# Complex Systems and Renormalization Group Explanations

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Despite the close connection between the central limit theorem and renormalization group (RG) methods, the latter should be considered fundamentally distinct from the kind of probabilistic framework associated with statistical mechanics, especially the notion of averaging. The mathematics of RG is grounded in dynamical systems theory rather than probability, which raises important issues with respect to the way RG generates explanations of physical phenomena. I explore these differences and show why RG methods should be considered not just calculational tools but the basis for a physical understanding of complex systems in terms of structural properties and relations.

**1. Introduction.** In order to assess the explanatory power provided by renormalization group (RG) methods, we need to look carefully at the type of problems that arise in complex systems. There are many definitions of complex systems, as well as different approaches for studying them. Typically, questions revolve around how their parts give rise to collective, stable behaviors, as well as how to understand indirect effects—how behavior in one part of the system can affect other more remote parts. The question of how complex systems are formed and evolve via pattern formation requires multiscale descriptions using mathematical tools such as scaling laws, RG methods, and a variety of other techniques. These specific methods also encompass the more general issue of relating fine-scale descriptions to larger-scale ones, specifically determining which fine-scale parameters are relevant on larger scales, as is the case with proteins that exhibit patterns of secondary, tertiary, and quaternary structures.

If we start from a microscopic scale with the goal of explaining a system's evolution at larger scales, two problems need to be addressed. First, the scale of interest needs to be specified; second, one needs to show that the

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more or less chaotic motion of the elements will contribute to the collective phenomena observed at the macro scale. This, of course, is also part of the overall characterization of emergence that is often associated with (among other things) the way stable, complex properties arise out of a multiplicity of relatively simple interactions. In these cases specifying the relationship between the microscopic and macroscopic scales is often a difficult problem. Describing a complex system at the microscopic level requires an enormous amount of information, typically more than one is capable of handling. Consequently, the macro description necessitates the strong compression of data, which effectively renders much of the microscopic information irrelevant for large-scale behavior. However, in order to deal effectively with complex systems, one must still choose which relevant quantities or degrees of freedom are necessary for the problem at hand. A further problem involves establishing relations among these quantities that will generate predictions regarding their evolution. Unfortunately, the motions of both the relevant and irrelevant degrees of freedom in a complex system are usually strongly coupled, which means that accurate predictions will automatically involve an accurate determination of all the irrelevant degrees of freedom as well—an essentially hopeless task.

A solution to this problem was provided by RG methods, which allow for a "thinning down" of the degrees of freedom. These methods also facilitate linkages in the behavior of a system across many different scales and in cases where fluctuations on many different scales interact. The RG is often used in statistical physics where we have many-body systems that exceed the technical possibilities of the mathematical calculus of mechanics. Explanations in statistical mechanics (SM) involve probability theory, which is used to treat large populations, especially the relation between the microscopic properties of individual atoms/molecules and the macroscopic bulk properties of materials.While there are interesting connections between probability theory and RG, especially with respect to the central limit theorem (CLT), I want to claim that characterizing the relationship in this way masks the fundamental features of RG, especially its role in treating complex systems. By 'fundamental' here I mean not only its mathematical significance in calculating the values of properties of these systems but also the underlying physical behavior it treats. Highlighting the connection between RG and dynamical systems theory emphasizes the "structural" features of RG that are crucial for illustrating its role in explaining how and why complex systems exhibit the types of stable behavior patterns they do. In other words, explicating the structural aspects of RG enables us to better appreciate the transition from its role as a calculational tool to an explanatory framework.

