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Equilibrium Price Dynamics of Emission Permits

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Abstract

This article presents a stochastic equilibrium model for environmental markets that allows us to study the characteristic properties of emission permit prices induced by the design of today's cap-and-trade systems. We characterize emission permits as highly nonlinear contingent claims on economy-wide emissions and reveal their hybrid nature between investment and consumption assets. Our model makes predictions about the dynamics and volatility structure of emission permit prices, the forward price curve, and the implications for option pricing in this market. Empirical evidence from existing emissions markets shows that the model explains the stylized facts of emission permit prices and related derivatives.

I. Introduction

Global warming is generally accepted as one of the greatest challenges that society faces in the 21st century, bringing the goal to limit and reduce

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anthropogenic greenhouse gas emissions to the top of political agendas worldwide. One of the most promising approaches to tackle this challenge is the use of market-based instruments. Based on the seminal work of Coase (1960), the idea is to oblige polluters to acquire a right for each ton of greenhouse gas they emit and to limit the amount of available rights according to a predefined cap. Starting from an initial allocation, these rights, called *emission permits*, can be traded among polluters and investors (*cap and trade*). The market mechanism leads to an efficient allocation of permits under perfect market conditions, such that the emission limit is achieved at the lowest cost for the economy.

As emission trading schemes are introduced all over the world,[1] a sizable financial market for emission permits and related derivatives has evolved, calling for a deeper understanding of how market prices behave for this new asset class. Against this background, it is the goal of this article to study the characteristic *price dynamics* of emission permits originating from today's emission trading systems. Based on a stochastic equilibrium model for emissions markets, we show that emission permits can be characterized as highly nonlinear contingent claims that are written on economy-wide emissions as a nontradable underlying. This characterization gives rise to several results on the properties of emission permit prices, their volatility structure, and the permit forward price curve, and it also has important implications for option pricing in this market.

Our results are of importance for at least three categories of economic agents. First, polluting companies are exposed to the price risk of emission permits as a new production factor for their businesses and engage in derivatives markets to enhance their risk management. Second, emission permits and related derivatives also provide a promising option for diversification to financial investors and appear as a natural hedge against the temperature risk priced in equity returns (Bansal, Kiku, and Ochoa (2017)). Third, it is of great interest for policymakers and regulators to understand the relation of the system's design and the induced stochastic behavior of emission permit prices, which is a key determinant of polluting companies' investments in environment-friendly technologies.

Our model considers an economy of companies that is regulated by an emission trading system. Companies have stochastic greenhouse gas emissions that they can reduce by implementing costly abatement measures, and they receive an initial number of emission permits that are tradable in the market. We particularly account for the design features of today's state-of-the-art emission trading systems, which are organized as a number of consecutive compliance periods. At the end of each compliance period, the regulated companies are obliged to cover their emissions with emission permits, and a penalty is incurred in the case of noncompliance. Additionally, companies have to deliver the lacking permits in the following compliance period. While it is allowed to transfer unused permits from one period to the following one (*banking*), the reverse direction (*borrowing*) is prohibited. Given these rules, each company optimizes its abatement and trading

[1] Most prominently, emissions trading has been introduced in the European Union, in several states of Canada and the United States, in New Zealand, in South Korea, and in several provinces and cities in China.

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strategy to find an optimal trade-off between implementing abatement measures, trading permits in the market, and taking the risk of penalty payments.

We study the equilibrium price dynamics of emission permits within the theoretical framework of our model and highlight how the marginal permit value derives from the specific design of the emission trading system. Particularly, an emission permit can be characterized as a strip of European binary options written on economy-wide emissions. In contrast to classical financial options, however, the dynamics of this nontradable underlying is not exogenously given, as companies obviously have influence through their emissions abatement. Abatement reduces the companies' greenhouse gas emissions, such that an emission permit is worth less than the corresponding strip of binary options in a scenario where no abatement is possible. We show theoretically that this option analogy is key to the price dynamics of emission permits and their volatilities and also determines the term structure of forward prices in this market.

Based on a calibration of the model to the EU Emissions Trading System (EU ETS), the world's largest environmental market, we refine our theoretical results to obtain the following testable predictions: First, similar to the "leverage effect" for stocks as options on the firm value (Geske (1979)), we obtain a negative relation between permit prices as (strips of binary) options on economy-wide emissions and their volatilities. Second, our model predicts a forward price curve that is in contango within compliance periods but generally backwardated across them, with the degree of backwardation reflecting the probability of a permit shortage in the ongoing period. The intuition is that emission permit forwards with maturity in the next compliance period cannot be used for compliance in the ongoing period, such that holding the spot permit provides an additional benefit to the owner, as with classical commodities. To the contrary, there is no such advantage compared with forward contracts maturing in the same period because companies need to cover their emissions with permits only at the very end of the compliance period. In other words, the usage option embedded in the spot asset (Routledge, Seppi, and Spatt (2000)) can only be exercised at the end of a compliance period, and emission permits are pure investment assets before that date. This hybrid nature between investment and consumption assets leads to the different shape of the forward price curve within and across compliance periods. Third, the volatility smile of options on emission permits is downward-sloping for most realistic emissions scenarios, which is in direct accordance with the negative relation of permit prices and volatilities explained previously and thus also a result of the permits' payoff structure induced by the system's design. The downward slope is, furthermore, most pronounced when the underlying permit prices are low.

We verify the model predictions using data from two important emissions markets, in particular from Phase II of the EU ETS and from the U.S. Acid Rain Program for sulfur dioxide emissions. These data confirm all our testable predictions.

Our article contributes to the connection of two originally separate strands of literature. In the finance literature on commodity markets and the assessment of commodity-related securities and projects (e.g., Schwartz (1997), Routledge et al. (2000), Casassus and Collin-Dufresne (2005), and Trolle and Schwartz (2009)), it is well recognized that economic decisions such as risk management and

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investment decisions depend critically on the assumed stochastic behavior of commodity prices. Although emission permits are often considered as a novel commodity, it is at least questionable that approaches developed for classical commodities like oil, gas, metals, and agricultural goods transfer to this new type of asset. On the other side, the environmental economics literature devotes much attention to analyzing the equilibrium outcomes of emission trading systems dependent on important design features and in comparison to conventional command-and-control mechanisms (e.g., Montgomery (1972), Cronshaw and Kruse (1996), Rubin (1996), and Schennach (2000)). Theoretical works in this area, however, build on a deterministic model framework, which precludes the investigation of economic questions that arise in consequence of price uncertainty.[2]

A newly evolving literature links these two strands by developing equilibrium models for emissions markets under uncertainty. A few studies investigate agents' optimal strategies and the properties of permit prices within stochastic frameworks that employ a stylized representation of an emission trading system and abstract from the specific design features of today's systems. In particular, these approaches consider emissions trading in a setting of one finite compliance period (Seifert, Uhrig-Homburg, and Wagner (2008), Carmona, Fehr, and Hinz (2009), Carmona, Fehr, Hinz, and Porchet (2010), and Chesney and Taschini (2012)) or under symmetric rules for banking and borrowing (Cetin and Verschuere (2009), Kijima, Maeda, and Nishide (2010)).[3] Because the value of emission permits only arises due to the regulatory rules of the system, we argue that it is crucial to account for the specific design of today's cap-and-trade systems to make clear-cut predictions about the behavior of permit prices and to derive implications for related derivatives. Indeed, this article shows that the design of state-of-the-art emission trading systems induces a number of characteristic features that are represented by emission permit spot, forward, and option prices.

The remainder of the article is organized as follows: The next section presents a stochastic equilibrium model for emissions markets, based on which we characterize the corresponding spot and forward price dynamics. In Section III, we calibrate our model to the EU ETS and derive empirically testable predictions within the calibrated setting. Section IV tests and confirms these predictions based on data from the EU ETS and the U.S. Acid Rain Program for sulfur dioxide emissions. Section V concludes.

II. Theoretical Model

This section proposes a stochastic equilibrium model for environmental markets that particularly accounts for the design of today's emission trading systems. We first introduce the model setup and summarize our notations in Table 1. Based on the model equilibrium, we characterize emission permit prices, their volatility structure, and the term structure of forward prices in emissions markets.

[2] Although Schennach (2000) takes a first step in this direction by introducing uncertainty into her model, she does not elaborate on the resulting stochastic dynamics of emission permit prices.

[3] An exception is Carmona and Fehr (2011), who incorporate the most important regulatory rules of current emission trading systems into a mathematical model for multiple linked permit markets.

TABLE 1
Model Notations

Table 1 summarizes and explains the notations used in the theoretical model. The superscript index i generally refers to company i in the economy, and by omitting the i we refer to an economy-wide version of the respective variable. The subscript index k stands for the kth compliance period of the emission trading system. Time dependence is indicated by a t as a subscript or in parentheses.

