J. Plasma Physics (2009), *vol.* 75, *part* 5, *pp.* 587–591. © 2009 Cambridge University Press
doi:10.1017/S0022377809990110 Printed in the United Kingdom

A note on the trapped electron dust grain current

ABDERREZAK BERBRI and MOULOUD TRIBECHE

Plasma Physics Group, Theoretical Physics Laboratory,
Faculty of Sciences–Physics, University of Bab-Ezzouar, USTHB,
B.P. 32, El Alia, Algiers 16111, Algeria
(mouloudtribeche@yahoo.fr)

(Received 21 September 2008, first published online 29 June 2009)

Abstract. It is shown that when the non-isothermal trapped electron current is rederived based on the orbit-limited motion theory, the variable dust charge can be expressed in terms of the Lambert function. One can then take advantage of this new transcendental function to illustrate how variable-charge nonlinear trapped dust modes can be investigated semi-analytically.

1. Introduction

A dusty plasma is a normal electron–ion plasma with an additional highly charged component of small micron or sub-micron sized extremely massive charged particulates (dust grains). Dust and plasma coexist in a wide variety of cosmic and laboratory environments. They are ubiquitous in different parts of our solar system, namely in planetary rings, in circumsolar dust grains, in the interplanetary medium, in cometary comae and tails, in asteroid zones, in the Earth's ionosphere and magnetosphere, and in interstellar molecular clouds [1, 2]. Besides these, dust particles have been observed in low-temperature plasmas, like those used in plasma processing and plasma crystals. Such mixtures of electrons, ions, and charged grains distinguish themselves from ordinary plasmas in various ways. Unique and novel features of dusty plasmas when compared with the usual electron–ion plasmas are the existence of a new, ultra-low-frequency regime for wave propagation and the high charging of the grains which can fluctuate due to the collection of plasma currents onto the dust surface. It has been shown both theoretically and experimentally that the presence of these extremely massive and highly charged dust grains in a plasma can modify the behavior of the usual waves and instabilities. On the other hand, it has been shown that the dust-charge dynamics introduces new eigenmodes, such as dust–acoustic (DA) modes [3], dust ion–acoustic (DIA) modes [4], dust Bernstein–Greene–Kruskal (DBGK) modes [5], etc. Dust grains become charged due to different processes, such as collection of charged particles from the surrounding plasma, photoionization, secondary electron emission, sputtering by energetic ions, etc. In low-temperature laboratory plasmas, the collection of electrons and ions is the prevailing charging process. The grain charge becomes a new self-consistent dynamical variable and represents an additional degree of freedom.

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Some recent theoretical work focused on the effects of particle trapping on different types of linear and nonlinear collective processes in charge-varying dusty plasmas [6–20]. It has been demonstrated that the presence of such trapped particles can significantly modify the wave-propagation characteristics in collisionless charge-varying dusty plasmas. However, for the sake of mathematical simplicity and to keep their analysis more tractable analytically, [6–20] made use of the charging current expressions derived for thermal electrons and ions, overlooking the fact that the ions or electrons depart from their thermodynamic equilibrium. The effects of electron or ion non-isothermality on the dust charging enter then through the particle densities and the electrostatic potential appearing in these predetermined currents. The aim of this paper is to show that when the non-isothermal trapped electron current is rederived based on the orbit-limited motion theory, the variable dust charge can be expressed in terms of the Lambert function. One can then take advantage of this new transcendental function to illustrate how variable-charge nonlinear trapped dust modes can be investigated semi-analytically.

2. Theoretical model

Let us consider an unmagnetized dusty plasma having electrons, positive ions, and isolated dust grains of densities n_e, n_i, and n_d, respectively. For simplicity, all the grains are assumed to have the same negative charge, equal to $q_d = -eZ$, and to be spheres of radius r. The ions are assumed to be in thermal equilibrium, with the density

$$n_i = n_{i0} \exp\left(-\frac{e\phi}{T_i}\right). \tag{1}$$

To model the hot electron distribution in the presence of trapped particles, we refer to a possible three-dimensional equilibrium state electron velocity distribution function which solves the collisionless Vlasov equation with a population of trapped particles [21]. Thus, we choose

$$F_e(v) = F_e(v_x, v_y, v_z) = F_{ef}(v_x, v_y, v_z) + F_{et}(v_x, v_y, v_z), \tag{2}$$

