] solved the boundary-value problem at each time instant by the boundary element method and integrated numerically the obtained distribution of the vertical velocity on the free surface to evaluate the shape of the free surface and the splash-up height at the next time instant. Owing to the flow singularity at the intersection points, the nonlinear Bernoulli equation predicts negative and unbounded pressures close to these points. Zhao *et al.* [17] suggested integration of the hydrodynamic pressure distribution only over the part of the wetted surface where the pressure is positive. Within this approach analytical results were obtained by Mei *et al.* [8] for the entry problems of a wedge, a circular cylinder and ship sections of Lewis forms; this is for the problems with known conformal mapping of the flow domain onto a half-plane. For an entering body of general shape, the solution can only be found numerically. Nevertheless, the boundary-value problems of the generalized Wagner approach are much simpler than those within the original formulation, making the approach very attractive in practice. Numerical results by Zhao *et al.* [17] agree well with experiments and numerical simulations of the impact problems for both the slamming force and the pressure distribution. Some disagreements for the analytical results by Mei *et al.* [8] occur for blunt wedges with deadrise angles less than 20 degrees (see Figure 5 from Mei *et al.* [8]) and at a short initial stage of a circular cylinder entry (see Figure 11 from Mei *et al.* [8]). This might be attributed to the fact that the pressure distributions were obtained by Mei *et al.* [8] in analytical form and were integrated over the total wetted region, including small vicinities of the intersection points, where the pressure is negative. However, in general, this approach is more accurate than the Wagner one. This makes it possible to think that the real shape of the entering body is the most important factor affecting the hydrodynamic force. The non-linearity of the Bernoulli equation enters the generalized Wagner approach very naturally due to the variation of the flow domain in time. The hydrodynamic force on the entering body is calculated in the papers by Zhao *et al.* [17] and Mei *et al.* [8] in slightly different ways – negative pressure regions are not taken into account by Zhao *et al.* [17]. Comparison of the hydrodynamic force as a function of the deadrise angle given by the generalized Wagner approach for the wedge entry problem with the similarity solution by Zhao & Faltinsen [16] shows that the

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predictions by Zhao *et al.* [17] are almost identical to the fully non-linear ones for deadrise angles smaller than 20 degrees, and slightly overestimate them for deadrise angles from 20 to 40 degrees; predictions by Mei *et al.* [8] slightly underestimate the fully non-linear hydrodynamic force for deadrise angles smaller than 20 degrees and are almost identical to them for deadrise angles from 20 to 40 degrees. This indicates that the details of the liquid flow and the pressure distribution close to the intersection points may give a reasonable contribution to the total hydrodynamic force. The latter subject was intensively studied by Oliver [10], who derived the first-order correction to the hydrodynamic force with the help of the composite expansion of the pressure distribution in the impact region.

Another semi-analytical model of impact was developed by Vorus [13]. The model is very original. The idea by Vorus was to neglect the geometrical nonlinearity of the impact problem but account for the nonlinear terms in the boundary conditions. The non-linear boundary conditions are imposed on the initial level of the liquid. The nonlinear Bernoulli equation is used to evaluate the pressure on the entering body. Positions of the intersection points are determined from the condition that the hydrodynamic pressure is equal to zero at these points. Separation of the flow near the intersection points is taken into account. The liquid free surface is divided into two parts. The first part is formed by the liquid particles, which escaped onto the free surface from the surface of the entering body. The second part is formed by the liquid particles, which were on the liquid boundary before the impact occurred. Numerical results obtained within the Vorus model by Xu [15] are in good agreement with the experimental ones.

Comparison of the models developed by Zhao *et al.* [17] and Vorus [13] is perhaps confusing, both models essentially improve the classical Wagner approach with respect to predictions of both the pressure distribution and the hydrodynamic force on the entering body. On the other hand, the models are very different. The model by Zhao *et al.* [17] stresses the point that the actual shape of the wetted surface has to be taken into account and along this surface the pressure must be calculated by using the nonlinear Bernoulli equation. The model by Vorus [13] stresses the point that nonlinearity of the free surface boundary conditions, especially close to the intersection points, is more important than the real shape of the flow region; the wetted area can be approximated with a flat disc and on this disc the pressure is calculated using the nonlinear Bernoulli equation. One can see that the common feature in these models is the nonlinear Bernoulli equation, while the other features are rather different. This observation suggests that these different features of the models are not probably so important and can be neglected without great loss of accuracy, but that the hydrodynamic pressure along the wetted area must be evaluated taking account for the nonlinear terms in the Bernoulli equation, with only positive pressures having a physical meaning; this idea is not new and it forms the base of the so called Logvinovich model.

Logvinovich [7] worked within the flat-disc approximation due to Wagner [14]. He recognized that the Wagner solution cannot be used close to the contact points, where the pressure takes its maximum value. The idea was to correct the velocity potential distribution given by Wagner, in order to account to some extent for some important features of the flow and pressure near the periphery of the wetted region. He argued that an additional term has to be added to the velocity potential distribution given by Wagner

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and the Wagner solution has to be used only in a part of the wetted area, where the nonlinear Bernoulli equation predicts positive hydrodynamic pressures. The additional term is evaluated from the condition that the flow velocity at the point of zero pressure is double that of the contact point. The velocity of the contact point is calculated by using the Wagner condition. Logvinovich [7] also discussed in which way the Wagner theory has to be understood and in which way the results of this theory can be used. He argued, in particular, that the results are less accurate if the boundary conditions are imposed at the splash-up height. Logvinovich [7] applied his model to both the wedge and cone entry problems, and showed good agreement of the numerical predictions with experimental results. The model was applied later on by Shorygin [12] to the problem of inclined cone entry and again the theoretical predictions were found to be in good agreement with the experimental data.