Notation	Explanation
Emission Trading System	
n	Number of compliance periods of the emission trading system
T_k	End of compliance period k of the emission trading system
a_k^i	Amount of period k emission permits allocated to company i
q_k^i	Cumulative emission permits allocated to company i up to period k
p_k	Penalty per ton of exceeding emissions at the end of period k
r	Annual interest rate of the economy
Trading Emission Permits	
$\theta_{k,t}^i$	Amount of period k emission permits bought by company i at time t
Q_k^i	Cumulative emission permits allocated to and bought by company i up to period k
$S_k(t)$	Price of period k emission permits at time t
Business-as-Usual Emissions and Abatement	
y_t^i	Business-as-usual emission rate of company i at time t
$\mu_y^i(t)$	Drift of business-as-usual emission rate of company i at time t
$\sigma_y^i(t)$	Volatility of business-as-usual emission rate of company i at time t
e_t^i	Short-term emission shocks of company i at time t
σ_e^i	Volatility of short-term emission shocks of company i
x_t^i	Cumulative emissions of company i up to time t
C^i	Abatement cost function of company i
ξ_t^i	Emissions abatement of company i implemented at time t

A. Setup

Assumption 1. Emission Trading System. We consider an economy given by a set of companies I whose emissions are regulated under a cap-and-trade system with n consecutive compliance periods $[0,T_1],[T_1,T_2],\ldots,[T_{n-1},T_n]$. The rules of the emission trading system are exogenously given and fixed at time 0. In particular, each company $i \in I$ receives an allocation $(a_1^i,\ldots,a_n^i)$ of emission permits for the different periods of the system at time 0 (i.e., a_k^i is the initial number of *period k permits* that are valid for compliance in period $[T_{k-1},T_k]$, with $T_0=0$). The companies are obliged to cover the emissions realized during period k by a sufficient number of period k permits by the end of the period, T_k. For enforcement, a penalty p_k is imposed for each ton of exceeding emissions. In addition, lacking permits have to be delivered in the following compliance period, effectively reducing the number of period $k+1$ permits. In a similar way, leftover permits not needed for compliance in a period are banked into the next period. It is not allowed, however, to borrow permits that are allocated for a future compliance period and use them in the ongoing period. After $[T_{n-1},T_n]$, the last period of the system, leftover permits and obligations to later delivery are forfeited.

Assumption 2. Trading Activities. Emission permits are tradable in secondary markets, and we denote the time t price of period k permits by $S_k(t)$. At each point in time, company i chooses the amount $\theta_{k,t}^i$ of period k permits that it buys or sells in the market, adjusting its time t holdings to $a_k^i + \int_0^t \theta_{k,s}^i\, ds$.

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Assumption 3. Business-as-Usual Emissions. We assume that without abatement, company i's emissions are driven by two components: i) a persistent business-as-usual emission rate y^i following the Itô process

$$y_t^i = y_0^i + \int_0^t \mu_y^i(s)\,ds + \int_0^t \sigma_y^i(s)\,dZ_s^i \tag{1}$$

with deterministic time-dependent drift $\mu_y^i(t)$ and volatility $\sigma_y^i(t) > 0$, and ii) short-term emission shocks e^i given by

$$e_t^i = \sigma_e^i w_t^i \tag{2}$$

with $\sigma_e^i > 0$. Z^i is a standard Wiener process, w^i is a standard Gaussian white-noise process, and we assume that the increments of $W_t^i = \int_0^t w_s^i\,ds$ and Z_t^i are uncorrelated. Whereas y^i captures deterministic patterns of the emission rate, such as seasonalities, as well as fluctuations that are permanent in nature, e^i represents temporary shocks, such as the outage of a carbon-friendly production unit that is instantaneously replaced by a more polluting one. Accordingly, the prevailing business-as-usual emission rate y^i drives the expected *future* emissions, in contrast to e^i, which only affects the instantaneous contribution to realized emissions.[4] Across companies (i.e., for all $i_1, i_2 \in I$), we allow the increments of $W_t^{i_1}$ and $W_t^{i_2}$ and of $Z_t^{i_1}$ and $Z_t^{i_2}$ to be correlated, and we assume zero correlation for the increments of $W_t^{i_1}$ and $Z_t^{i_2}$.

Assumption 4. Abatement. To actively manage the risk of paying penalties, a company can reduce its emissions through emission abatement measures. To this end, company i decides about the abatement measures ξ_t^i that it implements at t. This reduces company i's instantaneous emission rate to $y_t^i + e_t^i - \xi_t^i$, and its cumulative emissions up to time t result as

$$x_t^i = \int_0^t (y_s^i + e_s^i - \xi_s^i)\,ds. \tag{3}$$

We assume that abating at an instantaneous rate of ξ_t^i costs $C^i(\xi_t^i)$, where $C^i : \mathbb{R} \to \mathbb{R}_0^+$ is a differentiable and strictly convex abatement cost function, as motivated by detailed bottom-up studies (e.g., Klepper and Peterson (2006), Nauclér and Enkvist (2009)) noting that marginal abatement costs increase at least linearly.

Assumption 5. Risk-Neutral Cost Minimization. Firms are price-takers[5] and maximize their utility by finding an optimal trade-off between implementing abatement measures, trading permits in the market, and taking the risk of penalty payments.

[4] Short-term emission shocks e^i are considered in our model mostly for technical reasons. Without these shocks, a regular model solution cannot be guaranteed (see Carmona, Delarue, Espinosa, and Touzi (2013)).

[5] The assumption of price-taking firms is in line with the large number of companies regulated under today's emission trading systems. For example, there are more than 5,500 firms regulated under the EU ETS (see Calel and Dechezleprêtre (2016)).

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Under the assumption of risk neutrality,[6] company i minimizes overall costs by solving the optimization problem

$$\min_{(\theta^i,\xi^i)} \mathbb{E}_0 \left\{ \int_0^{T_n} e^{-rt} C^i(\xi_t^i)\, dt + \sum_{j=1}^{n} \int_0^{T_j} e^{-rt} S_j(t) \theta_{j,t}^i\, dt + \sum_{j=1}^{n} e^{-rT_j} p_j (x_{T_j}^i - Q_j^i)^+ \right\}, \tag{4}$$

with constant risk-free interest rate r and optimal trading strategy $\theta^i = (\theta_1^i, \ldots, \theta_n^i)$, $\theta_k^i = (\theta_{k,t}^i)_{t\in[0,T_k]}$ and abatement strategy $\xi^i = (\xi_t^i)_{t\in[0,T_n]}$. $\mathbb{E}_t\{\cdot\}$ denotes the expectation based on time t information, and we assume that agents have homogeneous beliefs.[7] The first term of the objective in problem (4) represents the costs of implementing abatement measures, and the second term describes the costs of company i's trading strategy. The last term incorporates possible penalties at the end of each compliance period, where $x_{T_k}^i - Q_k^i$ is the amount of lacking or leftover permits in period k, with $Q_k^i = \sum_{j=1}^{k} (a_j^i + \int_0^{T_j} \theta_{j,s}^i\, ds)$.

Note that $x_{T_k}^i - Q_k^i$ results from company i's actions and the rules of the cap-and-trade system. To understand this expression step by step, let us consider a company i whose emissions (net of abatement) during the n different compliance periods are specified by random variables $x_{0,T_1}^i, x_{T_1,T_2}^i, \ldots, x_{T_{n-1},T_n}^i$. We first ignore the company's trading activities. If company i's overall emissions during period 1 are larger than a_1^i, the exceeding $(x_{0,T_1}^i - a_1^i)$ tons of emissions are penalized, and company i has to deliver the lacking permits in period 2, decreasing the number of period 2 permits by the same amount. If, to the contrary, $x_{0,T_1}^i < a_1^i$, then company i banks the leftover period 1 permits into period 2, adding $(a_1^i - x_{0,T_1}^i)$ to the amount of period 2 permits. In summary, company i pays a penalty of $p_1(x_{0,T_1}^i - a_1^i)^+$, and the number of period 2 permits is altered to $a_2^i + (a_1^i - x_{0,T_1}^i)$. Analyzing the same for period 2, we see that penalties are incurred if $x_{T_1,T_2}^i - (a_2^i + a_1^i - x_{0,T_1}^i) > 0$. This can be reinterpreted in the sense that the amount of exceeding emissions at the end of period 2 is the difference between company i's cumulative emissions from 0 up to T_2, $x_{0,T_2}^i = x_{0,T_1}^i + x_{T_1,T_2}^i$, and the cumulative amount of permits $q_2^i = a_1^i + a_2^i$ that is allocated for periods 1 and 2. In general terms, the amount of lacking or leftover permits in period k results as the difference between company i's cumulative emissions up to T_k and the cumulative allocation $q_k^i = \sum_{j=1}^{k} a_j^i$ for all compliance periods up to k, that is, $x_{T_k}^i - q_k^i$. Incorporating the trading strategy alters the cumulative amount of permits q_k^i to Q_k^i, as defined previously.

[6] We discuss the implications of risk aversion in Section B of the Internet Appendix (available at www.jfqa.org).

[7] It might be an interesting avenue for future research to explore the implications of heterogeneous beliefs about future emissions in environmental markets. Disagreement about future emissions should affect the dynamics of aggregate abatement and give rise to a stochastic volatility component of permit prices and emissions, in analogy to the production economy setting studied by Baker, Hollifield, and Osambela (2016).

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B. Emissions Market Equilibrium

We solve the model for equilibrium permit prices that clear the market when all companies $i \in I$ choose optimal trading and abatement strategies according to the objective in (4). To begin with, we apply the stochastic maximum principle in conjunction with dynamic programming to characterize the optimal trading and abatement strategy of a company for given permit prices $S_1, \ldots, S_n$.[8]

Proposition 1. Optimality Conditions. For an optimal trading and abatement strategy (θ^i, ξ^i), a company's instantaneous marginal abatement costs are equal to the permit price of the ongoing compliance period,

$$\frac{\partial C^i}{\partial \xi^i}(\xi_t^i) \quad = \quad S_k(t), \quad t \in [T_{k-1}, T_k]. \tag{5}$$

Further, the present value of a company's expected overall penalty for an additional ton of emissions in period k is equal to the price of period k emission permits, $k = 1, \ldots, n$, that is,[9]

$$\sum_{j=k}^{n} e^{-r(T_j - t)} \mathbb{P}_t\{x_{T_j}^i > Q_j^i\} p_j \quad = \quad S_k(t), \quad t \in [0, T_k]. \tag{6}$$

The first condition arises due to the fact that companies can achieve compliance equally by abating emissions or by buying additional permits in the market. Both actions reduce the amount of uncovered emissions at the end of the ongoing compliance period. Consequently, the marginal cost of both actions is equal for an optimal strategy because otherwise, a company could improve by abating more emissions and buying less permits, or *vice versa*. This result is central to deterministic models for cap-and-trade systems developed in the environmental economics literature (e.g., Cronshaw and Kruse (1996), Rubin (1996)).[10]

The second condition characterizes the marginal value of an emission permit for an individual company, which is induced by the system's penalty mechanism and the rules on banking, borrowing, and later delivery. If a company is short of permits for a particular compliance period k, then an additional permit saves a penalty payment and additionally also reduces the number of permits to be delivered later, which effectively increases the amount of permits in the next compliance period. In case of a permit surplus, an additional permit does not avoid any penalty payment, but it can be banked and adds to the available amount of permits in the following compliance period as well. The same logic again applies for the following compliance periods. Overall, the marginal value of a period k emission permit for a company equals the sum of discounted penalties for period k and all following compliance periods weighted by the probability that penalties

[8] All proofs are provided in Section C of the Internet Appendix.