I begin with a brief account of RG methods before moving on to highlight the relationship between RG and probability theory in SM, especially the connection with the CLT. I conclude with a discussion of RG and dynamical

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systems theory, paying particular attention to how the emphasis on structural relations, as opposed to simply ignoring irrelevant degrees of freedom, enables us to clarify the way RG explains universal behavior—the fact that different microscopic systems look exactly the same near critical point. Although the values of the critical exponents describing this behavior are independent of microscopic details, universality involves more than that. What RG shows is that the critical exponents are properties attached to the flow around the (attractive) fixed points. Further, once the true RG trajectory has been approximated, it is impossible to reverse the flow back to the microscopic origin. In that sense RG exposes not only the fact that universality involves independence or autonomy but also the necessity of that autonomy.

**2. What Is RG and How Does It Work?** The theory of critical phenomena deals with continuous or second-order phase transitions in macroscopic systems (e.g., magnetic transitions and superfluid He). The Ising model is perhaps the simplest model that exhibits such a phase transition, which involves collective behavior on large scales near the critical temperature $T_c$. At the transition the correlation length $\xi$—the range over which fluctuations in one region of space are correlated with those in another and the scale on which collective behavior is observed—becomes infinite. The problem is that near $T_c$ these systems depend on two different length scales, the microscopic scale given by atoms or lattice spacing and the dynamically generated scale given by the correlation length that characterizes macro phenomena. In many classical systems one can simply decouple these different scales and describe the physics by effective macroscopic parameters without reference to the microscopic degrees of freedom. In SM this approach became known as mean field theory (MFT; Landau 1937) and assumes that the correlations between stochastic variables at the micro scale could be treated perturbatively with the macro expectation values given by quasi-Gaussian distributions in the spirit of the CLT.[1]

MFT predicted a universality of the singular behavior of thermodynamic quantities at $T_c$, meaning that they diverged in exactly the same way; for instance, $\xi$ always diverges as $(T-T_c)^{-1/2}$. It assumed that these properties were independent of the dimension of space, the symmetry of the system, and the microphysical dynamics. However, it soon became apparent that experimental and theoretical evidence contradicted MFT (e.g., Onsanger's [1944] exact solution to the two-dimensional Ising model). Instead, critical behavior was found to depend not only on spatial dimensions but on symmetries and some general features of the models. The fundamental difficulty

1. I discuss the relationship between RG methods and the CLT further in sec. 2.

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with MFT stems from the very problem it was designed to treat—criticality. The divergences at $T_c$ were an indication that an infinite number of stochastic degrees of freedom were in some sense relevant to what happens at the macro level, but these fluctuations on all length scales would add up to contradict the predictions of MFT.

This is unlike the typical case where physical systems have an intrinsic scale or where other relevant scales of the problem are of the same order. In these contexts phenomena occurring at different scales are almost completely suppressed with no need for any type of renormalization. Such is the case with planetary motion: it is possible to suppress, to a very good approximation, the existence of other stars and replace the size of the sun and planets by point-like objects. Further, in nonrelativistic quantum mechanics we can ignore the internal structure of the proton when calculating energy levels for the hydrogen atom. However, in MFT we have exactly the opposite situation: divergences appear when one tries to decouple different length scales. The divergence of $\xi$ makes it impossible to assume that a system of size $L$ is homogenous at any length scale $l << L$, and because $\xi$ also represents the size of the microscopic inhomogeneities in the system, its divergence prevents the statistical fluctuations from being treated perturbatively. Hence, it is impossible to use statistical averaging techniques for these types of systems.

This failure of MFT is interesting from the point of view of generic explanations, especially in light of the discussion in Humphreys (2014), where we have an example of a stable distribution failing to explain generic critical behavior. Because the statistical averaging procedures cannot accommodate the way inhomogeneities in the microscopic distributions contribute to large-scale cooperative behavior, the task was to explain how short-range physical couplings could generate this type of behavior at the macro level and how to predict it. What the situation seemed to imply was that it was not the values of specific physical quantities that were relevant but rather the features of their dependence with respect to the size $N$ of the system and the control parameters $K$.[2] In other words, the way micro features cooperated to produce universal behavior was the object of explanation, a general feature that could be separated from more specific aspects of the microscopic dynamics. RG methods provided a solution to such problems by determining, in a recursive manner, the effective interactions at a given scale and their relation to those at neighboring scales.