However, the Logvinovich model was developed only for the vertical entry of a blunt and symmetric rigid body. It is not clear in which way the velocity potential in the contact region provided by the Wagner approach can be modified to account for either three-dimensional effects or flexibility of the body. Moreover, it is not clear in which way the model by Logvinovich can be justified mathematically. Note that all models described in this section are still far from being justified. The model by Zhao *et al.* [17] is more promising from this point of view. However, even in this model the predicted pressure is unbounded in the impact region and the assumption that only positive pressure matters in calculation of the hydrodynamic force looks to be rather arbitrary. It is hard to believe that this assumption can be justified by introducing an inner small region on the periphery of the contact spot. One may expect that the formal asymptotic analysis of the entry problem by means of the method of matched asymptotic expansions including higher-order terms and matching properly the outer expansion (in the main flow region) with the inner expansion (in the jet region) would be more promising. Up to now even the second-order asymptotic solution of the two-dimensional impact problem has not been obtained. Once the higher-order asymptotic solution would be found, the semi-heuristic models described in this paper could be justified or declined on the basis of this formal asymptotic solution.

In the next section the formulation of the two-dimensional entry problem is given together with the discussion of the Bernoulli equation. In § 3 approximate formulae for the pressure distribution in the contact region and the hydrodynamic force acting on the symmetric entering body are derived in such a way that generalization of the obtained results to the three-dimensional case and to elastic body impact are obvious. Numerical results are collected in § 4 and § 5, where different analytical models are compared to each other, to experimental data and to numerical results obtained within fully nonlinear potential theory. It is shown that the Logvinovich model provides quite reasonable predictions of the hydrodynamic loads on an entering body and can be recommended for practical use.

2 Formulation of the problem

In this section a general formula for the two-dimensional pressure distribution along the wetted part of a blunt body entering liquid vertically is derived. The liquid is assumed

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ideal and incompressible. The body is symmetric and blunt. The liquid flow is assumed plane, symmetric and potential. The liquid region is of infinite depth. Initially the liquid is at rest, its free surface is horizontal ($y=0$) and the body touches the free surface at a single point ($x=0$) taken as the origin of the Cartesian coordinate system Oxy. The position of the body is described by the equation $y=f(x)-h(t)$, where $h(t)$ is the prescribed penetration depth and the function $f(x)$ describes the body shape, $f(-x)=f(x)$, $f(0)=0$. The velocity potential $\varphi(x,y,t)$ of the flow originated by the entering contour satisfies the Laplace equation in the flow domain, the kinematic and dynamic boundary conditions on the liquid free surface and the body boundary condition on the entering contour. It is important to notice that the geometry of the flow region is unknown in advance and must be determined as a part of the solution. Gravity effects and surface tension are neglected in the present analysis. We shall obtain the pressure distribution along the contact region and the hydrodynamic force acting on an entering contour. Only the initial stage of the impact is considered, when the penetration depth $h(t)$ is small compared with the horizontal dimension of the wetted part of the body $D(t)$. The latter is possible if and only if the deadrise angle of the body in the contact region is small.

To evaluate the pressure distribution $P(x,t)$ along the wetted part $D(t)$ of the blunt body during the initial stage, we assume that both the velocity potential $\phi(x,t)$ in the contact region, where $\phi(x,t)=\varphi(x,f(x)-h(t),t)$, and the dimension of this region are known. The hydrodynamic pressure $p(x,y,t)$ in the flow domain is given by the Cauchy-Lagrange integral

$$p(x,y,t)=-\rho\left(\varphi_t+\frac{1}{2}|\nabla\varphi|^2\right), \tag{2.1}$$

where ρ is the liquid density. By using the definitions

$$P(x,t)=p(x,f(x)-h(t),t), \tag{2.2}$$

$$\phi(x,t)=\varphi(x,f(x)-h(t),t) \tag{2.3}$$

and the boundary condition on the body surface

$$\varphi_y=\varphi_x f'(x)-\dot{h}(t), \tag{2.4}$$

we obtain

$$P(x,t)=-\rho\left[\phi_t+\frac{f'(x)\dot{h}}{1+f_x^2}\phi_x+\frac{1}{2}\frac{\phi_x^2-\dot{h}^2}{1+f_x^2}\right]. \tag{2.5}$$

The dot stands for the time derivative. Note that this is $\phi(x,t)$ which is calculated within the fully nonlinear potential theory by the boundary element method (see Iafrati & Battistin [3]). For a wedge entering liquid at a constant velocity V, we have $f(x)=|x|\tan\gamma$, where γ is the wedge deadrise angle, and $h(t)=Vt$; equation (2.5) then takes the form

$$P(x,t)=-\rho\left[\phi_t+\frac{1}{2}V\sin(2\gamma)\phi_x+\frac{1}{2}\cos^2\gamma\left(\phi_x^2-V^2\right)\right]. \tag{2.6}$$

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The hydrodynamic force $F(t)$ acting on the entering contour is given by the integral

$$F(t) = \int_{D(t)} P(x,t)\,dx. \tag{2.7}$$

Equations (2.5) and (2.7) are exact within the potential flow theory. The equations require the function $\phi(x,t)$, which can be numerically evaluated at each time step. On the other hand, it would be helpful to obtain approximately the distribution $\phi(x,t)$, to evaluate analytically both the pressure $P(x,t)$ and the force $F(t)$ on the basis of this approximation and finally to compare the approximate predictions with both numerical and experimental results. Such an analysis may be of help for future more rigorous studies, with the aim of quantifying the main contributions to the dynamic characteristics of impact and to derive approximate formulae suitable for practical use.

3 Asymptotic analysis

To evaluate approximately the distribution of the velocity potential $\phi(x,t)$ in the contact region and the dimension of this region, we consider the initial stage of impact, during which the horizontal size of the wetted part of the entering blunt contour grows starting from zero, but is less than a prescribed quantity $2L$. The penetration depth h at the initial stage is of the order of $f(L)$ and is small compared to the horizontal size of the contact region L for blunt bodies. The ratio $\varepsilon = f(L)/L$ is the small parameter in the present asymptotic study. Asymptotic analysis of the flow and the pressure distribution at the initial stage of the impact has been performed in detail by Howison *et al.* [2] and Oliver [10] among others. The main purpose of the present study is not to improve the asymptotic results but to show the asymptotic origins of the analytical models discussed in § 1.