[9] $\mathbb{P}_t\{\cdot\}$ denotes the probability conditional on time t information.

[10] This result is also shown within stochastic equilibrium models for one compliance period in the literature (see, e.g., Seifert et al. (2008) and Carmona et al. (2009), (2010), as well as Carmona et al. (2013) for the risk-averse case). In the context of multiple compliance periods, it is important to note that marginal abatement costs are equal to the permit price for the period in which the abatement is actually realized, which is the ongoing period in our case.

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are incurred. Each company's optimal trading and abatement strategy then aligns the risk of paying penalties in such a way that this marginal value is equal to the permit price S_k.[11]

C. Permit Prices

Proposition 1 implies that in equilibrium, marginal abatement costs and the probability of penalties to accrue are equalized over all companies in the economy, in line with the emissions market mechanism first formalized by Montgomery (1972). In a situation where marginal abatement costs differ across companies, companies with lower marginal abatement costs extend their abatement activities and sell emission permits to companies with higher abatement costs, enabling them to cut back their abatement actions. If they agree on a permit price between their respective marginal abatement costs, both companies are able to profit from these actions, implying that such a situation cannot persist in equilibrium. In the same way, the market is not in equilibrium if some companies have uncovered emissions at the end of a compliance period T_k while others have remaining permits. Taking a global point of view, this condition implies that one, and thus every, company's emissions exceed the company's permit holdings exactly when economy-wide cumulative emissions $x_{T_k} = \sum_{i \in I} x^i_{T_k}$ exceed economy-wide permit holdings $Q_k = \sum_{i \in I} Q^i_k$. Because the market-clearing condition holds in equilibrium, individual trading strategies cancel out in Q_k, and we have $Q_k = q_k = \sum_{i \in I} q^i_k$. Altogether, the following result arises from the condition given by equation (6).

Proposition 2. Equilibrium Permit Prices. In equilibrium, the price of a period k emission permit is given by

$$S_k(t) = \sum_{j=k}^{n} e^{-r(T_j - t)} \mathbb{P}_t\{x_{T_j} > q_j\} p_j \tag{7}$$

for $t \in [0, T_k]$. That is, an emission permit is a strip of European binary call options written on cumulative economy-wide emissions.

This proposition characterizes emission permits created in the context of environmental trading schemes as a financial derivative. The price of an emission permit consists of one value component for each compliance period of the system, representing the probability that penalties accrue for that period because the economy is short of emission permits. At the end of the period, T_k, this value is equal to the penalty for noncompliance if cumulative emissions x_{T_k} exceed the cumulative allocation q_k, and it is 0 otherwise. Thus, each value component has a payoff function that is identical to a European binary call option with maturity T_k and strike q_k, written on the cumulative emissions $x_t = \sum_{i \in I} x^i_t$ of the whole economy. In our framework, the dynamics of this nontradable underlying is given by

$$dx_t = (y_t - \xi_t)\, dt + \sigma_e\, dW_t, \tag{8}$$

[11] For the special case of one single compliance period, this result is shown by Carmona et al. (2009), (2010).

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where $y_t = \sum_{i \in I} y_t^i$ is the economy-wide business-as-usual emission rate following

$$dy_t = \mu_y(t)\,dt + \sigma_y(t)\,dZ_t, \tag{9}$$

and $\xi_t = \sum_{i \in I} \xi_t^i$ is economy-wide abatement. The parameter σ_e and the functions $\mu_y(t)$ and $\sigma_y(t)$ result from aggregating the individual emissions processes (1), (2), and (3) such that W_t and Z_t are standard Wiener processes with uncorrelated increments. A distinctive feature of economy-wide emissions x_t as a nontradable underlying is that market participants can and obviously do influence its state through their abatement actions ξ_t. Because abatement reduces the companies' emissions, an emission permit is worth less than the corresponding strip of binary options in a scenario where no abatement is possible.

The specific form of permit prices revealed by Proposition 2 obviously affects their precise distributional properties but also finds expression in major structural features. First, there is a time-dependent upper bound for emission permit prices,

$$S_k(t) \leq \sum_{j=k}^{n} e^{-r(T_j - t)} p_j, \tag{10}$$

corresponding to a scenario of permit shortage for all compliance periods. In such a situation, one permit less means an additional penalty in the current period and also in all following compliance periods due to the later delivery rule. This upper bound of emission permit prices obviously depends on the number of compliance periods in the system. Prices close to the upper bound are a sign for the regulator that under the implemented regulation, companies expect their emissions to exceed the cap for all compliance periods even after abatement, such that emission-reduction goals will most likely not be achieved. In that case, the regulator could raise the penalties to incentivize additional abatement and increase the chance of meeting the targets, driving prices away from the (now increased) upper bound. Another possible measure is to introduce subsidies for environment-friendly technologies in addition to the emission trading system, such that abatement costs are effectively reduced and the reduction goals are more likely to be achieved.

Second, the single value components of an emission permit are pulled to one of the two values (0 or the penalty) by the end of the corresponding compliance period. This effect is weak as long as the period end is far and the uncertainty about penalties to accrue is high, such that medium values are attained with significant probabilities. As uncertainty decreases, the convergence to one of the two possible values becomes stronger. As a direct consequence, the prices of period k and period $k+1$ permits are either identical at the end of period k or differ by the amount of the penalty, that is

$$S_k(T_k) - S_{k+1}(T_k) = 1_{\{x_{T_k} > q_k\}} p_k. \tag{11}$$

This means that there is a smooth transition when period k permits are converted into period $k+1$ permits by banking if the economy is in permit surplus. In contrast, if the economy is short of permits, prices decrease by the penalty, similar to the drop in the value of a coupon bond after a coupon payment date.

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Understanding an emission permit as a strip of binary options also sheds light on the volatility structure of permit prices. Given the dynamics in equations (8) and (9) of cumulative emissions and the prevailing emission rate, we obtain local volatilities by applying Itô's lemma.

Proposition 3. Local Volatility. The relative local volatility of emission permit prices, as given by

$$\sigma_{S_k} = \frac{\sqrt{\left(\frac{\partial S_k}{\partial x}\sigma_e\right)^2 + \left(\frac{\partial S_k}{\partial y}\sigma_y\right)^2}}{S_k}, \tag{12}$$

is state and time dependent.

Due to the binary option characteristics of emission permits, the volatility of each of the single value components, $S_k - S_{k+1}$, clearly depends on the time to the end of the compliance period, T_k, and is almost surely zero at the end of the period.[12] The state dependence becomes most evident when comparing scenarios of extremely high and medium emissions. In the first case, prices are close to the upper bound of equation (10) and absolute volatility (i.e., the numerator of expression (12)) is almost 0, because a marginal change in cumulative emissions or the prevailing emission rate hardly changes the probability of penalties. Compared to that, relative volatility is clearly higher for medium emissions because absolute volatility is positive and prices are far away from the upper bound. For very low emissions, however, we do not get a clear indication of the volatility behavior because absolute volatilities go to 0, but prices approach 0 as well. We shed light on this aspect by considering the detailed volatility structure for a calibrated setting in Section III.

D. Forwards

It is market convention to consider the emission permits of the actual ongoing compliance period as *spot permits*, such that spot permits are period 1 permits until the end of compliance period 1, then period 2 permits until T_2, and so on. This view makes practical sense because only period 1 permits are usable for compliance in period 1, and they are converted into period 2 permits by banking at the end of the period. In accordance with that, *permit forwards* deliver spot permits at maturity. Whereas a forward contract with maturity $\bar{t} \in [0, T_1)$ delivers a period 1 permit, forwards maturing at $\bar{t} \in (T_1, T_2)$ deliver period 2 permits. Based on this convention, we characterize the forward price curve in emission permit markets and discuss the applicability of classical convenience yield models.

Consider, at $t \in [0, T_1)$, an *intraperiod* forward contract with maturity $\bar{t} \in [t, T_1)$. Because period 1 permits cannot be used for compliance before T_1, they are pure investment assets before that date, and the storability of permits directly implies the standard cost-of-carry relation

$$F(t, \bar{t}) = e^{r(\bar{t}-t)} S_1(t), \tag{13}$$

[12] Because $\sigma_e > 0$, Section 3.3 of Carmona et al. (2013) applies to our model, which implies that the cumulative emissions exactly hit the cap with zero probability, $\mathbb{P}_t\{x_{T_k} = q_k\} = 0$.

