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Anticipating additional exoplanets in multi-planetary systems

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Abstract. We present a search for the presence of additional exoplanets in previously identified multi-planetary systems, using the Titius-Bode (TB) relation, a logarithmic distance between planets in our solar system. We analyze 229 multi-planetary systems, each containing a minimum of three confirmed planets, by modeling the TB relation employing the Markov Chain Monte Carlo (MCMC) technique. Our findings indicate that ∼53% of these systems exhibit a logarithmic spacing relation more pronounced than that observed in the solar system. We predict the existence of 426 supplementary exoplanets within these 229 systems, identifying 197 candidates through interpolation and 229 through extrapolation. Among the predicted planets, 47 are located in the habitable zone of their respective host stars. Within this subset, five planets have a maximum radius below $1.25R_{\oplus}$ and an estimated mass between 0.1 and $2M_{\oplus}$. We minimize inaccuracies in the orbital period estimation for the anticipated planets compared to the earlier studies.

Keywords. planets and satellites: detection, planets and satellites: dynamical evolution and stability, planets and satellites: formation, planets and satellites: general

1. Introduction

The Titius-Bode (TB) law, which has been known for more than 200 years, describes a roughly simple logarithmic spacing between planets in our solar system. Its classical relation is as follows:

$$a_n = 0.4 + 0.3 \times 2^n, \tag{1}$$

where a_n represents the semi-major axis of the nth planet in AU. The planet Mercury corresponds to $n = -\infty$, Venus to $n = 0$, Earth to $n = 1$ and so on (Nieto & Davis 1974). The undetected objects in our solar system were accurately anticipated using the TB relation. This relation is thought to be useful for making similar predictions in exoplanet systems. A pioneering application of the TB relation to multiple-exoplanet systems was carried out by Poveda & Lara (2008) on the five-planet 55 Cnc system. They found that the semi-major axes of the five observed planets fit well with a simple exponential TB connection, and they further predicted two additional planets at distances of 2 and 15 AU from the host star. A study by Bovaird & Lineweaver (2013) (hereafter, Bovaird & Lineweaver (2013)) examined a sample of 68 multiple-exoplanet systems, each with at least four planets, including confirmed and candidate systems. They discovered that most

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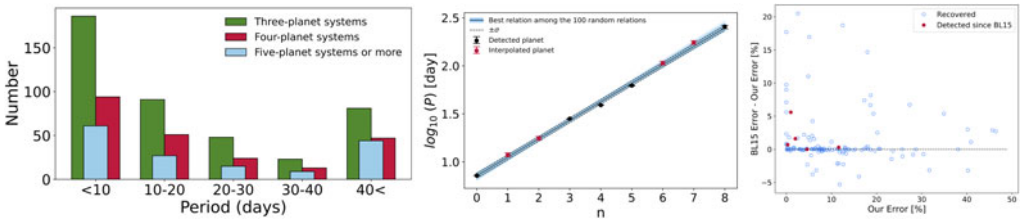


Figure 1. Left panel: distribution of the orbital period of exoplanets in 229 systems hosting different numbers of planets. Middle panel: the TB relation applied to the data from the GJ 667 C system. For this system, the fourth step is the best combination of detected and predicted planets with a γ value of 2.6. The two black dashed lines show $\pm 1\sigma$ uncertainties around the best scaling relation (solid black line). Right panel: Comparison of the uncertainty of the orbital period between Bovaird, Lineweaver, & Jacobsen (2015) and this study. The blue points indicate various planet removal scenarios in the systems, while the five red points represent detected planets after Bovaird, Lineweaver, & Jacobsen (2015) predictions. We achieve improved precision in uncertainties of the orbital period.

of these exoplanetary systems adhered to the TB relation more closely than the solar system. Using the Bovaird & Lineweaver (2013) method, Bovaird, Lineweaver, & Jacobsen (2015) (hereafter, Bovaird, Lineweaver, & Jacobsen (2015)) predicted the existence of 228 additional planets within 151 Kepler multiple-exoplanet systems, finding an average of 2 ± 1 planets in the HZ of each star. Due to intrinsic noise from the host stars, the planets' radii, their highly inclined orbits, and the detection methods, they established a completeness factor, which led them to conclude that not all predicted planets would be detected. The goal of this research is to test the adherence of 229 multiple-exoplanet systems to the TB relation and to predict whether any more undiscovered planets will exist in these systems. Moreover, we highlight the predicted exoplanets located within the HZ of their host stars. Finally, we compare our predictions with those of Bovaird, Lineweaver, & Jacobsen (2015).