We introduce the non-dimensional variables, which are denoted by the same symbols as the original variables but with a bar. The quantity L is taken as the length scale, εL as the displacement scale, $\varepsilon L/V_0$ as the time scale, where $V_0 = h'(0)$ is the scale of the flow velocity, the scale of the hydrodynamic pressure is $\rho V_0^2/\varepsilon$ and the scale of the velocity potential is LV_0. The non-dimensional velocity potential $\bar{\varphi}(\bar{x},\bar{y},\bar{t})$ satisfies the following equations:

$$\Delta\bar{\varphi} = 0 \qquad (\text{in} \quad \bar{\Omega}(\bar{t})), \tag{3.1}$$

$$\frac{\partial\bar{\varphi}}{\partial\bar{t}} + \varepsilon\frac{1}{2}|\nabla\bar{\varphi}|^2 = 0, \tag{3.2}$$

$$\frac{\partial\bar{\eta}}{\partial\bar{t}} + \varepsilon\frac{\partial\bar{\eta}}{\partial\bar{x}}\frac{\partial\bar{\varphi}}{\partial\bar{x}} = \frac{\partial\bar{\varphi}}{\partial\bar{y}} \qquad (\bar{y} = \varepsilon\bar{\eta}(\bar{x},\bar{t})), \tag{3.3}$$

$$\frac{\partial\bar{\varphi}}{\partial\bar{y}} = \varepsilon\bar{f}'(\bar{x})\frac{\partial\bar{\varphi}}{\partial\bar{x}} - \frac{\partial\bar{h}}{\partial\bar{t}} \qquad (\bar{y} = \varepsilon[\bar{f}(\bar{x}) - \bar{h}(\bar{t})]), \tag{3.4}$$

$$\bar{\varphi} \to 0 \qquad (\bar{x}^2 + \bar{y}^2 \to \infty), \tag{3.5}$$

where $h = \varepsilon L\bar{h}$, $\bar{f}(\bar{x}) = f(\bar{x}L)/(\varepsilon L)$ and the equation $y = \varepsilon L\bar{\eta}(x/L, tV_0/(\varepsilon L))$ describes the

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free surface shape. The function $\bar{\eta}(\bar{x},\bar{t})$ may be a multivalued function of $\bar{x}$. Equation (2.1) takes in the non-dimensional variables the form

$$\bar{p}(\bar{x},\bar{y},\bar{t}) = -\left(\frac{\partial\bar{\varphi}}{\partial\bar{t}} + \varepsilon\frac{1}{2}|\nabla\bar{\varphi}|^2\right). \tag{3.6}$$

The outer solution of the boundary-value problem (3.1)–(3.5) is formally sought in the form

$$\bar{\varphi}(\bar{x},\bar{y},\bar{t}) = \bar{\varphi}^{(w)}(x_1,y_1,\bar{t}) + \varepsilon\bar{\varphi}_1(x_1,y_1,\bar{t}) + O(\varepsilon^2), \tag{3.7}$$

$$\bar{\eta}(\bar{x},\bar{t}) = \bar{\eta}^{(w)}(x_1,\bar{t}) + \varepsilon\bar{\eta}_1(x_1,\bar{t}) + O(\varepsilon^2), \tag{3.8}$$

where $x_1 = \bar{x}$ and the new vertical variable $y_1 = \bar{y} - \varepsilon\bar{d}(\bar{t})$ has been introduced, in order to include into the analysis the ideas of the generalized Wagner approach. In this approach the linearization is performed at the splash-up height but not at the initial level of the liquid as in the classical Wagner theory, where $\bar{d}(\bar{t}) = 0$. The leading order terms are designated in the asymptotic formulae (3.7) and (3.8) with the subscript w in order to indicate that in the leading order as $\varepsilon \to 0$ the velocity potential is given by the Wagner approach.

Equation (2.3) provides

$$\begin{aligned}\phi(x,t) &= V_0L\bar{\varphi}(\bar{x},\varepsilon[\bar{f}(\bar{x}) - \bar{h}(\bar{t})],\bar{t})\\ &= V_0L[\bar{\varphi}^{(w)}(x_1,\varepsilon[\bar{f}(\bar{x}) - \bar{h}(\bar{t}) - \bar{d}(\bar{t})],\bar{t}) + \varepsilon\bar{\varphi}_1(x_1,\varepsilon[\bar{f}(\bar{x}) - \bar{h}(\bar{t}) - \bar{d}(\bar{t})],\bar{t}) + O(\varepsilon^2)].\end{aligned} \tag{3.9}$$

We neglect in equation (3.9) the term $\varepsilon\bar{\varphi}_1(x_1,\varepsilon[\bar{f}(\bar{x}) - \bar{h}(\bar{t}) - \bar{d}(\bar{t})],\bar{t})$, the higher order terms, which are represented in this equation by $O(\varepsilon^2)$, and expand the term $\bar{\varphi}^{(w)}(x_1,\varepsilon[\bar{f}(\bar{x}) - \bar{h}(\bar{t}) - \bar{d}(\bar{t})],\bar{t})$ around $y_1 = 0$ as

$$\bar{\varphi}^{(w)}(x_1,\varepsilon[\bar{f}(\bar{x}) - \bar{h}(\bar{t}) - \bar{d}(\bar{t})],\bar{t}) = \bar{\varphi}^{(w)}(x_1,0,\bar{t}) + \varepsilon\frac{\partial\bar{\varphi}^{(w)}}{\partial y_1}(x_1,0,\bar{t})[\bar{f}(\bar{x}) - \bar{h}(\bar{t}) - \bar{d}(\bar{t})] + O(\varepsilon^2). \tag{3.10}$$

Substituting expansion (3.7) into the body boundary condition (3.4) and taking the leading order part of this condition as $\varepsilon \to 0$, we obtain

$$\frac{\partial\bar{\varphi}^{(w)}}{\partial y_1}(x_1,0,\bar{t}) = -\frac{\partial\bar{h}}{\partial\bar{t}}. \tag{3.11}$$