where

$$F_{ef}(v) = n_{e0}\left(\frac{m_e}{2\pi T_{ef}}\right)^{3/2} \exp\left[-\frac{1}{2}\left(\frac{v^2}{T_{ef}/m_e} - \frac{2\phi}{T_{ef}/e}\right)\right] \quad \text{for } v_e > \sqrt{\frac{2e\phi}{m_e}},$$

$$F_{et}(v) = n_{e0}\left(\frac{m_e}{2\pi T_{ef}}\right)^{3/2} \exp\left[-\frac{1}{2}\beta_e\left(\frac{v^2}{T_{ef}/m_e} - \frac{2\phi}{T_{ef}/e}\right)\right] \quad \text{for } v_e \leqslant \sqrt{\frac{2e\phi}{m_e}}. \tag{3}$$

Here, ϕ is the electrostatic potential, $\mp e$ are the electron and ion charge, T_j the temperatures, and m_j the masses, where $j = e, i$ denote electrons and ions, respectively. The subscript '0' stands for equilibrium values and 'f' ('t') represents the free (trapped) electron contribution. Here, β_e is a parameter determining the number of trapped electrons and its magnitude is defined as the ratio of the free hot electron temperature T_{ef} to the hot trapped electron temperature T_{et}, i.e. $|\beta_e| = T_{ef}/T_{et}$. Charge neutrality at equilibrium requires $f = n_{i0}/n_{e0} = 1 + Z_0 n_{d0}/n_{e0}$.

In the standard orbit-limited probe model for a dust grain [22], the latter is charged by the plasma currents at the grain surface. The charging current originates from electrons and ions hitting the grain surface. Accordingly, the variable dust

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charge $q_{\rm d} = -eZ$ is determined self consistently by

$$v_{\rm d}\frac{dq_{\rm d}}{dx} = I_{\rm e} + I_{\rm i}, \tag{4}$$

where $I_{\rm e}$ and $I_{\rm i}$ are the average microscopic electron and ion currents entering the dust grains. The grain current from the thermal ions is [23, 24]

$$I_{\rm i} = \pi r^2 e(8T_{\rm i}/\pi m_{\rm i})^{1/2} n_{\rm i} W_{\rm i}, \tag{5}$$

where

$$W_{\rm i} = 1 - eq_{\rm d}/rT_{\rm i} \tag{6}$$

for $q_{\rm d} < 0$.

For a non-isothermal distribution such as (2), one should rederive the electron–dust charging current by using the orbit-limited motion theory. The latter is obtained by averaging the effective collision cross section $\sigma_{\rm e}(v, q_{\rm d}) = \pi r^2(1 + 2eq_{\rm d}/m_{\rm e}Cv^2)$ for electrons impacting the dust grains over the electron distribution, where $C = r(1 + r/\lambda_{\rm De}) \simeq r$ is the effective grain capacitance. Thus, we have

$$\begin{aligned} I_{\rm e} &= -e\iiint F_{\rm e}(v) v\sigma_{\rm e}(v, q_{\rm d})\, d^3v \\ &= -4\pi e\left\{\int_{v_{\rm min}}^{\sqrt{2e\phi/m_{\rm e}}} v^3\sigma_{\rm e}F_{\rm et}\, dv + \int_{\sqrt{2e\phi/m_{\rm e}}}^{\infty} v^3\sigma_{\rm e}F_{\rm ef}\, dv\right\}, \end{aligned} \tag{7}$$

where $v_{\rm min} = (-2eq_{\rm d}/m_{\rm e}C)^{1/2}$ is the minimum speed an electron must have in order to arrive at the grain surface and $v_{\rm min} < \sqrt{2e\phi/m_{\rm e}}$ has been assumed. After performing the integrals in (7), one can obtain the following electron charging current:

$$\begin{aligned} I_{\rm e} &= -en_{\rm e0}\pi r^2(8T_{\rm ef}/\pi m_{\rm e})^{1/2}\left\{e^{\beta_{\rm e}(Q_{\rm d}+\Psi)} + Q_{\rm d}\left(1 - \frac{1}{\beta_{\rm e}}\right) + \Psi\left(1 - \frac{1}{\beta_{\rm e}}\right) + 1 - \frac{1}{\beta_{\rm e}^2}\right\} \\ &= -en_{\rm e0}\pi r^2(8T_{\rm ef}/\pi m_{\rm e})^{1/2}\left\{e^{\beta_{\rm e}(Q_{\rm d}+\Psi)} + \left(1 - \frac{1}{\beta_{\rm e}}\right)\left(Q_{\rm d} + \Psi + 1 + \frac{1}{\beta_{\rm e}}\right)\right\}, \end{aligned} \tag{8}$$