But how, exactly, does this process work? The real-space RG approach used to treat critical phenomena stems from the Wilson (1971a, 1971b)–

2. A control parameter is one that appears in the governing equations of a system and measures the effects of an exterior influence such as temperature, pressure, field intensity, etc.

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Kadanoff (1966) method, which involves scaling relations on a lattice of interacting spins (e.g., ferromagnetic transition) and transformations from a site lattice with the Hamiltonian $H_a(S)$ to a block lattice with Hamiltonian $H_{2a}(S)$.[3] If one starts from a lattice model of lattice size $a$, one can sum over degrees of freedom at size $a$ while maintaining their average on the sublattice of size $2a$ fixed. Starting from a Hamiltonian $H_a(S)$ on the initial lattice, one can generate an effective Hamiltonian $H_{2a}(S)$ on the lattice of double spacing. This transformation is repeated as long as the lattice spacing remains small compared to the correlation length. The key idea is that the transition from $H_a(S)$ to $H_{2a}(S)$ can be regarded as a rule for obtaining the parameters of $H_{2a}(S)$ from those of $H_a(S)$. The process is then repeated with the lattice of small blocks being treated as a site lattice for a lattice of larger blocks, with each block considered as a new basic entity. One then calculates the effective interactions between them and constructs a family of corresponding Hamiltonians. The coarse-graining process provides the bridge from the micro to the macro levels and each state in between. Moving from small to larger block lattices gradually excludes the small-scale degrees of freedom such that for each new block lattice one constructs effective interactions and finds their connection with the interactions of the previous lattice. The iterative procedure associated with RG results in the system's Hamiltonian becoming more and more insensitive to what happens on smaller length scales.

We can see already how this procedure differs from those associated with MFT. Although it involves a process similar to averaging the ensemble over small-scale correlations, as the length scale changes so do the values of the different parameters describing the system. Instead of using the ensemble to calculate an average, as in SM, we use RG to transform one ensemble into another one with different couplings. Each transformation increases the length scale so that the transformation eventually extends to information about the parts of the system that are infinitely far away. This divergence of the correlation length means that the system "loses memory" of its microscopic structure and begins to display new long-range macroscopic correlations. Hence, the infinite spatial extent of the system is crucial in that it determines the thermodynamic singularities included in the calculation. But again, the way in which this process takes place is vital for understanding the differences between RG methods and statistical averaging.

The change in the parameters is implemented by a beta function:

$$\{\tilde{J}_k\} = \beta(\{J_k\}),$$

3. The contrast is with the momentum space approach put forward by Gell-Mann and Low (1954) for quantum field theory.

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which induces what is known as an RG flow on the *J*-space. The phase transition is identified as the place where the RG transformations bring the flow to a fixed point, an ensemble where further iterations produce no changes in either the values of the parameters or the correlation length. These fixed points give the possible macroscopic states of the system at a large scale. Hence, although the correlation length diverges at critical point, using the RG equations reduces the degrees of freedom, which, in effect, reduces the correlation length. What this means is that the micro physics has been "transformed" via RG in a way that detaches it from the stable macro behavior.

In order to fully understand how the fixed points become the focal point for explanation, it is important to stress that the basis of the idea of universality is that the fixed points are a property of transformations that are not particularly sensitive to the original Hamiltonian. What the fixed points do is determine the kinds of cooperative behavior that are possible, with each type defining a universality class. The coincidence of the critical indices in very different phenomena was inexplicable prior to RG methods, which were successful in showing that the differences were related to irrelevant observables—those that are "forgotten" as the scaling process is iterated. However, the important issue is not simply the elimination of irrelevant degrees of freedom; it is the existence of cooperative behavior characterized by the fixed points that serves as the explanatory foundation of universality.