Equations (3.10) and (3.11) yield

$$\bar{\varphi}^{(w)}(x_1,\varepsilon[\bar{f}(\bar{x}) - \bar{h}(\bar{t}) - \bar{d}(\bar{t})],\bar{t}) = \bar{\varphi}^{(w)}(x_1,0,\bar{t}) - \varepsilon\frac{\partial\bar{h}}{\partial\bar{t}}[\bar{f}(\bar{x}) - \bar{h}(\bar{t}) - \bar{d}(\bar{t})] + O(\varepsilon^2). \tag{3.12}$$

Equations (3.9) and (3.12) lead to the approximate formula

$$\phi(x,t) \approx V_0L\left\{\bar{\varphi}^{(w)}(x_1,0,\bar{t}) - \varepsilon\frac{\partial\bar{h}}{\partial\bar{t}}[\bar{f}(\bar{x}) - \bar{h}(\bar{t}) - \bar{d}(\bar{t})]\right\}. \tag{3.13}$$

Formally speaking, the latter formula is not asymptotic, because the first order term $\varepsilon\bar{\varphi}_1(x_1,\varepsilon[\bar{f}(\bar{x}) - \bar{h}(\bar{t}) - \bar{d}(\bar{t})],\bar{t})$ was neglected in (3.9) without any justification. However,

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equation (3.13) provides the leading order asymptotics of the function $\phi(x,t)$, which is a trivial observation. Equation (3.13) can be turned to the first order asymptotic formula if we add the term $V_0 L\varepsilon\bar{\varphi}_1(x_1,0,\bar{t})$ to its right-hand side. However, with the latter term the equation becomes rather complicated because one needs to obtain the solution of the impact problem up to the second order and evaluate the function $\bar{\varphi}_1(x_1,0,\bar{t})$ in the contact region. The second-order analysis of the impact problem has not been performed yet.

Equation (3.13) is used below for approximate evaluation of the pressure distribution in the contact region. Formally speaking, we cannot expect that this equation correctly describes the velocity potential close to the periphery of the contact region, where the non-linear effects are of major importance. Correspondingly, the pressure distribution evaluated with the help of (3.13) and (2.5), as well as the hydrodynamic force given by (2.7), could be rather different from their exact values. Fortunately, the following analysis shows that this is not the case.

In the original dimensional variables equation (3.13) takes the form

$$\phi(x,t) \approx \varphi^{(w)}(x,0,t) - \dot{h}(t)[f(x) - h(t) - d(t)], \tag{3.14}$$

where $\varphi^{(w)}(x,y,t)$ is the solution of the classical Wagner problem

$$\Delta\varphi^{(w)} = 0 \qquad (y < 0), \tag{3.15}$$

$$\varphi^{(w)} = 0 \qquad (y = 0, |x| > c(t)), \tag{3.16}$$

$$\varphi^{(w)}_y = -\dot{h}(t) \qquad (y = 0, |x| < c(t)), \tag{3.17}$$

$$\varphi^{(w)} \to 0 \qquad (x^2 + y^2 \to \infty). \tag{3.18}$$

Within the Wagner approach the contact region $D(t)$ corresponds to the interval $-c(t) < x < c(t)$, $y = 0$, where the function $c(t)$ is determined with the help of the so called Wagner condition [2, 10]. The Wagner condition can be written as (see Korobkin [4])

$$\int_0^{\pi/2} f[c(t)\sin\theta]d\theta = \frac{\pi}{2}h(t). \tag{3.19}$$

It should be noted that the dimension $2c(t)$ of the contact region is independent of the function $d(t)$. In the case of a wedge equation (3.19) gives

$$c(t) = \frac{\pi h(t)}{2\tan\gamma}.$$

The solution of the boundary-value problem (3.15)–(3.18) is well-known (see Howison *et al.* [2] and Oliver [10] for details). The solution provides the velocity potential in the contact region as

$$\varphi^{(w)}(x,0,t) = -\dot{h}(t)\sqrt{c^2(t) - x^2} \qquad (|x| < c(t)). \tag{3.20}$$

Equations (3.14) and (3.20) yield

$$\phi(x,t) \approx -\dot{h}(t)\sqrt{c^2(t) - x^2} - \dot{h}(t)[f(x) - h(t) - d(t)]. \tag{3.21}$$

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This is the approximate formula, which is used to evaluate the right-hand side of equation (2.5). By substituting (3.21) into (2.5), we obtain

$$P(x,t)=\rho\dot{h}^2(t)\left[\frac{\dot{c}}{\dot{h}}\frac{c}{\sqrt{c^2-x^2}}-\frac{1}{2}\frac{c^2}{c^2-x^2}\frac{1}{1+f_x^2}\right.$$
$$\left.-\frac{1}{2}\frac{f_x^2}{1+f_x^2}-\frac{\dot{d}}{\dot{h}}+\frac{\ddot{h}}{\dot{h}^2}(\sqrt{c^2-x^2}+f(x)-h(t)-d(t))\right]. \tag{3.22}$$

The approximate pressure distribution (3.22) is not integrable in the contact region owing to the second term. This defect is due to improper modelling of the flow close to the periphery of the contact region. Small vicinities of the contact points have to be considered in more details and both the flow and the pressure distribution there have to be corrected by using asymptotic methods (see Howison *et al.* [2]). Another approach to evaluation of the hydrodynamic force by using an equation similar to (3.22) was suggested by Wagner [14]. His idea was to use the formula for the hydrodynamic pressure only there, where $P(x,t)>0$. In this approach the inner regions around the contact points do not matter. It is not clear at present how this approach can be justified but the approach indeed provides rather good predictions of both the hydrodynamic force on the entering body and the pressure distribution. It should be noted that different authors derived the non-linear equations for the hydrodynamic pressure in different forms. Logvinovich (1969) did not account for the body shape in the Bernoulli equation. His formula for the pressure distribution can be derived from (3.22) if one takes $f(x)\equiv 0$ in this formula. The formula used by Wagner [14] is rather different from (3.22). According to Wagner, one need only to substitute equation (3.20) into (2.1).