2. Data and method

We incorporate physical characteristics of exoplanets, including their orbital periods, radii, and masses, alongside the masses and radii of their respective stars, sourced from the NASA Exoplanet Archive and the Extrasolar Planets Encyclopedia. Furthermore, we apply conservative and optimistic boundaries of the HZ of accessible stars, as provided by the Habitable Zone Gallery (Kane & Gelino 2012). There are 230 known multi-planetary systems, each consisting of at least three confirmed planets, totaling 818 planets. Of these, 142 systems have three planets, 59 systems have four, 20 systems host five planets, and seven systems contain six planets. The Kepler-132 system has four planets, where two of them, Kepler-132 b and Kepler-132 c, possess nearly identical orbital periods of 6.1782 days and 6.4149 days, respectively. However, it has been determined that Kepler-132 b and Kepler-132 c cannot orbit the same star (Lissauer et al. 2014). Therefore, we exclude Kepler-132 from our analysis, reducing our sample to 229 systems. The distribution of the orbital period of 813 member exoplanets in 229 multiple-exoplanet systems is shown in the left panel of Figure 1.

We use the TB relation to predict additional planets in multiple-exoplanet systems. The TB relation expressed through the orbital periods as follow:

$$P_n = P\alpha^n, n = 0, 1, 2, ..., N-1. \quad (2)$$

Let P_n denote the orbital period of the nth planet, with P and α as the fitting parameters. The solar system serves as a benchmark to assess the adherence of exoplanetary systems

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to the TB relation. In logarithmic terms, the TB relation is expressed as:

$$\log P_n = \log P + n \log \alpha = b + m \times n, n = 0, 1, 2, ..., N-1. \tag{3}$$

Given that $b = \log P$ and $m = \log \alpha$ represent the intercept and slope of the relationship, respectively, the χ^2/dof for a system with N planets is computed as:

$$\frac{\chi^2(b,m)}{N-2} = \frac{1}{N-2} \sum_{n=0}^{N-1} \left[\frac{(b + m \times n) - \log P_n}{\sigma} \right]^2, \tag{4}$$

N-2 represents the degrees of freedom (N planets, 2 fitted parameters, b and m), while σ measures the system's density. The density σ is given by $\sigma = 0.273\ S_p$ (see Figure 5 in Mousavi-Sadr, Gozaliasl, & Jassur (2021) (hereafter Sadr2021)), where S_p is defined as the average logarithmic period spacing between planets:

$$S_p = \frac{\log P_{N-1} - \log P_0}{N}, \tag{5}$$

Consider N as the number of planets within the system, where P_{N-1} and P_0 denote the largest and smallest orbital periods, respectively. By selecting an appropriate σ, $\chi^2/\mathrm{dof} = 1$ is calibrated for the solar system. An extrapolated planet beyond the outermost observed planet is predicted only if the system's planets adhere to the TB relation more accurately than those in the solar system. For cases where $\chi^2/\mathrm{dof} > 1$, indicating poorer adherence, interpolation begins. Initially, we incorporate a new planet chosen from 5,000 hypothetical candidates with random periods between the innermost and outermost detected planets, examining all potential intervals between adjacent planets to recalculate χ^2/dof. The specific planet minimizing χ^2/dof is selected. Up to 10 new specific planets may be inserted iteratively, considering all possible intervals and combinations. To determine the optimal combination of observed and predicted planets, we use the highest value of the parameter γ, which measures the enhancement in χ^2/dof per added planet, defined as:

$$\gamma = \frac{\left(\frac{\chi_i^2 - \chi_j^2}{\chi_j^2}\right)}{n_{ins}}, \tag{6}$$

where χ_i^2 and χ_j^2 are the χ^2 values before and after inserting of n_{ins} planets. The analysis of the system GJ 667 C is presented in Figure 1 (middle panel), where we have inserted four additional planets into the system. We independently analyze the data for each system, determining the uncertainty of the optimal parameters via the MCMC method. The geometric transit probability (P_{tr}) for predicted planets is approximated by $d_s/2a_p$, where d_s represents the stellar diameter and a_p denotes the planet's orbital radius (Koch & Borucki 1996). The upper limits on the mass or radius of the predicted planets are derived by selecting the lowest signal-to-noise ratio (SNR) from the observed planets within that system and applying it to the predicted planet's orbital period. For transiting planets, the maximum radius is calculated by:

$$R_{max} = R_{minSNR} \left(\frac{P_{predicted}}{P_{minSNR}} \right)^{0.25}, \tag{7}$$

and for the planets detected by radial velocity, the maximum mass is calculated by:

$$M_{max} = M_{minSNR} \left(\frac{P_{predicted}}{P_{minSNR}} \right)^{\frac{7}{6}}. \tag{8}$$

Here, R_{minSNR}, M_{minSNR}, and P_{minSNR} denote the radius, mass, and orbital period of the planet with the minimum SNR, respectively. We employ the mass-radius relationship from Bashi et al. (2017) to interchange radius and mass values.

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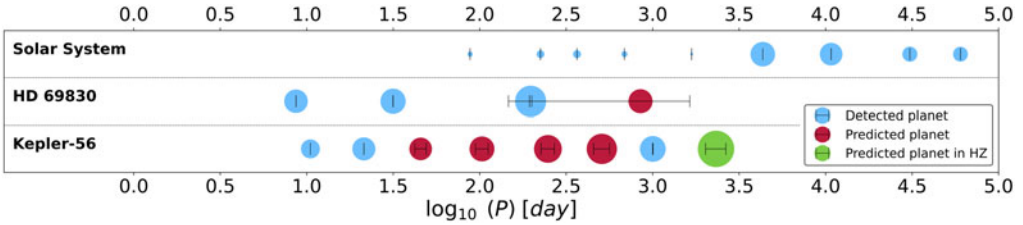


Figure 2. Orbital periods and scaled radii of planets in the solar system and two exoplanet systems with predicted planets. A list of all systems is provided in Figures 7 and 8 of Sadr2021.

