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Particle acceleration in plasmas by perpendicularly propagating waves

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Abstract. The acceleration of particles to high energy by relativistic plasma waves has received a great deal of attention lately. Most of the particle-acceleration schemes using relativistic plasma waves rely either on intense terawatt or petawatt lasers or on electron beams as the driver of the acceleration wave. These laboratory experiments have attained accelerating fields as high as 1 GeV cm^{-1} with the electrons being accelerated to about 100 MeV in millimetre distances. In space and astrophysical plasmas, relativistic plasma waves can also be important for acceleration. A process that is common to both laboratory and space plasmas is the surfatron concept, which operates as a wave acceleration mechanism in a magnetized plasma. In this paper, we present test-particle results for the surfatron process.

Dedication

This paper is dedicated to John Dougherty on the occasion of his 65th birthday.

1. Introduction

In view of John Dougherty's seminal contributions to both laboratory and space plasma physics, we wish to contribute to this special issue in his honour a brief discussion of a subject that may also be relevant to both of these fields. The study of acceleration processes remains one of the most important areas of research in both laboratory and astrophysical plasmas. In this paper, we shall concentrate on a particular process that occurs in both space and laboratory plasmas, namely the surfatron acceleration scheme (Katsouleas and Dawson 1983). This scheme for particle acceleration was proposed to overcome the dephasing limit suffered by relativistic plasma-wave accelerators such as the plasma beat-wave accelerator (PBWA), the laser wakefield accelerator (LWFA) and the plasma wakefield accelerator (PWFA). In these schemes, a relativistic plasma wave is generated using either intense laser or electron beams (Tajima and Dawson 1979; Chen et al. 1985).

Laboratory experiments have attained accelerating fields as high as 100 GeV cm^{-1}, with electrons being accelerated to about 100 MeV in millimetre distances (Gordon et al. 1998). In these experiments, one of the limiting factors to the final energy gain is associated with dephasing. This occurs when a resonant particle gains energy and starts to move faster than the wave. It out-runs the wave and

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reaches the bottom of the accelerating potential well, after which it experiences a decelerating force and loses energy.

To overcome the dephasing limit, Katsouleas and Dawson (1983) suggested applying a static magnetic field **B** perpendicular to the direction of propagation of the electrostatic wave. Particles trapped by the wave electric field propagate at the phase velocity of the wave $v_{\rm ph}$, perpendicular to the magnetic field. These trapped particles see an electric field $\mathbf{v}_{\rm ph} \times \mathbf{B}$, which accelerates them parallel to the wavefront until the $q\left(\mathbf{v}_{\rm ph} \times \mathbf{B}\right)$ force is large enough to overcome the trapping field of the wave. The maximum energy that a particle can gain from the plasma beat-wave accelerator in the absence of a magnetic field is $2\left(\omega^2/\omega_p^2\right) mc^2$, where ω is the laser frequency, ω_p is the plasma frequency, m is the particle mass and c is the speed of light.

In the relativistic surfatron, the particle can in principle gain energy indefinitely. In reality, it can only gain that which is in the wave to begin with, although in practice the wave front has only a finite width, which will ultimately limit the maximum energy gain. In space and astrophysical plasmas, wave acceleration is usually stochastic in nature, with the time for acceleration being long in comparison with the change in the phase of the field. In laboratory plasmas, the acceleration can be either stochastic or non-stochastic. For non-stochastic (sometimes referred to as 'coherent') acceleration, the time for acceleration is short in comparison with the time for the field to change. A laboratory example of coherent wave acceleration is the laser or electron-beam plasma accelerator.

In astrophysics, an important problem in cosmic-ray acceleration in shocks is the source of the low-energy seed population that is subsequently accelerated to higher energies by the Fermi process. A possible acceleration mechanism for these lower-energy particles arises from resonance with plasma waves propagating in a magnetic field by the surfatron process. Closer to home, the Sun produces copious amounts of energetic particles. The observations of γ rays due to nuclear collisions require protons of GeV energies, and hard MeV X rays from bremsstrahlung require relativistic electrons. The observations of γ rays followed closely by the detection of GeV protons requires a prompt acceleration process. Acceleration of electrons and ions by large-amplitude magnetosonic waves to relativistic energies has been demonstrated in numerical simulations (Lembege and Dawson 1989). Simulations of particle acceleration with intense electron magnetic pulses in underdense electron–positron plasmas surrounding pulsars demonstrate non-stochastic acceleration (Ashour-Abdalla et al. 1981).