We consider two main possibilities for the function $d(t)$. The first one, $d(t)\equiv 0$, corresponds to the linearization of the original problem around the initial liquid level, $y=0$. The second one, $d(t)=f[c(t)]-h(t)$, corresponds to the linearization at the splash-up height. The second choice is used within the generalized Wagner approach (see Mei *et al.* [8]), but in the present analysis we do not account for the real shape of the body in calculations of the velocity potential. Note that $\dot{d}/\dot{h}=0$ in the first case and $\dot{d}/\dot{h}=f'(c)\dot{c}/\dot{h}-1$ in the second one.

To demonstrate the main developments, the problem of a wedge penetrating liquid at constant velocity V is considered. In this case equation (3.22) provides

$$P(x,t)=\frac{1}{2}\rho V^2\left[\frac{\pi}{\tan\gamma}\frac{c}{\sqrt{c^2-x^2}}-\cos^2\gamma\frac{c^2}{c^2-x^2}-\sin^2\gamma-2\frac{\dot{d}}{\dot{h}}\right]. \tag{3.23}$$

The Original Logvinovich Model (OLM) is obtained from (3.23) if we neglect the second order terms with respect to small deadrise angle γ and take $d(t)\equiv 0$. In this model the pressure in the contact region is given as

$$P(x,t)=\frac{1}{2}\rho V^2\left[\frac{\pi}{\tan\gamma}\frac{c}{\sqrt{c^2-x^2}}-\frac{c^2}{c^2-x^2}\right]. \tag{3.24}$$

The modified Logvinovich model (MLM) is obtained from (3.23) if we take $d(t)\equiv 0$ but keep the second order terms with respect to the wedge deadrise angle γ. In this model the

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pressure in the contact region is given as

$$P(x,t)=\frac{1}{2}\rho V^2\left[\frac{\pi}{\tan\gamma}\frac{c}{\sqrt{c^2-x^2}}-\cos^2\gamma\frac{c^2}{c^2-x^2}-\sin^2\gamma\right]. \tag{3.25}$$

The Generalized Wagner Model (GWM) in combination with the flat-disc approximation is obtained from (3.23) if we take $d(t)=f[c(t)]-h(t)$, but keep the second order terms with respect to the wedge deadrise angle γ. For the wedge entry problem $d(t)=(\pi/2-1)h(t)$ and $\dot{d}/\dot{h}=\pi/2-1$. In this model the pressure in the contact region is given as

$$P(x,t)=\frac{1}{2}\rho V^2\left[\frac{\pi}{\tan\gamma}\frac{c}{\sqrt{c^2-x^2}}-\cos^2\gamma\frac{c^2}{c^2-x^2}-\sin^2\gamma+2-\pi\right]. \tag{3.26}$$

Due to the second term in (3.23), the obtained pressure distribution tends to minus infinity as $x\to\pm c(t)$. The pressure is positive on the interval $-a(t)<x<a(t)$, where the function $a(t)$ is the solution of the equation $P[a(t),t]=0$. It is convenient to define the ratio $\xi=a(t)/c(t)$ and to solve the latter equation with respect to ξ. We obtain

$$a(t)=\xi c(t),\qquad \xi=\sqrt{1-X^2}, \tag{3.27}$$

where X is given by different formulae for different models

$$X=\frac{2\tan\gamma}{\pi}\qquad\text{(OLM)}, \tag{3.28}$$

$$X=\frac{\sin(2\gamma)}{\pi[1+\sqrt{1-4\pi^{-2}\sin^4\gamma}]}\qquad\text{(MLM)}, \tag{3.29}$$

$$X=\frac{\sin(2\gamma)}{\pi[1+\sqrt{1-4\pi^{-2}\sin^2\gamma(\sin^2\gamma+\pi-2)}]}\qquad\text{(GWM)}. \tag{3.30}$$

It can be seen that for small γ formulae (3.28)–(3.30) are asymptotically equivalent to each other. However, we are going to use equations (3.24)–(3.26), which were derived for small deadrise angles, for moderate angles, where differences between quantities defined by equations (3.28)–(3.30) are not necessary small.

We may expect that the obtained formulae for the pressure distribution provide reasonable predictions if the length of the negative pressure interval, $c(t)-a(t)$, is small compared to $c(t)$. Some limitations are obvious from equations (3.27) and (3.28). Equation (3.27) shows that X must be smaller than unity and then (3.28) yields $\gamma<\arctan(\pi/2)$, which is the wedge deadrise angle is smaller than 57 degrees. However, the negative pressure interval is smaller than 10% of the total contact region only for the wedge deadrise angle smaller than 35 degrees. This is the limit, within which the predictions by the Original Logvinovich Model (OLM) will be compared to both numerical and experimental results. Two other models, MLM and GWM, have no such limits and may be formally used

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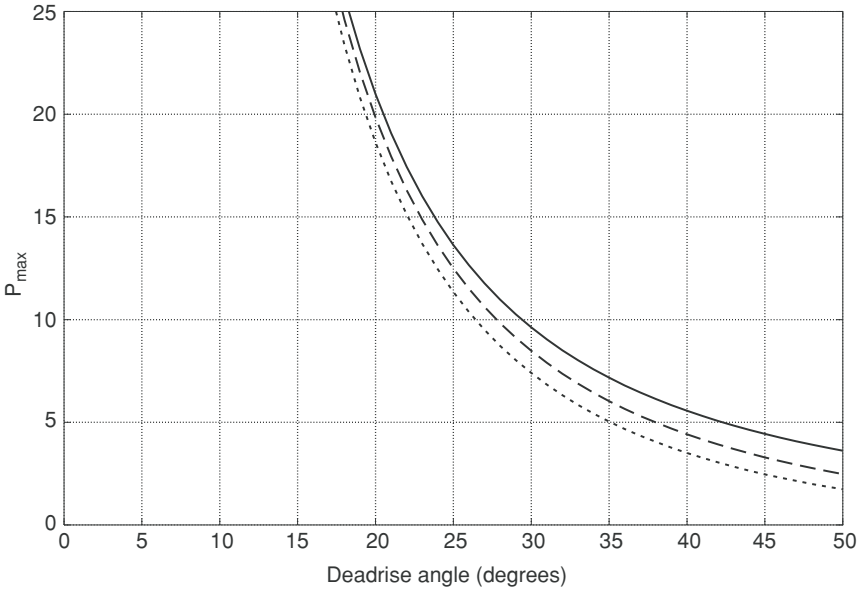


FIGURE 1. Maximum non-dimensional pressure $P_{max}/\frac{1}{2}\rho V^2$.

for any deadrise angles even if the formulae were derived under the small deadrise angle assumption.