In a general sense the elimination of unwanted degrees of freedom coincides with the suppression of information related to explanation at different levels, a strategy that is common in all areas of physics and is embedded in the statistical averaging procedures in SM. What is different in the context of RG is the way information is suppressed and what the end result is. A significant feature of RG is that it illustrated how, in the long-wavelength/large-space-scale limit, the scaling process in fact leads to a fixed point when the system is at critical point, with very different microscopic structures giving rise to the same long-range behavior. What this means is that RG equations show that critical point phenomena have an underlying order. Indeed, what makes the behavior of these phenomena predictable, even in a limited way, is the existence of certain scaling properties that exhibit universal behavior.[4]

The important point that distinguishes RG from previous renormalization methods used in the 1940s to rid quantum electrodynamics of infinities is that the number and type of relevant parameters are determined by the

4. I have not mentioned the issue of reduction in my discussion of RG, primarily because it is not immediately relevant for the general point I want to make. However, for an excellent treatment of the relation between reduction and renormalization see Batterman (2010).

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outcome of the renormalization calculation.[5] Assuming that a fixed point is reached, one can find the value that defines the critical temperature, and the series expansions near the critical point provide the values of the critical indices.[6] In that sense RG methods provide us with physical information concerning how and why different systems exhibit the same behavior near critical point. They determine these universality classes by proving the existence and universality of scaling laws, laws that provide the mathematical foundation for observed experimental behavior. The next step in the explanatory story is to show how these scaling properties deliver physical information about the systems we are interested in.

**3. Renormalization Group, Statistical Mechanics, and Probability Theory.** An application of RG methods involves transferring the problem from a study of a particular system $S$ to a study of scale transformations such that the results depend only on the scaling properties. What that requires is a shift away from the phase space of the system to a space of models or Hamiltonians. The transformations take place within this functional space with each element corresponding to a physical system with some fixed value of the control parameter(s) $K$. Rather than study the equilibrium state of $S$ within a specified model (computing the value of state functions and variations with respect to variables and parameters), the focus is on the transformation of the model and its parameters in connection with a change in scale of the description of $S$. This allows for the calculation of quantitative and universal results from properties of the renormalization flow.

From this very brief description we can see the shift away from methods used in SM that are grounded in probability theory toward a more structurally based approach to dealing with large-scale features of phenomena. However, there are also similarities. Khinchin (1949) was responsible for the first systematic use of probabilistic methods in SM and showed, using the CLT, that the Boltzmann distribution of the single-molecule energy in systems of weakly correlated molecules is universal. In other words, it is independent of the form of the interaction provided that it is short range (Jona-Lasinio 2001). Although the CLT has a number of variants, in very general terms it states that the sum of many independent random variables tends to a Gaussian, whatever the original distribution might have looked like. When

5. In earlier versions parameters such as mass and charge were specified at the beginning and the change in length scale simply changed the values from the bare values appearing in the basic Hamiltonian to renormalized values. The old renormalization theory was a technique used to rid quantum electrodynamics of divergences but involved no "physics."

6. Phase transitions in thermodynamic systems are associated with the emergence of power-law distributions of certain quantities whose exponents are referred to as the critical exponents of the system.

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we sum many random variables, the details of the distributions of the individual variables become unimportant; instead, simple generic behavior emerges. Put differently, the Gaussian distribution is a fixed-point function for large sums.[7] As we have seen above, failures of MFT could be traced to the existence of strongly correlated variables, cases for which the CLT fails to apply. However, we can derive the CLT from within the framework of RG, so what exactly is the connection between them? Attention to these connections is also important for Humphreys's (2014, 14) point that while the distributions that appear in the generalized CLT are the only fixed points of RG transformations, the fact that the latter are widely used in condensed matter physics does not require us to think that they have explanatory priority. However, what the relation between RG and CLT reveals is that at least in the context of critical phenomena RG does, in fact, emerge as the more powerful explanatory framework.