Production of energetic electrons and ions at low-Mach-number (Ohsawa 1986) and high-Mach-number shocks demonstrate the importance of coherent surfatron acceleration in astrophysical shocks, covering everything from the Earths' bow shock to supernovae remnants. Acceleration of particles by plasma propagating perpendicular to the magnetic field is of fundamental importance in laboratory space and astrophysical plasmas.

In this paper, we shall outline the main principles involved, and demonstrate the acceleration process associated with plasma waves propagating perpendicular to the ambient magnetic field by the non-stochastic surfatron process.

2. Surfatron acceleration

Waves such as Bernstein waves, lower-hybrid magnetosonic and upper-hybrid waves, which all propagate with a component of their electric field perpendicular

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to the ambient magnetic field, all have a common damping mechanism resulting in energization of particles perpendicular to the magnetic field. The energization mechanism can be of a stochastic or non-stochastic nature. Karney (1979) described stochastic heating of ions by lower-hybrid waves as a random interaction of waves and particles, with a resonant point in the particles' cyclotron orbit where the particles receive kicks allowing them to diffuse in velocity space. At a particular amplitude, known as the stochastic threshold, significant heating is possible. This threshold is given by Karney (1979) as

$$\frac{E}{v_{\rm ph}B} = \frac{1}{4}\left(\frac{\omega}{\omega_c}\right)^{1/3},$$

where E is the wave-field amplitude, ω is the wave frequency, $v_{\rm ph}$ is the phase velocity and ω_c is the cyclotron frequency. For wave amplitudes below this threshold, we have the linear non-stochastic regime, for $\frac{1}{4}(\omega/\omega_c)^{1/3} < E/v_{\rm ph}B < 1$, we are in the stochastic regime identified by Karney (1979), and for $E/v_{\rm ph}B > 1$, we are in the non-stochastic nonlinear regime. Most attention has been paid to the stochastic regime resulting in plasma heating. The non-stochastic regime has been concerned with particle acceleration for both non-relativistic and relativistic particles. The essential elements of surfatron acceleration in the relativistic non-stochastic regime were derived by Katsouleas and Dawson (1983), although it had been used by a number of authors in different areas (Cairns 1971; Sagdeev and Shapiro 1973; Sugihara and Midzeino 1976; Lembege and Dawson 1983; Ohsawa 1990). It has been used to explain electron energization observed in simulations of stimulated Raman scattering in magnetized plasmas (Dawson et al. 1983), and more recently to overcome the main weakness of diffusive shock acceleration by providing an acceleration mechanism for low-energy particles known as the injection problem (Lee et al. 1996).

A test-particle description of the surfatron acceleration model can easily be described using relativistic equations for the particle. Considering a magnetic field in the z direction and a wave with a longitudinal electric field moving in the y direction, the equations of motion for an electron are

$$\frac{dP_x}{dt} = \frac{-eBP_y}{m_0\gamma}, \tag{1}$$

$$\frac{dP_y}{dt} = \frac{eBP_x}{m_0\gamma} - eE\sin(kx-\omega t), \tag{2}$$

$$\frac{dy}{dt} = \frac{P_y}{m_0\gamma}, \tag{3}$$

where m_0 is the electron rest mass and P_i is the particle momentum ($=\gamma m v_i$). If ω_c is the cyclotron frequency (calculated using the rest mass ($=eB/m_0$), we can normalize time in units of $1/\omega_c$, length in units of c/ω_c and momentum in units of m_0c; we also transform the y coordinate such that $y \to y - \alpha t$, where $\alpha = \omega/k$ is the wave phase speed. Then we obtain the dimensionless equations of motion