4 Validation of the analytical models

The maximum pressure $P_{max}(t)$ in the contact region between the liquid and the entering wedge can be obtained from equations (3.24)–(3.26) as

$$P_{max} = \frac{1}{2}\rho\dot{c}^2(t) \qquad \text{(OLM)}, \tag{4.1}$$

$$P_{max} = \frac{1}{2}\rho\dot{c}^2(t)\left[\cos^{-2}\gamma - \frac{V^2}{\dot{c}^2}\sin^2\gamma\right] \qquad \text{(MLM)}, \tag{4.2}$$

$$P_{max} = \frac{1}{2}\rho\dot{c}^2(t)\left[\cos^{-2}\gamma - \frac{V^2}{\dot{c}^2}(\sin^2\gamma + \pi - 2)\right] \qquad \text{(GWM)}. \tag{4.3}$$

The maximum non-dimensional pressure $P_{max}/\frac{1}{2}\rho V^2$ is independent of time and is depicted in Figure 1 as a function of the wedge deadrise angle γ. Within the classical Wagner approach the maximum pressure is calculated with the help of the method of matched asymptotic expansions and is equal to $\frac{1}{2}\rho\dot{c}^2(t)$, which is identical to (4.1) provided by the OLM. In Figure 1 the solid line corresponds to the MLM, the dashed line to GWM and the dotted line to the OLM. Numerical analysis by Iafrati & Battistin [3] for the wedge with deadrise angle of 20 degrees entering the liquid at constant velocity provides the maximum non-dimensional pressure about 17.0 and the analytical values by the three

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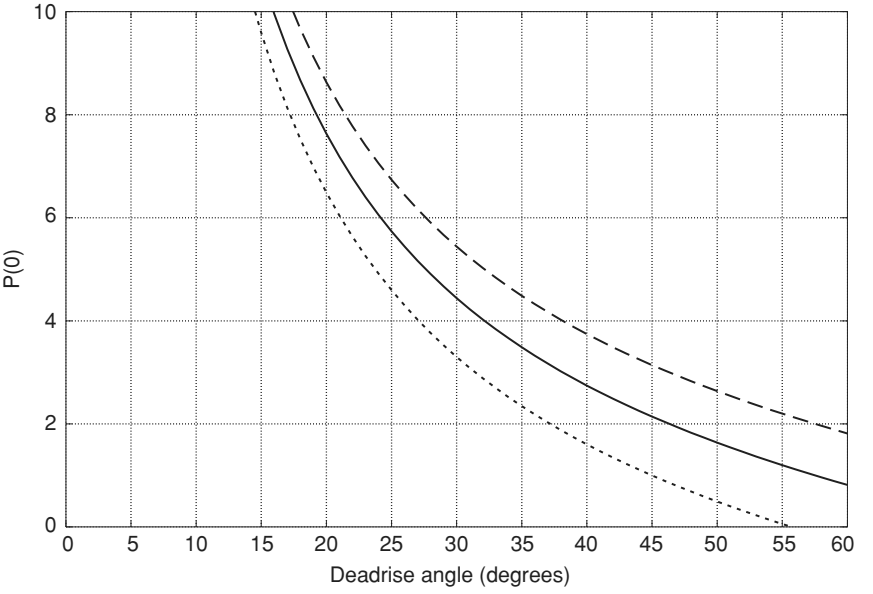


FIGURE 2. Non-dimensional pressure $P(0,t)/\frac{1}{2}\rho V^2$ at the centre of the contact region.

models under consideration are equal to 18.6 (OLM), 19.8 (GWM) and 21(MLM). Both the OLM and the Wagner asymptotic solution provide the best fit to the numerical result.

The non-dimensional pressure $P(0,t)/\frac{1}{2}\rho V^2$ at the centre of the contact region is independent of time and is depicted in Figure 2 as a function of the wedge deadrise angle γ. The models under consideration provide

$$P(0,t) = \frac{1}{2}\rho V^2\left[\frac{\pi}{\tan\gamma} - 1\right] \qquad \text{(OLM and MLM)}, \tag{4.4}$$

$$P(0,t) = \frac{1}{2}\rho V^2\left[\frac{\pi}{\tan\gamma} + 1 - \pi\right] \qquad \text{(GWM)}. \tag{4.5}$$

In Figure 2 the solid line corresponds to the MLM, the dashed line to the classical Wagner model and the dotted line to the GWM. Numerical analysis by Iafrati & Battistin [3] for a wedge with deadrise angle of 20 degrees entering the liquid at constant velocity provides the non-dimensional pressure at the wedge tip of about 7.6 and the analytical values by the three models under consideration are equal to 7.6 (OLM and MLM) and 6.5 (GWM). The classical Wagner approach gives 8.6.