Without going through the specific steps (see Sornette 2000), the RG derivation of CLT shows that the Gaussian distribution is a fixed point in the sense that it is form stable under successive renormalizations. This is essentially the notion of self-similarity that is crucial for scale invariance, which means that the resulting Gaussian function is identical to the initial Gaussian function after the appropriate shift and rescaling of the variables. This point was clarified by Cassandro and Jona-Lasinio (1978), who showed that scaling is strictly connected with the generalizations of limit theorems. In the original articles on RG by Wilson (1971a, 1971b) and others, no simple probabilistic interpretation was given, but what Cassandro and Jona-Lasinio argue is that the universality of critical behavior is connected with the universality of limit distributions obeying certain constraints (1978, 919), specifically a fixed-point equation. It is important to note, however, that CLT (especially the RG derivation) is the expression of a collective phenomenon that holds, strictly speaking, in the limit of infinite $N$ (although the Gaussian shape is a good approximation of the center of the probability distribution function for the sum if $N$ is sufficiently large) and hence says nothing about behavior of the tails for finite $N$.

From the perspective of explanatory priority we know that collective behavior in critical phenomena involves dependent variables; hence, clarifying the relation between CLT and RG involves generalizing CLT to the case of dependent variables where the random fields appearing in these new limit theorems for critical phenomena will display the necessary scaling properties. While a general theory of these limit theorems is not currently available,

7. The idea is nicely expressed by Gnedenko and Kolmogorov (1954), who claim that all epistemologic value of the theory of probability is based on the fact that large-scale random phenomena in their collective action create strict, nonrandom regularity.

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some examples have been given by Jona-Lasinio (2001) and others where a form of scaling is introduced from the beginning. While this suggests, at least for the present, an explanatory priority for RG methods, an understanding of the relation between probability theory and RG is important because it helps to highlight the deep statistical nature of critical universality, specifically the importance of distinguishing between physical couplings at the micro scale and the statistical correlations between these couplings. In other words, the essential feature in explaining the behavior of critical phenomena is not the physical couplings themselves but the statistical coupling of these couplings (Lesne 1998).

As Lesne (1998) points out, we can see how these are importantly different by considering the range $\lambda$ of the direct physical couplings between elementary constituents compared with the range $\xi$ of statistical correlations. In the Ising model $\lambda$, the lattice parameter, is the distance $a$ separating the nearest-neighbor spins on the lattice. It is a physical quantity that depends on the value and does not determine the critical behavior of the system. By contrast, $\xi$ diverges at $T_c$ in the Ising model; it is always greater than $\lambda$ and is a global statistical characteristic. In other words, a statistical organization of the constituents and their statistical couplings is present on every scale from $\lambda$ to $\xi$. More importantly, any elementary subsystems separated by any length $l$ between $\lambda$ and $\xi$ will have correlated statistical distributions whether or not they have any physical interaction. What this means is that the only characteristic scale of the global behavior of the system is the correlation length $\xi$; $\lambda$ (the microscopic scale) plays no role. When a large number of degrees of freedom exhibit collective, cooperative behavior, it is the statistical characteristics that determine the corresponding macroscopic critical behavior.[8]

Despite this emphasis on the statistical nature of RG transformations, any truly probabilistic interpretation will require limit theorems for the strongly correlated variables. Because these are not available, our understanding of the process relies on the existence of fixed points and the replacement of one ensemble description with another via a "structural" transformation rather than a straightforward averaging. As I noted above, in the case of SM the statistical ensemble defined by a Hamiltonian is used to calculate an average, but with an RG operation we move from one ensemble to another with a Hamiltonian that has different couplings from the original one. The mathematical foundation of these types of structural transformations comes from dynamical systems theory, which deals with changes under transformations and flows toward fixed points. Instead of deriving exact single solutions for a particular

8. In addition to deriving the CLT using RG techniques, the probabilistic approach to understanding critical behavior involves deriving scaling properties as universal consequences of collective behavior. The exponents are then related to the statistical properties of the configurations.