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where $F(t,\bar{t})$ is the forward price. To the contrary, this does not apply to *interperiod* forwards (i.e., contracts with maturity $\bar{t} \in (T_1, T_2)$ traded at $t \in [0, T_1)$). The forward contract delivers a period 2 permit, which can obviously not be used for compliance in period 1, whereas holding the spot (period 1) permits provides the option to use them at the end of period 1. Accordingly, we have the cost-of-carry relation $F(t,\bar{t}) = e^{r(\bar{t}-t)} S_2(t)$ for interperiod forwards, and the discount relative to holding the spot permit can be quantified through equation (7) as

$$F(t,\bar{t}) = e^{r(\bar{t}-t)} S_1(t) - e^{r(\bar{t}-T_1)} \mathbb{P}_t\{x_{T_1} > q_1\} p_1. \tag{14}$$

It follows that the backwardation of interperiod forwards, defined as the difference between the current spot price and the discounted forward price, $B(t,\bar{t}) = S_1(t) - e^{-r(\bar{t}-t)} F(t,\bar{t})$, is determined by the probability of permit shortage at the end of the ongoing compliance period. We summarize the implications of equations (13) and (14).[13]

Proposition 4. Forward Price Curve. The permit forward price curve has the following properties for all $t \in [0, T_1)$:

i) Forwards are in contango within the compliance period (i.e., for $\bar{t} \in [t, T_1)$).

ii) The backwardation of forwards with maturity in the following compliance period (i.e., $\bar{t} \in (T_1, T_2)$) is given by

$$B(t,\bar{t}) = e^{-r(T_1-t)} \mathbb{P}_t\{x_{T_1} > q_1\} p_1. \tag{15}$$

In particular, interperiod forwards are in contango if the probability of permit shortage at the end of the ongoing compliance period is 0, in weak backwardation if it is positive, and in strong backwardation if it is above $(e^{r(T_1-t)} - e^{-r(\bar{t}-T_1)}) S_1(t)/p_1$.

This proposition provides a direct link to the classical commodity literature. Holding the spot asset endows the owner with an embedded usage and timing option, as elaborated by Routledge et al. (2000) and Jarrow (2010), among others. However, whereas for classical commodities any point in time comes into question for exercising this usage option, emission permits can only be consumed at the end of a compliance period. Thus, this option is worthless within a compliance period, and holding the spot permit has no advantage compared with forwards maturing in the same period. To the contrary, the usage option becomes relevant if the forward's maturity is in the next compliance period. If the economy is short of permits, the option is exercised to save penalty payments. Otherwise, it is not exercised for a number of leftover permits that are banked into the next period. Consequently, interperiod forwards are in weak backwardation if the probability of permit shortage at the end of the compliance period is not exactly 0.

Since the seminal work of Brennan (1958), it is common to express the benefit of holding the spot asset rather than a forward contract as a convenience yield

[13] In line with Litzenberger and Rabinowitz (1995), we say that forwards with maturity $\bar{t}$ are in contango if $B(t,\bar{t}) \leq 0$ and in (weak) backwardation if the forward price is lower than the compounded spot price (i.e., $B(t,\bar{t}) > 0$). Strong backwardation means that the forward price is lower than the current spot price.

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(see, e.g., Schwartz (1997), Casassus and Collin-Dufresne (2005)). In general, a time-dependent stochastic instantaneous convenience yield δ_t changes the standard cost-of-carry relation to

$$F(t,\bar{t}) = \mathbb{E}_t\{e^{r(\bar{t}-t)-\int_t^{\bar{t}}\delta_s ds}\}S_1(t). \tag{16}$$

For given $t \in [0, T_1)$, we define $D_t(\bar{t}) := \int_t^{\bar{t}} \delta_s ds$ as the convenience yield from t to $\bar{t}$. From equations (13) and (14), we can directly derive necessary properties of D_t.

Proposition 5. Convenience Yields. The cumulative convenience yield D_t fulfills

i) $D_t(\bar{t}) = 0$ for intraperiod forwards (i.e., $\bar{t} \in [t, T_1)$), and

ii) $\mathbb{E}_t\{e^{-D_t(\bar{t})}\} = 1 - e^{-r(T_1-t)}\mathbb{P}_t\{x_{T_1} > q_1\}p_1/S_1(t)$ for interperiod forwards (i.e., $\bar{t} \in (T_1, T_2)$).

It is obvious that D_t has to "jump" in T_1 in order to fulfill the conditions of Proposition 5. Therefore, it is not possible to define an instantaneous convenience yield δ_t in such way that D_t has these properties. In particular, standard models like a mean-reverting stochastic convenience yield or a simple AR(4) process as used by Daskalakis, Psychoyios, and Markellos (2009) and Chevallier (2009) do not satisfy Proposition 5. These models inevitably lead to relative mispricing when intra- and interperiod forwards with two or more different maturities are considered.

E. Calculating Equilibrium Prices

Whereas the previous sections extensively characterize the emissions market equilibrium, we provide a strategy to explicitly solve for equilibrium permit prices in Section A of the Internet Appendix. In particular, we show in Proposition 6 that a competitive model equilibrium exists and can be obtained as the solution of a global problem, in which a central planner optimizes the economy-wide abatement strategy in light of possible penalties for exceeding emissions.[14] The resulting equilibrium price process is unique and can be computed as the solution of a system of partial differential equations (PDEs), as we show in Proposition 7. The explicit model solution allows us to derive specific empirical predictions from a calibrated model version in the next section.

III. Calibration and Empirical Predictions

We calibrate our model to the second compliance period (Phase II) of the EU ETS, the world's largest environmental market. Within the calibrated setting, we illustrate the specific features of emission permit prices and volatilities, and we generate empirically testable predictions by refining our theoretical results. Table 2 presents the calibrated parameter values. In the EU ETS, Phase II from

[14] Accordingly, the competitive emissions market equilibrium in our model is Pareto efficient, in line with the seminal idea of Coase (1960) and its formalization by Montgomery (1972). Deviations from Pareto efficiency can occur, for example, in the presence of imperfect competition in the permit market (e.g., Hahn 1984) or of transaction costs (e.g., Stavins 1995).

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TABLE 2
Parameter Values for an EU ETS Model Setting

In Table 2, we consider a model setting calibrated to Phase II of the EU Emissions Trading System (EU ETS), the compliance period from 2008 to 2012. Allocations and penalties for Phase II and the three following compliance periods from 2013 to 2020, 2021 to 2028, and 2029 to 2036 are chosen as outlined by the European Parliament (see MEMO/08/796). Parameters for the emissions processes given by equation (8) and equation (17) are inferred from data provided by the European Environment Agency (EEA). We consider a scenario of low abatement costs (with coefficient γ_1) compared with the case of higher costs (γ_2). r is the constant risk-free interest rate.

		Compliance Period k			
Parameter		1	2	3	4
End of compliance period	T_k	5	13	21	29
Allocation (million permits)	a_k	10,400	14,800	12,500	10,100
Penalty (euro)	p_k	100	122	149	182
Drift of emission rate	μ_y		0.80		
Volatility of emission rate	σ_y		28.00		
Volatility of emission shocks	σ_e		2.00		
Abatement cost coefficient (low)	γ_1		0.10		
Abatement cost coefficient (high)	γ_2		0.20		
Interest rate (per annum)	r		0.04		

2008 to 2012 is followed by consecutive compliance periods of 8 years in length from 2013 onward. Figures on the number of permits to be allocated for the different periods are reported by the European Commission, with each permit allowing for 1 ton of carbon dioxide (CO_2) equivalent greenhouse gas emissions. Penalties are 100 euros per ton of exceeding emissions in Phase II and increase "in accordance with the European index of consumer prices,"[15] which we assume as 2.5% per year. The parameters for the economy-wide emissions processes given by equations (8) and (9), of which we specify the latter one as an arithmetic Brownian motion

$$dy_t = \mu_y\, dt + \sigma_y\, dZ_t, \tag{17}$$

are extracted from yearly emissions data provided by the European Environment Agency (EEA) (see http://www.eea.europa.eu/). These data cover the CO_2 equivalent emissions of all industry sectors except land use, land-use change, and forestry (LULUCF) activities in the EU member states and are considered as a good proxy for emissions within the EU ETS (Ellerman and Buchner (2008)). We use data from 1995 to 2004 that are not biased by the introduction of the EU ETS. Further, we employ a quadratic abatement cost function

$$C(\xi_t) = \tfrac{1}{2}\gamma\xi_t^2 \tag{18}$$

for the aggregate economy,[16] and a linear interpolation of Europe's marginal abatement cost curve (Nauclér and Enkvist (2009), Cline (2011)) suggests that the cost coefficient γ is in the region of 0.1 to 0.2. In line with that, we consider two different scenarios, $\gamma_1 = 0.1$ and $\gamma_2 = 0.2$, which allows us to analyze the impact of abatement measures. Finally, we set the constant risk-free interest rate r to 4%, in line with the average nominal 10-year EuroSwap rate over the period from 2008 to 2012.

[15] See Directive 2009/29/EC of the European Commission.