Table 1. Exoplanets predicted to lie in the HZ of host stars, with estimated maximum mass and radius similar to Earth's.

Panel	Host name	Dis.[1]	Period (days)	a(AU)	ON[2]	R_{Max} ($R_\oplus$)	M_{Max} ($M_\oplus$)	P_{tr} (%)	HZ_{Cons} (AU)	HZ_{Opt} (AU)
1	GJ 3138	RV	$41.0^{+11.4}_{-9.4}$	$0.20^{+0.01}_{-0.03}$	2	1.0	1.2	1.12	0.22-0.42	0.17-0.44
2	Kepler-186	Tr	$41.2^{+1.7}_{-1.7}$	$0.19^{+0.01}_{-0.01}$	4	0.9	1.1	1.26	0.24-0.46	0.19-0.49
3	Kepler-186	Tr	$73.6^{+3.5}_{-3.5}$	$0.28^{+0.01}_{-0.01}$	5	1.0	1.4	0.85	0.24-0.46	0.19-0.49
4	Wolf 1061	RV	$61.9^{+2.5}_{-2.5}$	$0.20^{+0.01}_{-0.01}$	2	1.2	1.8	0.70	0.10-0.21	0.08-0.22
5	YZ Cet	RV	$7.2^{+0.4}_{-0.3}$	$0.04^{+0.01}_{-0.01}$	3 E	1.2	1.9	1.96	0.05-0.10	0.04-0.10

Notes:
[1]Discovery method of the system: 'Tr' and 'RV' represent transit and radial velocity, respectively.
[2]Orbital numbers (ON) followed by 'E' indicate the extrapolated planets.

3. Results

Our analysis reveals that 122 systems align with the TB relation more closely than the solar system without the need for interpolation. For the 107 systems that align less well with the TB relation compared to the solar system, we inserted up to 10 additional planets. Post-interpolation, these systems follow the TB relation as well or better than the solar system. Specifically, within these 107 systems, 50 needed one additional planet, 33 needed two, and the remaining 24 required more than two additional planets. Figure 2 shows the orbital periods and scaled radii of exoplanets in HD 69830 and Kepler-56 along with the solar system. For all systems studied, we also predict an extrapolated planet beyond the outermost observed one. In total, we foresee 426 potential additional exoplanets, 197 predicted through interpolations. Detailed information on all predicted exoplanets is provided in Sadr2021 and Mousavi-Sadr (2024). Notably, six planets predicted in Kepler-1388, Kepler-1542, Kepler-164, Kepler-374, Kepler-402, and Kepler-403 are labeled "Planetary Candidate" by the NASA Exoplanet Archive. Of the predicted exoplanets, 47 lie within the HZ of their host stars (see Table 2 in Sadr2021). In the Kepler-167 system, we predict three interpolated exoplanets, two within the conservative HZ with orbital periods of 157.0 and 373.5 days. Kepler-186 is projected to host two additional planets with periods of 41.2 and 73.6 days, residing within the optimistic and conservative HZ, respectively. Among these 47 HZ exoplanets, five have estimated maximum mass and radius within Earth's range as defined by Borucki et al. (2011) and Stevens & Gaudi (2013): the fourth and fifth planets of Kepler-186, the second planets of GJ 3138 and Wolf 1061, and the YZ Cet extrapolated planet (see Table 1). There are new planet detection for seven exoplanetary systems (Kepler-1388, Kepler-150, Kepler-1542, Kepler-20, Kepler-80, Kepler-82, and KOI-351) after the predictions made by Bovaird, Lineweaver, & Jacobsen (2015). We find that for five of them, the detected and predicted orbital periods agree very well within the errors.

We remove planets from systems with different combinations and apply our method and Bovaird, Lineweaver, & Jacobsen (2015)'s to recover the orbital periods of removed planets. As a result, our predictions agree roughly better than those made by Bovaird,

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Lineweaver, & Jacobsen (2015), indicating that the use of a precise modeling method and measurements could improve our predictions and uncertainties as well (see the right panel of Figure 1).

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Discussion

We present a list of 229 analyzed exoplanetary systems (123 of them have not been analyzed by Bovaird & Lineweaver (2013) and Bovaird, Lineweaver, & Jacobsen (2015) before) containing their unique TB relations and a total of 426 additional predicted exoplanets, of which 47 are located within HZ of their parent stars. Additionally, we predict that the maximum mass and radius of five of the predicted planets in HZ fall within the mass and radius range of Earth. As an important result, the planets of $\sim$53% of our sample follow a logarithmic spacing relation like the TB relation. Thus, to identify the possible dynamical or gravitational aspects of the TB relation, we must await exoplanet surveys such as TESS to examine more comprehensive data on multi-planetary systems.

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