The hydrodynamic force $F(t)$ acting on the entering wedge is given within the approaches under consideration by the formula

$$F(t) = 2\int_0^{a(t)} P(x,t)\,dx. \tag{4.6}$$

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Equations (3.24)–(3.26) yield

$$F(t) = \rho V^2 c(t)\left[\frac{\pi^2}{2\tan\gamma} - K(\gamma)\right], \tag{4.7}$$

where $K(\gamma) = 0$ in the classical Wagner approach,

$$K(\gamma) = \frac{\pi}{\tan\gamma}\left(\frac{\pi}{2} - \arcsin\xi\right) + \frac{1}{2}\ln\left[\frac{1+\xi}{1-\xi}\right] \qquad \text{(OLM)}, \tag{4.8}$$

$$K(\gamma) = \frac{\pi}{\tan\gamma}\left(\frac{\pi}{2} - \arcsin\xi\right) + \frac{1}{2}\cos^2\gamma\ln\left[\frac{1+\xi}{1-\xi}\right] + \xi\sin^2\gamma \qquad \text{(MLM)}, \tag{4.9}$$

$$K(\gamma) = \frac{\pi}{\tan\gamma}\left(\frac{\pi}{2} - \arcsin\xi\right) + \frac{1}{2}\cos^2\gamma\ln\left[\frac{1+\xi}{1-\xi}\right] + \xi(\sin^2\gamma + \pi - 2) \qquad \text{(GWM)}. \tag{4.10}$$

Note that in (4.8)–(4.10) the quantity ξ is calculated by using (3.28)–(3.30), respectively. The function $K(\gamma)$ describes the nonlinear effects, but only those which contribute to the hydrodynamic pressure. The nonlinear effects due to both the nonlinear terms in the boundary conditions and the variation of the flow domain geometry in time are not taken into account in (4.7).

For a finite wedge with the width of its upper base B the maximum of the hydrodynamic force occurs at the time instant t_m, when the jet root escapes on the free surface, $c(t_m) = B/2$. In the non-dimensional variables we obtain

$$\tau_m = Vt_m/B = \frac{1}{\pi}\tan\gamma, \tag{4.11}$$

$$F_{max} = F(t_m)/\left(\frac{1}{2}\rho V^2 B\right) = \frac{\pi^2}{2\tan\gamma} - K(\gamma), \tag{4.12}$$

where $K(\gamma)$ is defined by equations (4.8)–(4.10) for the different models.

It should be noted that the non-dimensional quantities τ_m and F_{max} depend only on the wedge deadrise angle γ and the non-dimensional time τ_m is equal to that obtained within the classical Wagner theory. The theoretical predictions are compared to the numerical results by Iafrati & Battistin [3] for wedges with deadrise angles of 20, 30 and 60 degrees. In Figure 3 the function $F_{max}(\gamma)$ is shown for different models. Curve 1 corresponds to the classical Wagner approximation and the numerical results by Iafrati & Battistin [3] are shown with crosses. It is seen that the Wagner theory overestimates the maximum hydrodynamic force, but correctly describes the tendency of the force to decay with the deadrise angle increase. Curve 2 is obtained for the MLM with the help of equations (3.29) and (4.9). This curve is rather close to the numerical results and can be used up to the deadrise angle of 60 degrees. The dotted curve shows the hydrodynamic force predicted by the OLM for deadrise angles smaller than 40 degrees. It is seen that the predictions by the OLM are almost identical to the numerical ones. The results obtained within the GWM are shown with curve 3, which is slightly lower than the numerical values.

We may conclude that the analytical models considered in this paper and based on linearization about the initial liquid level, $d(t) = 0$, provide reasonable predictions of both

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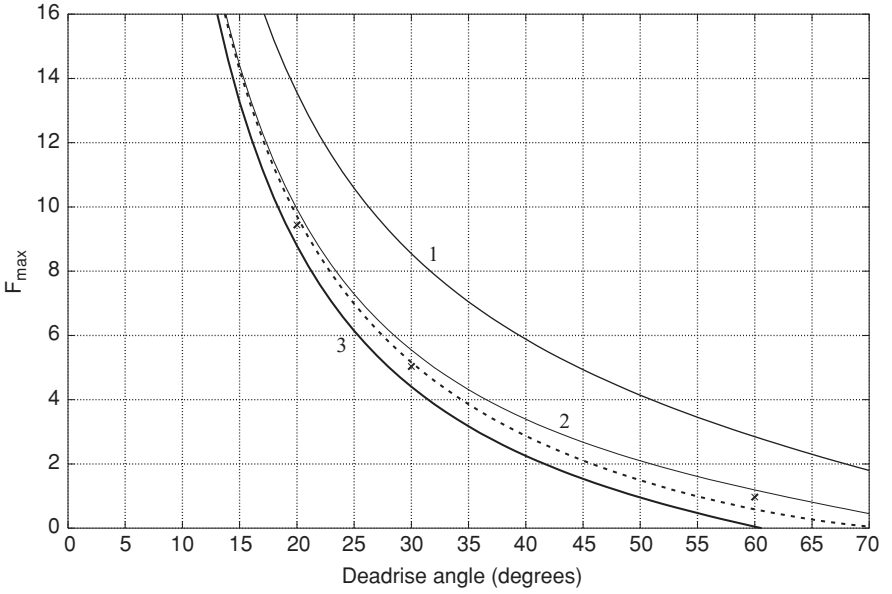


FIGURE 3. Function $F_{max}(\gamma)$ for different models.

the hydrodynamic pressure in the contact region and the hydrodynamic force acting on the entering wedge.

5 Water entry of circular cylinder

In this section we consider a circular cylinder with radius R entering liquid at constant velocity V. The original Logvinovich model is used to evaluate the hydrodynamic force on the entering cylinder. The analytical results are compared with the experimental ones by Armand & Cointe [1].