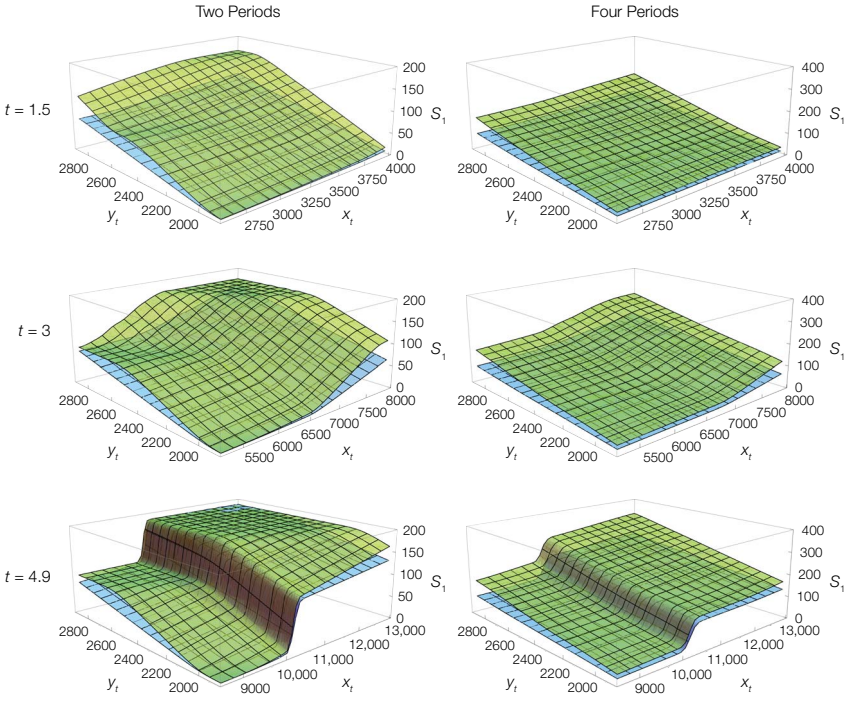
[16] The economy-wide abatement cost function C aggregates the companies' individual functions C^i, as defined in Proposition 6 and its proof, see Sections A and C of the Internet Appendix.

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To illustrate and refine our model results, we begin with the central model implication that emission permits can be characterized as a strip of binary options written on economy-wide emissions, as shown by Proposition 2. Figure 1 nicely illustrates this option characteristics by plotting permit prices S_1 dependent on realized emissions x_t and the business-as-usual emission rate y_t for different points in time within the compliance period $[0, T_1]$. The binary options character becomes clearly visible toward the end of the compliance period and translates to a strongly nonlinear and time-variant dependence of permit prices on emission levels when going back in time.

FIGURE 1
Permit Price S_1 in Emissions Market Equilibrium

The plots in Figure 1 show the permit price S_1 dependent on cumulative realized emissions x_t and the prevailing business-as-usual emission rate y_t. We consider a 2-period and a 4-period setting calibrated to the EU Emissions Trading System (EU ETS) and consider in each case 3 time points within the first compliance period: one and a half ($t = 1.5$) and 3 years ($t = 3$) after the compliance period's start and shortly before the period's end ($t = 4.9$). The green (upper) plots represent the high-abatement-cost scenario, and the blue (lower) plots represent the case of low abatement costs. Parameter values are chosen according to Table 2.

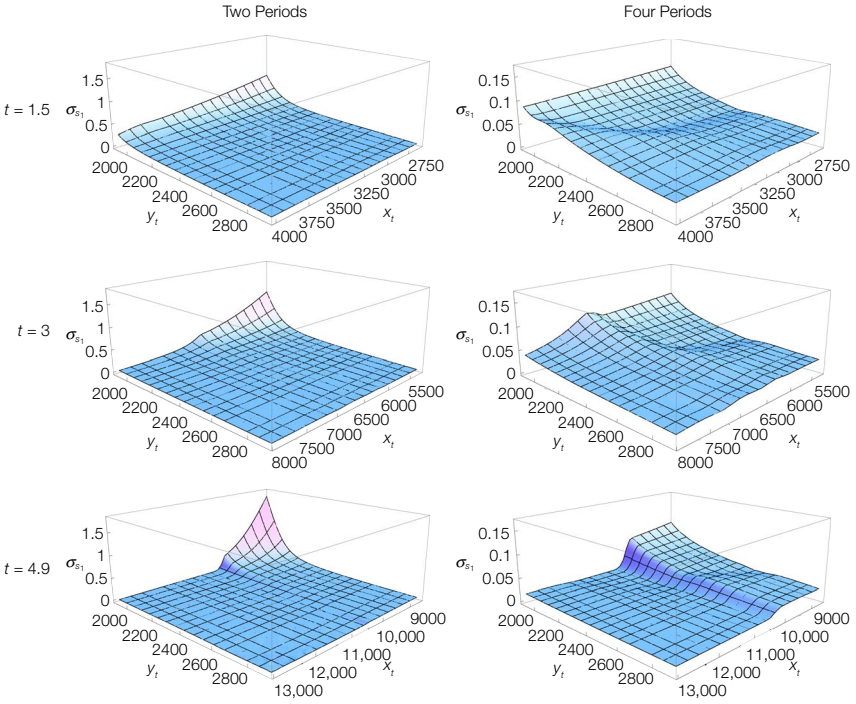


Proposition 3 shows that this option characteristic also implies a strong state and time dependence of permit price volatilities. Whereas we have specifically established that volatility goes to 0 when emission levels (and thus permit prices) are very high, our theoretical results are silent about the behavior of relative volatilities for very low emissions, when both permit prices and price sensitivities approach 0. Figure 2 sheds light on this point and shows that volatilities reach their highest level when emissions are low, suggesting that prices decrease faster for

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FIGURE 2
Relative Local Volatility σ_{S_1} in Emissions Market Equilibrium

The plots in Figure 2 show the relative local volatility σ_{S_1}, according to equation (12), dependent on cumulative realized emissions x_t and the prevailing business-as-usual emission rate y_t. We consider a 2-period and a 4-period setting calibrated to the EU Emissions Trading System (EU ETS) and consider in each case 3 time points within the first compliance period: one and a half ($t = 1.5$) and 3 years ($t = 3$) after the compliance period's start and shortly before the period's end ($t = 4.9$). Parameter values are chosen according to Table 2. We depict volatilities for the low-abatement-cost scenario with coefficient γ_1, noting that the level and shape are very similar for high abatement costs.



falling emissions than their sensitivity to emission changes. Altogether, these results lead to our first empirical prediction:

Prediction 1. Emission permit prices and their instantaneous volatilities are negatively correlated.

A negative relation between prices and volatilities, as we predict for emissions markets, is well established for equity markets, where it is known as the "leverage effect" (Black (1976), Christie (1982)). The technical analogy to emission permits is worth noting: Just like stocks are options on the firm value, emission permits are (strips of binary) options on economy-wide emissions. The resulting distributional properties explain the "leverage effect" in equity markets (Geske (1979)) and a similar effect for emission permits. A difference is that the payoff structure for emission permits as a strip of binary options is much more complex than that for a single option, and the resulting permit price distributions can be various when some of these binary options are in-the-money and some are out-of-the-money. Lower permit prices lead to higher permit price volatilities for

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the most part, but the relation is not completely monotone for medium-emission scenarios, as the plots for the 4-period setting show.

Moreover, characterizing emission permits as strips of binary options provides an understanding of how current and future compliance periods contribute to the value of an emission permit. This is, first, of great relevance for policymakers, who can, to incentivize additional abatement actions, add an additional value component to today's emission permits by announcing further consecutive compliance periods in the future. Additionally, Figure 2 reveals that a higher number of compliance periods leads to lower volatilities,[17] which is often desired, because stable prices increase the confidence in the trading system and tend to trigger early investment in environment-friendly technologies.

Second, understanding how the different compliance periods add to the permit price is also critical for market participants because it directly translates to the forward price curve in emissions markets. Proposition 4 shows that a high probability of permit shortage for the ongoing compliance period leads to a clear backwardation of the forward curve across periods, as the first period's value component is not present in emission permits for the second period. Only if the value component of the first period approaches 0, which is the case if the amount of allocated permits largely exceeds the expected emissions, a (near-)contango situation across periods is possible. Overall, we derive the following testable prediction:

Prediction 2. A contangoed forward price curve in emissions markets implies that the system is severely overallocated. Otherwise, forwards are backwardated across compliance periods, and the degree of backwardation correlates positively (negatively) with the expected amount of exceeding emissions (leftover permits).

Finally, our model results have important implications for option pricing in emissions markets. Options on emission permits are traded with considerable liquidity[18] and allow polluters to manage nonlinear exposures to permit price risk, for example, due to investments in environment-friendly production technologies that are operated only when permit prices are above a certain threshold. Because the underlying of emission permit options is a strip of European binary call options according to Proposition 2, the pricing problem is structurally similar to compound options first studied by Geske (1979). Accordingly, permit option prices deviate from the Black–Scholes benchmark, in which the underlying is assumed to follow a geometric Brownian motion, as a result of the emission permit price dynamics. We discuss the resulting pricing patterns relative to this benchmark in terms of the corresponding volatility smile. In light of Prediction 1, a downward-sloping volatility smile in emissions markets appears plausible due to the negative relation of emission permit prices and their instantaneous volatilities. The shape of the smile is, however, not immediately implied by instantaneous volatilities

[17] A higher number of compliance periods translates to lower permit price volatilities because of two effects: First, the value components attributable to compliance periods in the remote future react less sensitively to changes in today's cumulative emissions or the prevailing emission rate. Second, the tightening allocation in later compliance periods leads to higher values (and thus lower relative volatilities) for these value components, such that the value components coming from additional compliance periods weaken the high relative volatility from the current period and cause a smoothing effect.

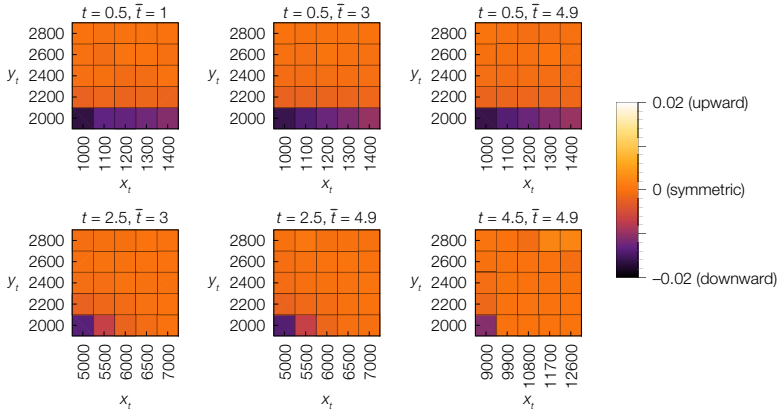
[18] In the EU ETS, emission permit options account for approximately 10% of the overall transaction volume (see Kossoy and Guigon (2012)).