which reduces to

$$I_{\rm e} = -en_{\rm e0}\pi r^2(8T_{\rm e}/\pi m_{\rm e})^{1/2}\exp(Q_{\rm d} + \Psi) \tag{9}$$

for Boltzmann-distributed electrons ($\beta_{\rm e} = 1$). The following normalized quantities are introduced:

$$\Psi = \frac{e\phi}{T_{\rm ef}}, \quad Q_{\rm d} = \frac{eq_{\rm d}}{rT_{\rm ef}}. \tag{10}$$

We note that the characteristic time for dust motion is of the order of tens of milliseconds for micrometer-sized grains, while the dust charging time is typically of the order of 10^{-8} s. Within the time of charging, the displacement of the grain is thus negligible compared to the spatial scale of the problem. It follows that the charging process can be treated as a local phenomenon, and the convective term on the left-hand side of the charging equation can be neglected. It follows that we have

$$I_{\rm e} + I_{\rm i} \simeq 0, \tag{11}$$

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and the grains are always at the floating potential. Thus, (11) leads to

$$Q_{\rm d} = -\Psi - {\rm W}\left\{ \frac{\exp[g(\Psi)]}{1 - 1/\beta_{\rm e} + \sqrt{\sigma\mu} f \exp(-\sigma\Psi)} \right\} + g(\Psi), \tag{12}$$

where

$$g(\Psi) = \frac{1 - \beta_{\rm e}^2 + \beta_{\rm e}^2 \sqrt{\mu/\sigma} f \exp(-\sigma\Psi)(1 + \sigma\Psi)}{\beta_{\rm e}\{\beta_{\rm e} - 1 + \sqrt{\mu\sigma} f \beta_{\rm e} \exp(-\sigma\Psi)\}}, \tag{13}$$

and $\mu = m_{\rm e}/m_{\rm i}$, $f = n_{\rm i0}/n_{\rm e0}$, and $\sigma = T_{\rm ef}/T_{\rm i}$. The Lambert function W [25], also called the omega function, is the inverse function of $h({\rm W}) = {\rm We}^{\rm W}$ and several well-known problems in electrostatics and in quantum mechanics can be solved with greater facility using it. Charge neutrality at equilibrium ($\Psi = 0$) requires the following constraint:

$$f = \sqrt{\frac{\sigma}{\mu}} \frac{\exp(Q_{\rm d0}) + (1 - 1/\beta_{\rm e}) Q_{\rm d0} + 1 - 1/\beta_{\rm e}^2}{1 - \sigma Q_{\rm d0}}. \tag{14}$$

3. Conclusion

To conclude, the grain current from non-isothermal trapped electrons is rederived based on the orbit-limited motion theory. The variable dust charge is then expressed in terms of the Lambert function and one can take advantage of this new transcendental function to illustrate how the variable-charge solitary dust acoustic waves can be investigated semi-analytically. Considering the wide relevance of nonlinear oscillations, our results should help to understand the salient features of coherent nonlinear structures in non-equilibrium plasmas which contain particle distributions that are far away from Maxwellian. Such non-isothermal particle distributions and associated electrostatic structures are frequently found in space and laboratory experiments. We hope that they should also enhance our knowledge of nonlinear dust phase-space vortices (holes) in charge-varying dusty plasmas and dusty plasma sheaths where voids are formed. It may be stressed here that a complete study of the present problem requires a self-consistent solution to the Vlasov–Poisson system in three-dimensional real and velocity space. This is a very difficult task because of inherent nonlinearity. In addition, the discussion of free and trapped particle orbits becomes much more complicated, being at least influenced by two constants of motion, the energy and the angular momentum [26]. All these aspects are of great importance but beyond the scope of the present paper, which we hope will represent a first step towards a more generally valid theory.

Acknowledgement

This work was supported in part by the Ministère de l'Enseignement Supérieur et de la Recherche Scientifique (Contract No. D00220060041).

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