The shape function of the cylinder is given by the equation $f(x) = R - \sqrt{R^2 - x^2}$. By substituting this equation into the Wagner condition (3.19), we obtain

$$\tau = 1 - \frac{2}{\pi}\mathrm{E}(\alpha), \tag{5.1}$$

where $\tau = Vt/R$ is the non-dimensional time, $\alpha = c(t)/R$ is considered as a parameter changing from zero to unity, $c(t)$ is the horizontal coordinate of the contact point, $\mathrm{E}(\alpha)$ is the complete elliptic integral of the second kind. Time differentiation of equation (5.1) provides the speed of the contact point in the parametric form

$$\frac{\dot{c}}{V} = -\frac{\pi}{2\mathrm{E}'(\alpha)}, \qquad \mathrm{E}'(\alpha) = [\mathrm{E}(\alpha) - \mathrm{K}(\alpha)]/\alpha, \tag{5.2}$$

where $\mathrm{K}(\alpha)$ is the complete elliptic integral of the first kind. The quantity ξ introduced by (3.27) is given within the OLM as

$$\xi(\alpha) = \sqrt{1 - \pi^{-2}[\mathrm{E}'(\alpha)]^2} \tag{5.3}$$

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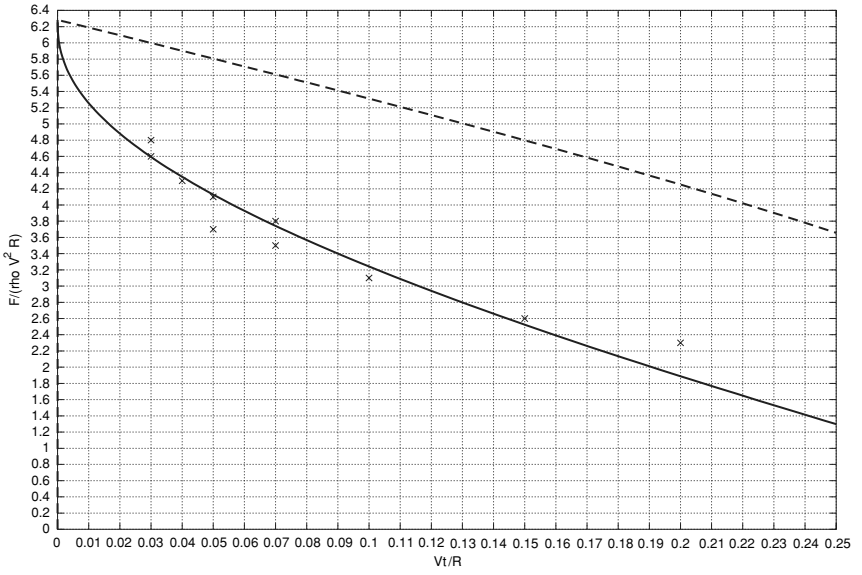


FIGURE 4. Force predicted by the OLM (solid line); prediction by the classical Wagner model (dashed line).

and the non-dimensional hydrodynamic force $F(t)$ is calculated in parametric form by using equations (4.7) and (4.8), which yield

$$\frac{F}{\rho V^2 R} = -\alpha\left[\frac{\pi}{\mathrm{E}'(\alpha)}\arcsin\{\xi(\alpha)\} + \frac{1}{2}\ln\left(\frac{1+\xi(\alpha)}{1-\xi(\alpha)}\right)\right]. \tag{5.4}$$

It can be seen that the OLM is not so complicated compared to the classical Wagner theory. To obtain the Wagner formula for the force, we need to replace $\arcsin\{\xi(\alpha)\}$ by $\frac{\pi}{2}$ and delete the log-term in (5.4). In Figure 4 the force predicted by the OLM is shown by the solid line and the prediction by the classical Wagner model is shown by dashed line. Experimental results by Armand & Cointe [1] are shown with crosses. The solid line in Figure 4 can be approximated by the analytical formula

$$\frac{F}{\rho V^2 R} = 6.28 + \sqrt{\tau}[0.84\ln\tau - 5.72] - 6.13\tau \tag{5.5}$$

with absolute error less than 0.2%. Equation (5.5) can be used in practical calculations.

Figure 4 demonstrates that the correction for non-linear terms in the Bernoulli equation in combination with the flat-disc approximation provides a reasonable prediction of the hydrodynamic force acting on an entering circular cylinder.

6 Conclusion

The derivations of the models OLM and MLM given in this paper are rational and can be generalized to the three-dimensional impact problem provided the corresponding Wagner problem has been solved and both the geometry of the contact region and the

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distribution of the velocity potential over this region are known. Scolan & Korobkin [11] derived several exact solutions of the Wagner problem with elliptic contact lines. The obtained solutions can be used together with OLM and MLM to improve the theoretical predictions of the hydrodynamic loads. In order to do this, one needs to repeat the analysis of § 2 and § 3 for a three-dimensional body, to replace equation (3.20) with the formula for the velocity potential distribution in an elliptic contact region (see Scolan & Korobkin [11]), to evaluate the pressure and distinguish the part of the contact region, where the hydrodynamic pressure is positive. Generally speaking, the region of positive pressure is not elliptic. Then one needs to integrate the pressure over the region, where it is positive, to find the total hydrodynamic force on the three-dimensional entering body. In the case of a free falling body both the pressure distribution and the geometry of the positive pressure region are strongly dependent on the body acceleration, which itself has to be determined as a part of the solution. The latter problem is much more complicated than that of body entry at constant velocity owing to the fact that the hydrodynamic force is a nonlinear function of the body acceleration. It is expected that the problem of free falling body can be effectively solved by iterations at each time step.

Similar analysis can be also performed for two-dimensional problems of elastic body impact onto the liquid free surface. In this problem the shape function $f(x,t)$ is dependent on elastic deflections of the body surface and, therefore, the equation for the pressure will be more complicated than equation (2.5) derived for a rigid body. Equation (3.20) for the velocity potential in the contact region will be also more complicated than in the rigid body case (see Korobkin & Khabakhpasheva [5] for details). However, the main features of the algorithm for computing the pressure distribution and the hydrodynamic force are essentially the same as for a rigid body problem. By using the modified Wagner theory presented in this paper improvement of theoretical predictions of the stress distribution in the elastic body during the impact stage is expected.

It should be noted that the OLM is well known and has been intensively tested for wedges and cones (see Logvinovich [7]). It was found that this model can be safely used for these bodies if their deadrise angles are smaller than 30 degrees. Figure 3 shows that accounting for the body geometry in the Bernoulli equation may improve the predictions of the hydrodynamic force for moderate deadrise angles. This observation suggests that the Modified Logvinovich Model should be perspective to be used in practical calculations of impact loads acting on an arbitrary blunt body.

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