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because it is also affected by the time-dependent volatility behavior until an option's expiry. Therefore, we perform an extensive simulation study to characterize the volatility smile within our calibrated model for a number of different time points, emissions scenarios, and option maturities, where the technical details are outlined in Section D of the Internet Appendix. Figure 3 illustrates the shape of the volatility smile for each of the scenarios considered, and we summarize these results in the form of the following prediction:

FIGURE 3
Slope of Simulated Implied Volatility Smiles

The heat maps in Figure 3 illustrate the slope of the implied volatility smile for emission permit options based on our calibrated model. We simulate option prices for different scenarios of realized emissions x_t, the prevailing emission rate y_t, time t, and the option's maturity $\bar{t}$. Each of the 25 squares in each plot corresponds to one emissions scenario, and the color indicates the slope of the smile according to the given legend. Details of the simulation study are explained in Section D of the Internet Appendix.



Prediction 3. The volatility smile in emissions markets is generally symmetric or downward-sloping, and the downward slope is most pronounced when emissions, and thus permit prices, are low.

Thus, the pricing patterns in permit option markets and the characteristic shape of the volatility smile have some structural similarities with equity options while standing in contrast to the often upward-sloping smile in commodity markets. It should be noted that the volatility smile shapes in our study are induced by the characteristics of permit price distributions under the real measure, and the (possibly state-dependent) risk premia of market participants are not incorporated. The inclusion of risk premia might reduce the downward slope of the smile predicted in our model if market participants perceive high permit prices as bad states of the economy.[19]

[19] We discuss the effects of risk aversion and the role of high emission states more broadly in Section B of the Internet Appendix.

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IV. Empirical Evidence

To test the predictions of our model, we use data from the second compliance period (Phase II) of the EU ETS from 2008 to 2012 and the U.S. Acid Rain Program for sulfur dioxide emissions, which has been in operation since 1995. For the emission permits of the EU ETS, which are called European Union Allowances (EUAs), a rich amount of data on futures and options with a range of different maturities is available. We obtain these data from the European Climate Exchange (ECX), the world's largest carbon futures exchange,[20] and select the yearly maturities in December for our analysis, which are the most liquid contracts in this market. As highlighted in Table 3, next-to-maturity December futures are traded with an average volume of 10,238 contracts per day and an open interest of 158,375 contracts on average, demonstrating the remarkable liquidity of this relatively new market. Trading volumes and open interest are lower but still considerable for longer-maturity futures and for option contracts, providing reliable grounds for an empirical analysis of our model predictions.

TABLE 3

Summary Statistics of EUA Futures and Option Data

Table 3 presents daily trading volumes and open interest for our sample of European Union Allowance (EUA) futures and options from Jan. 2008 to June 2012. Volumes and open interest are denoted in number of contracts and computed over all contracts that are traded at least once on a particular trading day. We consider EUA futures with yearly maturities in December, with a rollover date at the end of October. EUA options are written on EUA futures with the same maturity, and we restrict our sample to next-to-maturity December options with moneyness between 0.8 and 1.2. The data are provided by the European Climate Exchange (ECX), operated by Intercontinental Exchange (ICE).

	Volume		Open Interest	
Contract	Mean	Std. Dev.	Mean	Std. Dev.
EUA Futures				
Next to maturity	10,238.46	5,949.41	158,375.47	83,500.48
Second-next to maturity	2,468.27	2,218.19	115,194.20	90,442.44
Third-next to maturity	1,281.00	1,280.45	68,245.18	60,176.09
Fourth-next to maturity	543.04	743.69	25,174.79	24,061.82
EUA Options (next to maturity)				
Calls	576.10	609.40	7,435.73	4,811.21
Puts	528.99	503.20	7,029.01	4,642.82

As detailed in the following discussion, we consider the next-to-maturity December futures contract as the spot contract, and we use intraday data to compute realized volatilities for testing Prediction 1. Longer-maturity EUA futures allow us to investigate Prediction 2 on the slope of the forward price curve in emissions markets. For this prediction, we additionally extend our analysis to a unique sample of emission permit term structures from the U.S. Acid Rain Program for sulfur dioxide emissions. Finally, we test Prediction 3 on the volatility smile in emissions markets based on EUA options traded in the EU ETS.

[20]The ECX is operated by Intercontinental Exchange (ICE), http://www.theice.com. Daily data are provided directly by the European Climate Exchange (ECX), whereas we obtain intraday data from Thomson Reuters. We thank SIRCA for providing access to the Thomson Reuters DataScope Tick History archive, http://www.sirca.org.au.

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A. Emission Permit Prices and Volatilities

According to Prediction 1, volatilities in emissions markets are high when prices are low, and *vice versa*. We evaluate this prediction by analyzing daily EUA prices and volatilities during Phase II of the EU ETS. To calculate the volatility $\widehat{\sigma}(t)$ on day t, we employ a model-free approach based on intraday price data given by

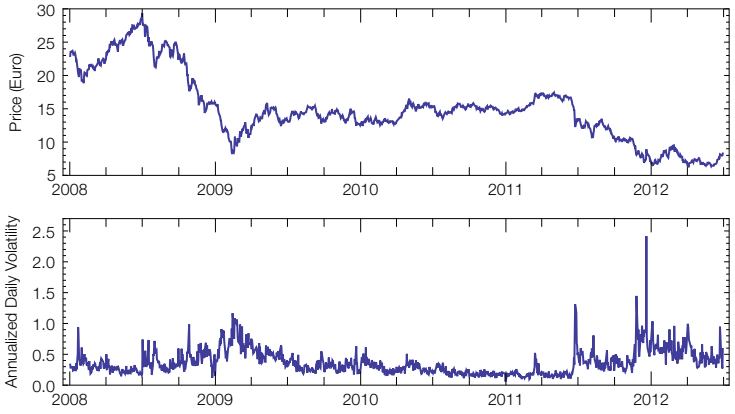
$$\widehat{\sigma}(t) \quad = \quad \sqrt{252 \sum_{i=1}^{N} R_{t,i}^2},$$

where $R_{t,i}$, $i = 1, \ldots, N$ are 15-minute intraday log returns, and we annualize according to a number of 252 trading days per year. Because intraday prices for actual spot EUAs are only limitedly available due to partially thin trading, we base our analysis on next-to-maturity EUA futures instead, which can be considered equivalently due to the standard cost-of-carry relation within a compliance period.[21] We construct a time series of intraperiod futures prices and volatilities from Jan. 2008 to June 2012 by taking the December EUA futures next to maturity and rolling over to the next contract after the last day of October.

The time-varying nature of permit price volatilities is clearly observable in Figure 4. Volatilities increase from the beginning of the sample until the first quarter of 2009, before declining to a comparably low level in 2010 and the first half of 2011. In the second half of 2011 and in 2012, daily volatilities are significantly

FIGURE 4
Prices and Realized Volatilities of EUA Futures

The plots in Figure 4 show the prices and realized volatilities of European Union Allowance (EUA) futures for our sample period from Jan. 2008 to June 2012. We construct the time series by considering the December futures next to maturity and rolling over to the next contract at the end of October. Daily prices are provided by the European Climate Exchange (ECX), operated by Intercontinental Exchange (ICE). Realized volatilities are computed on a daily basis using a model-free high-frequency approach based on 15-minute price data provided by the Thomson Reuters DataScope Tick History archive.



[21] The standard cost-of-carry relationship within a compliance period is predicted by our model (see Proposition 4). Because this result is empirically confirmed by the existing literature (see Uhrig-Homburg and Wagner (2009), Rittler (2012)), we refrain from an empirical evaluation here.

higher again. A comparison with the EUA price level clearly reveals that higher volatilities generally correspond to lower prices. Prices are lowest in the beginning of 2009 and in 2012, when volatilities reach their highest level. As reported in Table 4, the sample correlation of daily prices and volatilities is −0.464 and highly statistically significant, which clearly confirms the prediction.

TABLE 4
Statistical Test of Empirical Predictions

We test our empirical predictions on the correlation of permit prices and volatilities, the correlation of leftover permits and the backwardation of the forward price curve, and the correlation of permit prices and the slope of the volatility smile. Table 4 reports Spearman's rank correlation and the corresponding *p*-values for the respective time series. We use data from Phase II of the EU Emissions Trading System (EU ETS) to test Predictions 1 and 3, whereas Prediction 2 is tested based on data from the U.S. Acid Rain Program for sulfur dioxide emissions.