$$\frac{dp_x}{dt} = -\frac{p_y}{\gamma}, \tag{4}$$

$$\frac{dp_y}{dt} = \frac{p_x}{\gamma} - \beta\sin(ky), \tag{5}$$

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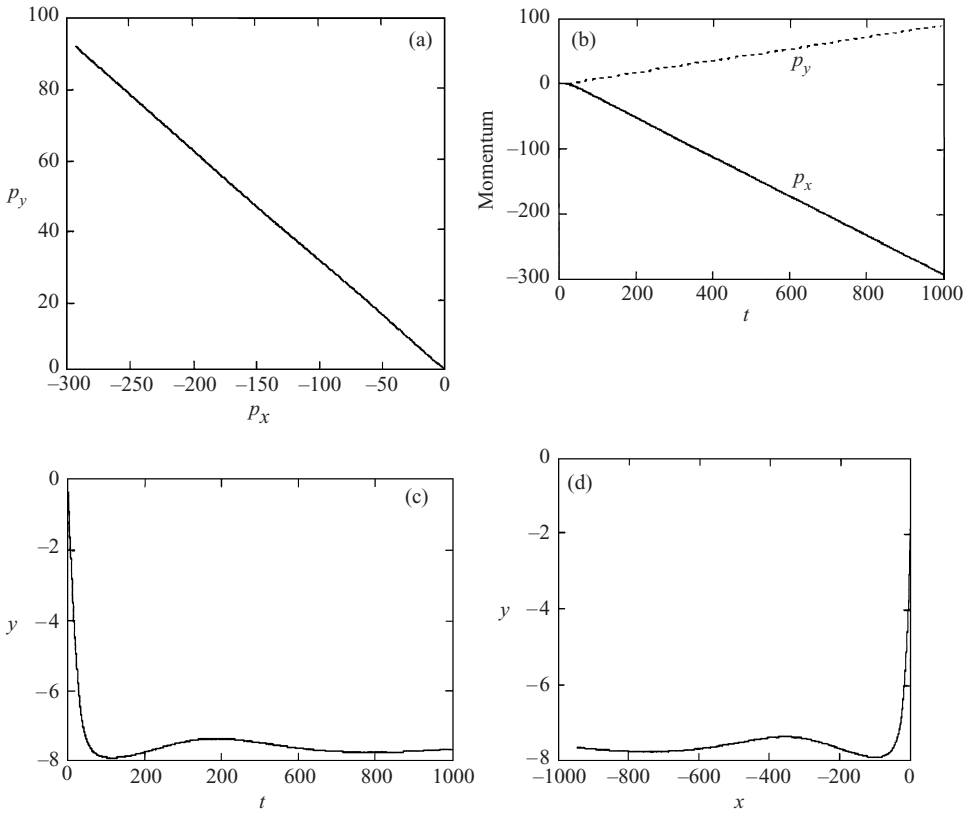


Figure 1. Numerical solutions of (4)–(6) for a wave phase speed $\alpha = \omega/k = 0.3$, $k = 0.2$, and a value of β (1.05) above the critical value of 1.048: (a) phase-plane diagram for the x and y momenta; (b) momenta as functions of time; (c) y as a function of time; (d) trajectory in the (x, y) plane.

$$\frac{dy}{dt} = \frac{p_y}{\gamma} - \alpha, \tag{6}$$

where

$$\beta = \frac{eE}{mc\omega_c} = \frac{E}{cB}.$$

Consider a solution to (4)–(6) in which there is steady acceleration of the particle. If $dy/dt = 0$, corresponding to no phase change of the particle with respect to the wave, then, from (4) and (6), we get $dp_x/dt = -\alpha$. If we assume that at the same time, $dp_y/dt = \eta$, and that we are in the asymptotic regime where