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model, the emphasis is on the geometrical and topological structure of ensembles of solutions. A further explication of these aspects of RG methods allows us to appreciate the generic structural approach to explanation that it provides. By focusing on large-scale structural behavior, we are able to see how RG techniques furnish an understanding of complex behavior that extends beyond calculating values for critical indices.

**4. RG and Dynamical Systems.** In general, explanations in the context of dynamical systems have the following goals: to describe the typical behavior of trajectories for many different types of systems as time (rather than space) tends to infinity, and to understand how this behavior changes under perturbations. What we want to know is the extent to which the system is stable—whether it is possible to deform a perturbed system in such a way that we can recover the original one. In that sense the goal is to answer general questions about, for example, whether there is any relation between the long-term behavior and initial conditions, rather than trying to find precise solutions to the equations defining the system itself, something that is often not possible. Thus, a simple definition of a dynamical system can be given in terms of a group of transformations on a topological space (manifold) with one parameter—time. An equation of evolution is then used to generate trajectories extended in time. The objective is to describe the fixed points—values of the variable(s) describing the steady states of the system that will not change over time. If a fixed point is attractive, nearby states will converge toward it. In addition to fixed points, there are also periodic points, states of the system that repeat themselves after several time steps. These two features are crucial since simple nonlinear dynamical systems often exhibit chaotic behavior that is more or less random and unpredictable.

Feigenbaum (1978) was responsible for showing that a class of dynamical systems could exhibit universal self-similar behavior that could be explained using RG. The difference with critical phenomena is that the spatial extension of the system is replaced by duration in time and **R**, the renormalization operator, acts on the evolution law for the system. Feigenbaum's focus was on the logistic map, which is one of the simplest forms of a chaotic process. As with any one-dimensional map, it is a rule for getting a number from a number. Mathematically, the logistic map is written as

$$x_{n+1} = r\,x_n(1 - x_n) \qquad (n = 0, 1, 2, \ldots),$$

where $x_n$ is a number between 0 and 1 and $r$ is a parameter that can be varied to change the character of the $x$ sequence. The logistic map has been used extensively to study variability in populations of different species (May 1976). The simplest behavior of an orbit of such a map is convergence to a fixed point. Some mathematical mappings involving a single linear parameter exhibit ap-

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parently random behavior (chaos) when the parameter lies within certain ranges. As the parameter is increased toward this region, the mapping undergoes bifurcations at precise values of the parameter. Thus, if $r$ (the driving parameter) is small, the values of the iteration eventually just repeat (system becomes periodic). As $r$ increases (3.0), there is oscillation between two values, a bifurcation results, and the periodicity doubles.[9] As $r$ increases, there is further bifurcation with oscillation between four values and so on. Chaos ensues when $r$ equals 3.57. Fiegenbaum discovered that the ratio of the difference between the values at which such successive period-doubling bifurcations occur tends to a constant of around 4.6692... (a ratio of convergence known as the Feigenbaum constant). Not only did he provide a mathematical proof of that fact, but he showed that the same behavior, with the same mathematical constant, would occur within a wide class of mathematical functions, prior to the onset of chaos.

What this means is that in the limit of aperiodic behavior, there is a unique and hence universal solution common to all the different systems undergoing period doubling. They are analogous to critical phenomena in that they are fixed-point theories, with the Feigenbaum constant $\delta$ (the rate of convergence/onset of complex behavior) viewed as a critical exponent. In other words, for all systems undergoing period doubling, $\delta$ has a universal value. In that sense the transition to chaos can be seen as a critical phase transition and treated using RG methods. In the application to dynamical systems this involves proving the scaling behavior of the bifurcation values, predicting the numerical value of $\delta$, and describing the associated universality class.