	Estimate	*p*-Value
Prediction 1. Correlation of permit prices and volatilities	−0.464	<0.001
Prediction 2. Correlation of leftover permits and backwardation	−0.426	0.088
Prediction 3. Correlation of permit prices and the slope of the volatility smile	0.730	<0.001

B. Forward Price Curve

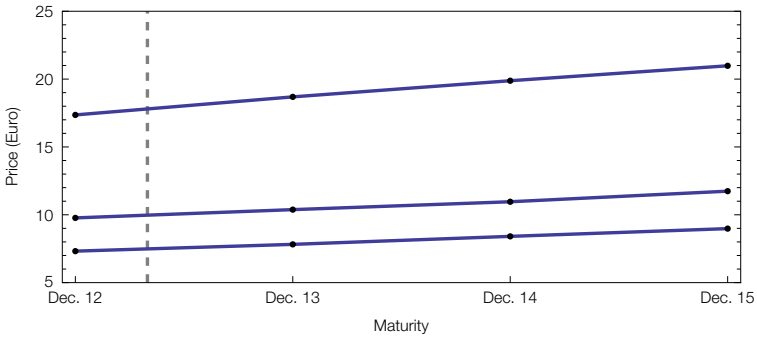
Prediction 2 states that the forward price curve across compliance periods is generally backwardated, and the backwardation increases in the expected amount of exceeding emissions. That means that a (near-)contango situation is only possible in case of a large overallocation of permits, whereas the backwardation should be particularly pronounced if there is a severe permit scarcity in the ongoing compliance period. For Phase II of the EU ETS, market reports clearly indicate a considerable permit surplus that was realized from 2011 on. Thus, the backwardation of EUA futures maturing in the following compliance period should be small according to our predictions. As illustrated by Figure 5 for 3 selected exemplary days, the forward price curve is indeed virtually in contango across the compliance periods during the years 2011 and 2012. Although this correspondence of low emissions expectations to the lack of backwardation in the EU ETS is in line with our prediction, the evidence could be strengthened by analyzing a scenario of high expected emissions as well. Unfortunately, EUA futures maturing in Dec. 2013 or later were not liquidly traded before 2011, such that interperiod futures data for a scenario with higher expected emissions are not available for the EU ETS.

Therefore, we additionally investigate data from the U.S. Acid Rain Program for sulfur dioxide (SO_2) emissions, which provides a unique sample to analyze the backwardation across compliance periods dependent on emission levels. Although the SO_2 market is extensively studied in the economics literature (e.g., Joskow, Schmalensee, and Bailey (1998), Schmalensee, Joskow, Ellerman, Montero, and Bailey (1998)), this aspect is not explored by existing research. The basic rules of the SO_2 emission trading system are very similar to those of the EU ETS and thus are in line with our model assumptions. In our terminology, every single year is a compliance period in the SO_2 system because borrowing is prohibited between different years. There is an SO_2 permit auction in the first quarter of each year, which provides prices for the spot permits of the ongoing

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FIGURE 5
EUA Futures Price Curves

The plot in Figure 5 illustrates the futures price curve for European Union Allowance (EUA) futures on three selected exemplary dates: May 18, 2011, Nov. 18, 2011, and Apr. 18, 2012. We consider the last December futures contract that matures in Phase II and the three next December maturities in Phase III to analyze the degree of backwardation across compliance periods. The dashed vertical line marks Apr. 30, 2013, the compliance deadline of Phase II.



year as well as for 7-year advance permits to be used not earlier than 7 years in the future. We further obtain data on the overall amount of leftover permits at the end of each year, which market participants banked into the next year.[22] Assuming rational expectations, these data proxy the ex ante expectation of emissions, such that a high (low) realized number of leftover permits also indicates low (high) ex ante expected emissions for that year. Overall, we obtain 17 yearly observations from 1995 to 2011 to analyze the backwardation in the SO_2 market,[23] as illustrated in Figure 6. In neat accordance with Prediction 2, the backwardation is strongest in the years 2005 and 2006 when the permit surplus is on a low level. In these cases, the ex ante probability of a permit shortage at the end of the ongoing compliance period is relatively high, and the spot emission permits can be used to avoid penalty payments. This is not possible, however, with 7-year advance permits, which leads to the backwardation. In the years before 2005, it is clearly observable how an advancing scarcity of permits leads to a higher backwardation, and in the same way, an increasing overallocation after 2006 reduces the observable backwardation more and more. Overall, we find a negative correlation between leftover permits and the degree of backwardation in the SO_2 market (see Table 4), which confirms Prediction 2.

C. Volatility Smile

Prediction 3 states that the volatility smile in emissions markets is symmetric or downward-sloping, with the strongest downward slope for scenarios when emissions (and therefore permit prices) are low. We investigate this result based on

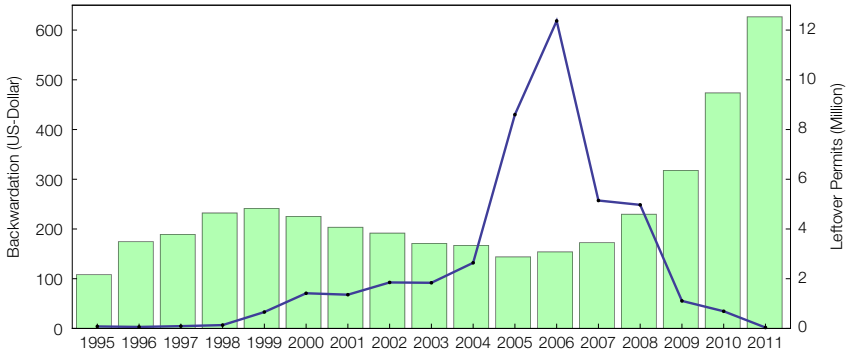
[22] Data on the yearly SO_2 permit auctions and on leftover permits are published by the U.S. Environmental Protection Agency (EPA) (see http://www.epa.gov).

[23] Starting in 2012, the circumstances of the Acid Rain Program fundamentally changed when the EPA replaced the Clean Air Interstate Rule (CAIR) with the Cross-State Air Pollution Rule (CSAPR). We exclude the years 2012 and later from our sample because the prices of spot and 7-year advance permits have virtually gone to 0 in consequence of the regulation changes.

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FIGURE 6
Backwardation in the U.S. Acid Rain Program

The plot in Figure 6 shows the interperiod backwardation in the U.S. Acid Rain Program for sulfur dioxide emissions for each year from 1995 to 2011 compared to the leftover permits at the end of the year. We obtain the backwardation (solid line) for a particular year as the price difference between spot permits and 7-year advance permits that are auctioned in the first quarter of that year. The leftover permits at the end of the year (bars) are taken as a proxy for the ex ante expectation at the time of the auction. Data are provided by the U.S. Environmental Protection Agency.



our sample of EUA options, focusing on the most liquidly traded next-to-maturity December options.[24] To compute volatility smiles, we take the difference of each option's implied volatility to the at-the-money implied volatility for each trading day and aggregate the resulting *relative* implied volatilities by moneyness through time.

Figure 7 illustrates the average volatility smiles for the years 2010, 2011, and 2012.[25] We observe that the smile is clearly downward-sloping for each of the 3 years. Moreover, the decrease of EUA prices from 2010 to 2012 is accompanied by an increasing downward slope of the volatility smile. This pattern is exactly in line with Prediction 3, showing that scenarios of very low realized and expected emissions translate to a strongly downward-sloping volatility smile. We formally test this prediction by considering the slope $(\widehat{IV}_{m^+} - \widehat{IV}_{m^-})/(\widehat{m}^+ - \widehat{m}^-)$ of the volatility smile for each month, where $\widehat{IV}_{m^+}$ ($\widehat{IV}_{m^-}$) is the average *relative* implied volatility, and $\widehat{m}^+$ ($\widehat{m}^-$) is the average moneyness of all options with moneyness $m > 1$ ($m < 1$) traded during that month.[26] As reported in Table 4, the

[24] As for futures contracts, we roll over to the December options of the following year at the end of October. Options traded at prices smaller than 0.06 euro and observations that violate simple no-arbitrage conditions are excluded from our sample, in line with the option pricing literature (e.g., Bakshi, Cao, and Chen (1997)), and we only consider options that are traded at least once on a particular trading day. We restrict our analysis to options with moneyness (defined as the strike divided by the underlying price) between 0.8 and 1.2, and we define the traded option that is closest to a moneyness of 1 on a particular day as at-the-money.

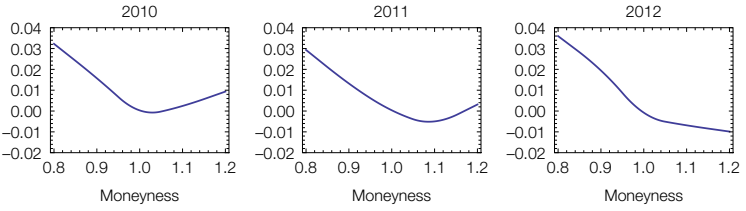
[25] Before 2010, the smile was completely flat because the market was still in an immature state, and investors simply priced options with the standard Black or Black–Scholes formula (see also Carmona and Hinz (2011), Hitzemann and Uhrig-Homburg (2018)). We therefore investigate Prediction 3 from 2010 onward.

[26] Note that our empirical procedure is consistent with the simulation study in Section III, where we define the smile's slope with respect to strike prices K as $(IV_{K^+} - IV_{K^-})/(K^+ - K^-)$ (see Section D of the Internet Appendix). Computing the slope with respect to moneynesses $m = K/F(t,\bar{t})$ in the empirical part better allows us to aggregate observations through time and effectively scales the slope

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FIGURE 7
Implied Volatility Smiles of EUA Options

The plots in Figure 7 show the average implied volatility smiles of European Union Allowance (EUA) options for the years 2010, 2011, and 2012. Smiles are calculated as the difference of each option's implied volatility and the at-the-money implied volatility for each trading day and aggregated by moneyness (defined as the strike divided by the underlying price) through time. We select the next-to-maturity December options for our analysis, with rollover at the end of October.



correlation between the slope of the volatility smile and the EUA price is 0.730 and highly statistically significant, providing strong support for Prediction 3.

Overall, we conclude that all our testable model predictions are confirmed by data from existing emissions markets. It can be seen as a natural next step to conduct a more rigorous analysis of our model by evaluating its fit to empirical emission permit prices and related derivatives compared with classical standard models for asset price dynamics. This approach is taken by Hitzemann and Uhrig-Homburg (2018). Indeed, the authors show that reduced-form variants of our model deduced from Proposition 2 are superior regarding both their historical model fit to futures prices and their option pricing performance.