$$p_x \approx -\alpha t, \tag{7}$$

$$p_y \approx \eta t, \tag{8}$$

$$\gamma = (1 + p_x^2 + p_y^2)^{1/2} \approx p_x^2 + p_y^2, \tag{9}$$

then (4) implies that

$$\alpha = \frac{-\eta}{(\alpha^2 + \eta^2)^{1/2}}, \tag{10}$$

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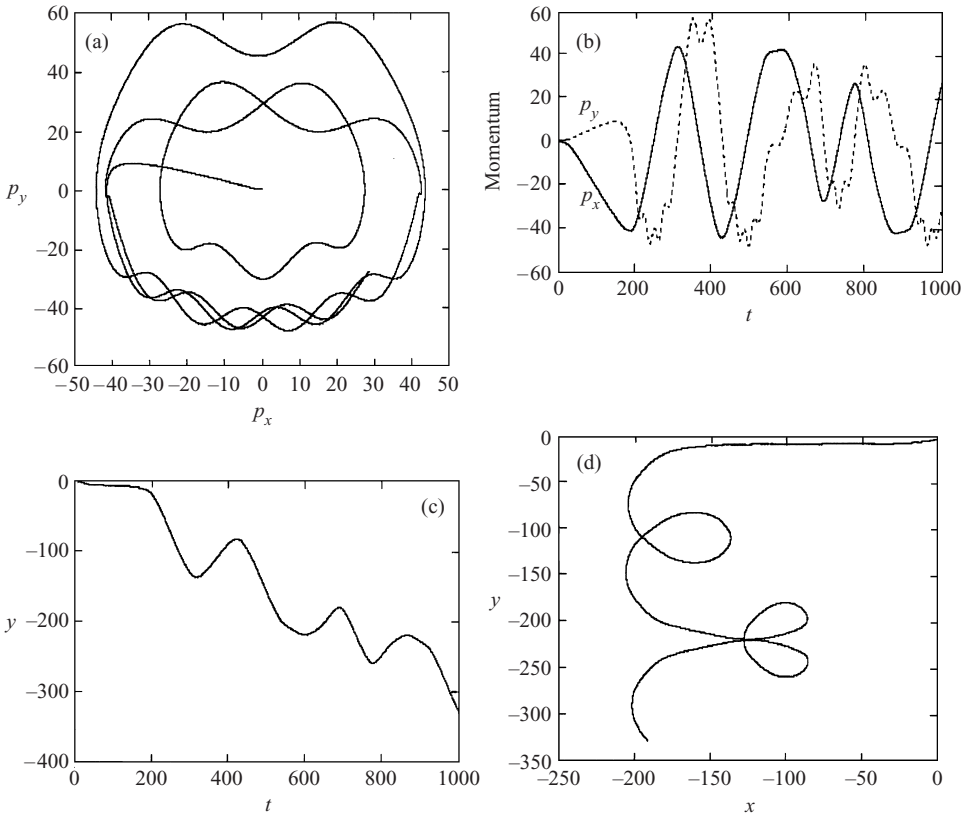


Figure 2. As Fig. 1, but for a value of β (1.045) below the critical value.

or

$$\eta = \frac{\alpha^2}{(1-\alpha^2)^{1/2}}. \tag{11}$$

We are assuming that $\alpha < 1$, i.e. that the wave phase velocity is less than c. For these relations to be consistent with (2), we need also

$$\eta = -\frac{\alpha}{(\alpha^2+\eta^2)^{1/2}} - \beta\sin(ky_0) \tag{12}$$

where y_0 is the constant value of y. Rearranging (12) gives

$$-\beta\sin(ky_0) = \frac{1}{(1-\alpha^2)^{1/2}}, \tag{13}$$

which has no possible solution unless

$$\beta \geqslant \frac{1}{(1-\alpha^2)^{1/2}}. \tag{14}$$

The condition (14) gives a threshold wave amplitude above which we may expect to find solutions in which the particle travels along the wave front, continuously gaining energy; the threshold value is $E \approx \gamma_{\rm ph} cB$, which is the relativistic trapping threshold, where $\gamma_{\rm ph} = (1-\alpha^2)^{-1/2}$. A trapped particle satisfying $E = \gamma_{\rm ph} cB$ can never detrap.