The first step is to define the relevant space $F$ of evolution maps on which the renormalization operator will act. This is directly analogous to the space of Hamiltonians for the spatial (position) case. Once the fixed points $R\varphi = \varphi$ are found, it is possible to show that the fixed-point equation

$$[\varphi \, o \, \varphi][x\varphi^2(0)] = \varphi^2(0)\, \varphi(x) \qquad \varphi \in F,$$

where $\varphi(0) = 1$ admits a unique solution in $F$. The equation expresses the exact self-similarity between $\varphi$ and its iterate $\varphi \, o \, \varphi$ at all the timescales between the trajectories that they generate. One then investigates the linear stability of $\varphi$ with respect to the renormalization action in order to determine the flow generated by $R$ in $F$. The renormalization picture in the space $F$ of unimodal maps is then related to the period-doubling scenario observed in most of the one-parameter families of such maps. Without going through the details of each of the results, the outcome is that the universality classes of

9. With $r$ between 0 and 1, the population will eventually die, independent of the initial population. With $r$ between 1 and 2, the population will quickly approach the value $r-1/r$ independent of the initial population. With $r$ between 2 and 3, the population will also eventually approach the same value but will first fluctuate around that value for some time.

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the period-doubling scenario are the set of all one-parameter families of unimodal maps that cross transversally the basin of attraction of $\varphi$ with respect to the renormalization action (see Feigenbaum 1978).

The renormalization procedures in each of these cases—SM and dynamical systems—illustrate the similarities between them (see Lesne 1998). However, what is more important than the analogies is the underlying structure that makes them possible. Each context, the Ising model in SM and different types of dynamical systems, has a specific structural law or constraint that embodies the information necessary to describe an equilibrium state or its evolution. In SM the relevant structural feature is the Hamiltonian, while in dynamical systems it is the evolution map. RG methods focus on the space defined in terms of the Hamiltonians or the evolution maps or whatever the relevant structural feature for the system is. Shifting away from individual "physical" models of the system with a specific microstructure defined on a phase space and replacing it with an emphasis on the way the structural features (the space of Hamiltonians or evolutionary maps) change under RG transformations illustrates how we can understand the relation between stable macro behavior and the micro level from which it arises.

This change of orientation/methodology results in a rather different epistemological situation. Emphasis on a model of a system $S$ fails to provide a way of focusing on the right degrees of freedom for the problem at hand. In contrast, the emphasis on renormalization flow/maps allows us to investigate the robustness of predictions via their reliance on structurally stable behavior. As Goldenfeld and Kadanoff (1999) point out, complexity can be defined as structure with variations. They point out that nature can produce complex structures even in simple situations and can obey simple laws in complex situations. This is exactly what RG so powerfully illustrates! Explaining the structural stability of complex systems in terms of structural constraints and how they transform might sound like an obvious strategy, but the processes involved in carrying it out were far from obvious until the advent of RG methods.

**5. Conclusions.** Steven Weinberg's (1981) paper "Why the Renormalization Group Is a Good Thing" stresses the fact that RG allows one to arrange a theory in such a way that only the relevant degrees of freedom appear in one's equations. While a reduction in degrees of freedom is an important goal, it is only part of the story. The way this reduction is achieved makes apparent how RG functions not only as a calculational tool but as the source of physical information as well. The latter is accomplished by showing how the process relies on the transformation of structural features of systems (the Hamiltonian in SM, the evolution map for discrete dynamical systems, etc.) rather than specific values for microscopic parameters. The only signature of short-distance behavior lies in the initial conditions of the RG flow,

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not in the flow itself. Ultimately, the objective is the determination of fixed points that provide the basis for the structural stability characteristic of cooperative behavior. Emphasizing the way RG analysis proceeds via these structural features, drawing out similarities between the use of RG in SM and dynamical systems, and highlighting its relationship to the CLT enable us to appreciate the power of RG methods in investigating properties of a wide variety of physical systems.

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