V. Conclusion

We develop a stochastic equilibrium model for environmental markets that explicitly accounts for the specific design features of today's cap-and-trade systems. Our model reveals how the design of current emission trading systems translates to characteristic features of emission permit prices and related derivatives. In particular, we show that an emission permit can be characterized as a strip of European binary options written on economy-wide emissions, and this option characteristic gives rise to several general properties of emission permit prices. Further, our model reveals the hybrid nature of emission permits between investment and consumption assets, which is reflected by a partly contangoed and partly backwardated forward price curve. We show that our model captures the stylized facts of emission permit prices and related derivatives according to empirical evidence from existing emissions markets.

By providing a consistent framework for the price dynamics of emission permits resulting from the design of current emission trading systems, our model may well be applied by policymakers to evaluate measures that aim at influencing permit prices by adjusting a running system. As a concrete example, consider the extensively discussed proposal to increase the emissions reduction goal of the European Union to 30% compared with 1990 emission levels, which equals a

by the current futures price $F(t,\bar{t})$. This scaling is conservative in that it reduces the downward slope when prices are low and therefore biases us against finding support for Prediction 3.

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retirement of about 1,400 million permits in Phase III from 2013 to 2020.[27] Our calibrated setting with two compliance periods and low abatement costs predicts that such an adjustment would lead to an increase of emission permit prices in the middle of Phase II by approximately 85% from 12.81 to 23.76 euros.[28] Thus, our model indicates that such a structural measure could be an effective tool against the prevailing low permit prices in the EU ETS. In addition, our framework also allows the inference of information about market expectations of future emissions from related derivatives, enabling the regulator to take this information into account for important policy decisions.

References

Baker, S. D.; B. Hollifield; and E. Osambela. "Disagreement, Speculation, and Aggregate Investment." *Journal of Financial Economics*, 119 (2016), 210–225.

Bakshi, G.; C. Cao; and Z. Chen. "Empirical Performance of Alternative Option Pricing Models." *Journal of Finance*, 52 (1997), 2003–2049.

Bansal, R.; D. Kiku; and M. Ochoa. "Price of Long-Run Temperature Shifts in Capital Markets." NBER Working Paper No. 22529 (2017).

Black, F. "Studies of Stock Price Volatility Changes." In *Proceedings of the 1976 Meetings of the American Statistical Association* (1976), 177–181.

Brennan, M. J. "The Supply of Storage." *American Economic Review*, 48 (1958), 50–72.

Calel, R., and A. Dechezleprêtre. "Environmental Policy and Directed Technological Change: Evidence from the European Carbon Market." *Review of Economics and Statistics*, 98 (2016), 173–191.

Carmona, R.; F. Delarue; G.-E. Espinosa; and N. Touzi. "Singular Forward–Backward Stochastic Differential Equations and Emissions Derivatives." *Annals of Applied Probability*, 23 (2013), 1086–1128.

Carmona, R., and M. Fehr. "The Clean Development Mechanism and Joint Price Formation for Allowances and CERs." In *Progress in Probability,* Vol. 63, Seminar on Stochastic Analysis, Random Fields and Applications (2011), 341–383.

Carmona, R.; M. Fehr; and J. Hinz. "Optimal Stochastic Control and Carbon Price Formation." *SIAM Journal on Control and Optimization*, 48 (2009), 2168–2190.

Carmona, R.; M. Fehr; J. Hinz; and A. Porchet. "Market Design for Emission Trading Schemes." *SIAM Review*, 52 (2010), 403–452.

Carmona, R., and J. Hinz. "Risk-Neutral Models for Emission Allowance Prices and Option Valuation." *Management Science*, 57 (2011), 1453–1468.

Casassus, J., and P. Collin-Dufresne. "Stochastic Convenience Yield Implied from Commodity Futures and Interest Rates." *Journal of Finance*, 60 (2005), 2283–2331.

Cetin, U., and M. Verschuere. "Pricing and Hedging in Carbon Emissions Markets." *International Journal of Theoretical and Applied Finance*, 12 (2009), 949–967.

Chesney, M., and L. Taschini. "The Endogenous Price Dynamics of Emission Allowances and an Application to CO_2 Option Pricing." *Applied Mathematical Finance*, 19 (2012), 447–475.

Chevallier, J. "Modelling the Convenience Yield in Carbon Prices Using Daily and Realized Measures." *International Review of Applied Financial Issues and Economics*, 1 (2009), 56–73.

Christie, A. A. "The Stochastic Behavior of Common Stock Variances: Value, Leverage and Interest Rate Effects." *Journal of Financial Economics*, 10 (1982), 407–432.

Cline, W. R. *Carbon Abatement Costs and Climate Change Finance*. Washington DC: Peterson Institute for International Economics (2011).

Coase, R. H. "The Problem of Social Cost." *Journal of Law and Economics*, 3 (1960), 1–44.

Cronshaw, M. B., and J. Kruse. "Regulated Firms in Pollution Permit Markets with Banking." *Journal of Regulatory Economics*, 9 (1996), 179–189.

[27] Report COM(2012) 652 from the European Commission on the state of the European carbon market in 2012 discusses this option as one of six possible structural measures for the EU ETS.

[28] We estimate the emission levels x_t and y_t in line with yearly emissions data published by the European Commission. Details of this calculation are available from the authors.

https://doi.org/10.1017/S0022109018000297 Published online by Cambridge University Press

Daskalakis, G.; D. Psychoyios; and R. N. Markellos. "Modeling CO_2 Emission Allowance Prices and Derivatives: Evidence from the European Trading Scheme." *Journal of Banking and Finance*, 33 (2009), 1230–1241.

Ellerman, A. D., and B. K. Buchner. "Over-Allocation or Abatement? A Preliminary Analysis of the EU ETS Based on the 2005-06 Emissions Data." *Environmental and Resource Economics*, 41 (2008), 267–287.

Geske, R. "The Valuation of Compound Options." *Journal of Financial Economics*, 7 (1979), 63–81.

Hahn, R. W. "Market Power and Transferable Property Rights." *Quarterly Journal of Economics*, 99 (1984), 753–765.

Hitzemann, S., and M. Uhrig-Homburg. "Empirical Performance of Reduced-Form Models for Emission Permit Prices." SSRN Working Paper No. 2297121 (2018).

Jarrow, R. A. "Convenience Yields." *Review of Derivatives Research*, 13 (2010), 25–43.

Joskow, P. L.; R. Schmalensee; and E. M. Bailey. "The Market for Sulfur Dioxide Emissions." *American Economic Review*, 88 (1998), 669–685.

Kijima, M.; A. Maeda; and K. Nishide. "Equilibrium Pricing of Contingent Claims in Tradable Permit Markets." *Journal of Futures Markets*, 30 (2010), 559–589.

Klepper, G., and S. Peterson. "Marginal Abatement Cost Curves in General Equilibrium: The Influence of World Energy Prices." *Resource and Energy Economics*, 28 (2006), 1–23.

Kossoy, A., and P. Guigon. *State and Trends of the Carbon Market*. Washington, DC: The World Bank (2012).

Litzenberger, R. H., and N. Rabinowitz. "Backwardation in Oil Futures Markets: Theory and Empirical Evidence." *Journal of Finance*, 50 (1995), 1517–1545.

Montgomery, W. D. "Markets in Licenses and Efficient Pollution Control Programs." *Journal of Economic Theory*, 5 (1972), 395–418.

Nauclér, T., and P.-A. Enkvist. *Pathways to a Low-Carbon Economy. Version 2 of the Global Greenhouse Gas Abatement Cost Curve*. New York, NY: McKinsey & Company (2009).

Rittler, D. "Price Discovery and Volatility Spillovers in the European Union Emissions Trading Scheme: A High-Frequency Analysis." *Journal of Banking and Finance*, 36 (2012), 774–785.

Routledge, B. R.; D. J. Seppi; and C. S. Spatt. "Equilibrium Forward Curves for Commodities." *Journal of Finance*, 55 (2000), 1297–1338.

Rubin, J. D. "A Model of Intertemporal Emission Trading, Banking, and Borrowing." *Journal of Environmental Economics and Management*, 31 (1996), 269–286.

Schennach, S. "The Economics of Pollution Permit Banking in the Context of Title IV of the 1990 Clean Air Act Amendments." *Journal of Environmental Economics and Management*, 40 (2000), 189–210.

Schmalensee, R.; P. L. Joskow; A. D. Ellerman; J. P. Montero; and E. M. Bailey. "An Interim Evaluation of Sulfur Dioxide Emissions Trading." *Journal of Economic Perspectives*, 12 (1998), 53–68.

Schwartz, E. S. "The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging." *Journal of Finance*, 52 (1997), 923–973.

Seifert, J.; M. Uhrig-Homburg; and M. W. Wagner. "Dynamic Behavior of CO_2 Spot Prices." *Journal of Environmental Economics and Management*, 56 (2008), 180–194.

Stavins, R. N. "Transaction Costs and Tradeable Permits." *Journal of Environmental Economics and Management*, 29 (1995), 133–148.

Trolle, A. B., and E. S. Schwartz. "Unspanned Stochastic Volatility and the Pricing of Commodity Derivatives." *Review of Financial Studies*, 22 (2009), 4423–4461.

Uhrig-Homburg, M., and M. Wagner. "Futures Price Dynamics of CO_2 Emission Allowances: An Empirical Analysis of the Trial Period." *Journal of Derivatives*, 17 (2009), 73–88.

https://doi.org/10.1017/S0022109018000297 Published online by Cambridge University Press