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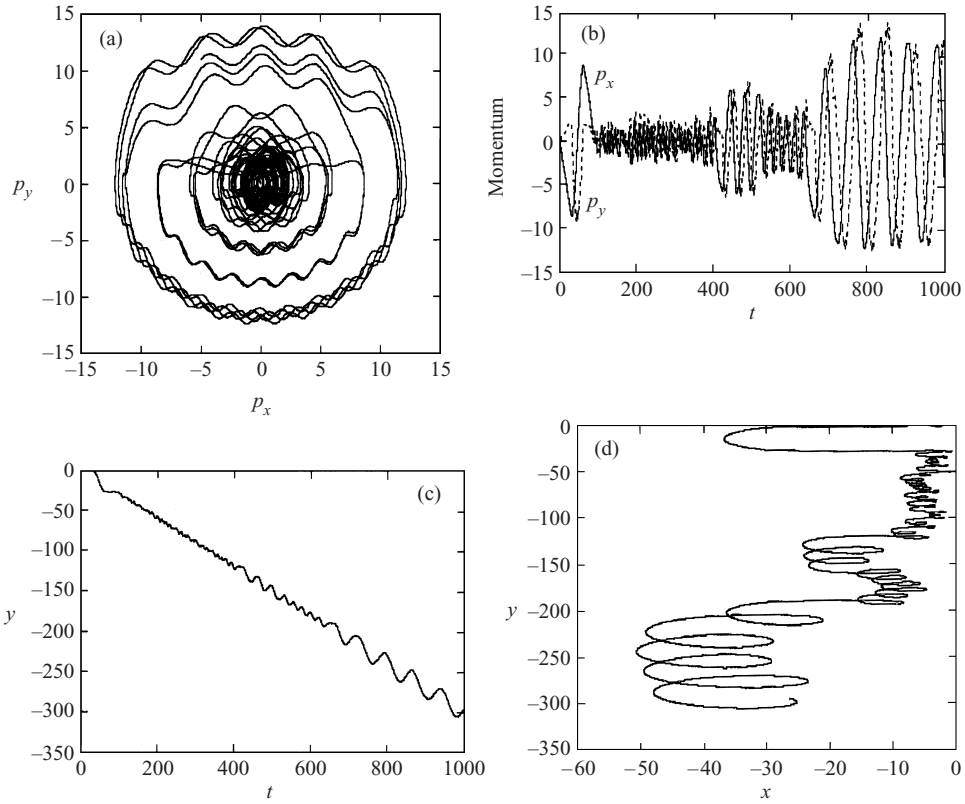


Figure 3. As Fig. 1, but for $k = 2$.

In the asymptotic regime that we have identified, the particle velocity is very close to c, and it travels at an angle to the wave direction so that its velocity component along the wave direction is ηc.

Numerical solutions of (4)–(6) are shown in Fig. 1 for $\alpha = 0.3$, for which the critical β is 1.048. The initial run has $\beta = 1.05$, and we show a phase-plane diagram for the momenta in (a), the momenta and y as functions of time in (b) and (c), and the trajectory in the (x, y) plane (d). Clearly the results approach an asymptotic solution of the form described above, with y tending to a constant value and the momenta and energy increasing linearly with time. The only constraint on the energy achieved is the extent of the system. If we drop the value of β to 1.045, below the critical value, we get a completely different behaviour as shown in Fig. 2. In this case, the wave no longer has a high enough amplitude for it to trap the particle, and from the graph of y in Fig. 2(c), it is clear that the particle falls behind the wave. The behaviour appears to be chaotic (Mendonça and Silva 1997), although this does not rule out some complicated type of periodicity.

For shorter wavelengths, the indications are that we need to have an amplitude further above the threshold for the steady acceleration to appear. It is likely that the threshold will depend on the initial conditions, in which case it is possible that there may be a steady-acceleration solution just above threshold, but that it requires more carefully chosen initial conditions to access it. The third solution (Fig. 3) has

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$k = 2$, instead of 0.2. We see that for $\beta = 1.05$, there is now a chaotic-type solution. However, the steady acceleration reappears when we go to $\beta = 1.1$.

3. Summary

The acceleration of particles is an important area of research, and the surfatron mechanism is important in laboratory, space and astrophysical plasmas. For example, plasma heating by magnetosonic waves is effective in accelerating ions, the electrons being only poorly accelerated. However, the electrons are found to be important in the heating process, as this relies on the electrostatic field E_x, resulting from the space-charge separation between the electrons and the ions, to accelerate ions up to the phase velocity of the wave, at which stage they become trapped ($v_{\rm ph} = eE_x/m_i\omega$). The trapped ions are accelerated in the y direction owing to the $ev_{\rm ph}B_z$ force, until this force is large enough to detrap them, by which time they can have velocities far in excess of $v_{\rm ph}$. This results in very strong acceleration of the ions. Electrons, on the other hand, can be effectively accelerated by upper-hybrid or electron Bernstein modes. In this paper, we have used a test-particle model to demonstrate the physical process rather than study a particular wave-particle process.

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