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RISK REALLOCATION IN DEFINED-CONTRIBUTION FUNDED PENSION SYSTEMS

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This paper explores the introduction of collective risk-reallocation elements into defined-contribution pension contracts. We consider status-contingent, age-contingent, and asset-contingent arrangements to reallocate risk among participants. Eliminating asset market risk for the retired raises their welfare, whereas it lowers the welfare of the workers, despite the fact that they benefit later from the same arrangement. Overall welfare falls. The welfare effects are largest when personal and pension portfolios are optimally chosen. Allowing for intragenerational heterogeneity, the highest-skilled retirees benefit most, whereas the highest-skilled workers lose most. Our main results are qualitatively robust to a number of model variations and extensions.

Keywords: Funded Pensions, Risk Reallocation, Defined Contribution, Intergenerational Welfare

1. INTRODUCTION

In many countries pension systems are being reformed [see OECD (2011)] because their sustainability is threatened. In particular, a number of countries have set up funded pension arrangements to complement or replace nonfunded, pay-as-you-go arrangements, whereas other countries already featuring a sizable funded pillar are reconsidering its design. Newly funded systems, such as those in Israel and Norway, tend to be of the defined-contribution (DC) type. In the case of the United Kingdom, defined-benefit (DB) funded pensions have on a large scale been replaced by individual funded DC arrangements,[1] whereas in the Netherlands an increasing number of companies are putting their pension funds at arm's length in order to prevent pension risks from spilling over onto the company's balance sheet. This leads to the so-called funded collective defined contribution (CDC) arrangements. Many observers feel that this will be a first step toward a system of individually funded DC arrangements.

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Experience from the United Kingdom and the United States has shown that the size of the DC pension benefit determined at retirement date is highly sensitive to the momentary levels of the interest rate and the stock markets. This is also confirmed by Burtless (2000), who shows that the annuity benefit of a male worker entering an individual retirement account plan at age twenty-two, investing his contributions in financial markets, and retiring at age sixty-two in 1975 would have been only two-fifths of what he would have received in 1969 when retiring at the same age. In addition, Agnew et al. (2003) shows that individuals fail to invest optimally in occupational DC accounts by investing all or nothing in equity, trading infrequently, and holding a disproportionate share of their pension wealth in their own employer's equity. Hence, pension fund participants run considerable risk in terms of the future benefits they may expect, and this raises the question of whether it is desirable to retain or reintroduce at least some collective elements in funded DC pension arrangements.

Feldstein and Ranguelova (1998, 2001) explore the replacement of the current U.S. social security program with a system of funded individual retirement accounts and argue that even with a much lower contribution rate than the projected future social security contribution rate, pension benefits are likely to be higher under the funded scheme. However, there is always a non-negligible chance that benefits under the funded scheme will be lower. Hence, Feldstein and Ranguelova (1998) discuss the possibility of removing the downward risk of retirees by taxing workers in the case where the downward risk materializes.

In this paper, we develop and simulate a quantitative many-generation overlapping generation model to explore and quantify the potential benefits of incorporating collective risk-reallocation features into a DC supplementary pension arrangement. The model explicitly considers a public pay-as-you-go first pillar, intragenerational heterogeneity, endogenous labor supply, and financial market and wage shocks. In particular, we focus mostly on the reallocation of financial market risks across the various generations participating in the fund. Those risks are mainly concentrated with retirees and older workers, because these groups hold the largest stocks of financial assets. However, precisely these groups have the shortest horizon to recover from potential negative shocks, whereas they have limited or no flexibility in terms of increasing their labor supply. Therefore, it is relevant to examine the potential welfare gains from reallocating some or all of the financial market risks from retired individuals to workers within a DC pension arrangement. We consider a variety of risk-reallocation schemes. The "status-contingent" scheme sets the exposure of retirees to financial market risk to zero and shifts this risk in a uniform way to the various working generations. That is, the return that working generations receive on their asset holdings is blown up by a constant factor. The "age-contingent" scheme differs from the status-contingent scheme in that vulnerability to financial market risks is made to fall with the age of the worker, the idea being that older workers have less time to recover from negative shocks and have less flexibility to make up for losses by increasing their labor supply. Moreover, older workers have larger financial asset

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holdings, which increases their vulnerability to bad shocks. We also consider an "asset-contingent" scheme, in which the increase in the sensitivity of individual compensation to financial market risk is larger for workers at lower ages and belonging to higher-skilled groups.

Our schemes are all intratemporally balanced. That is, the ex post transfers that take place after the stochastic shocks have hit in a given period sum to zero. Moreover, the schemes do not involve ex ante redistribution. However, they do lead to ex post redistribution of wealth across skill classes. Therefore, we introduce a second asset-contingent scheme that avoids resource flows among skill groups. That is, after this scheme is applied, the aggregate financial wealth of each skill group is unaffected.

We mostly focus on the case in which skill differences are absent. The aggregate welfare effects as measured through consumption-equivalent variations are rather similar across the various schemes at the level of the entire population currently alive, as well as at the level of the entire groups of workers and retirees. Retirees as a group benefit under all schemes that shift financial market risks to workers, whereas workers as a group are worse off, despite the fact that they benefit from the same risk-reallocation arrangement when they themselves retire. Welfare for the population at large also declines, indicating that risk-reallocation between generations to create better DC pensions is not very effective. The welfare effects are particularly large when both personal and pension portfolios are endogenously chosen. The reason is that in this case the retirees' pension portfolios are fully allocated to equity, implying that the working generations have to provide substantial insurance to guarantee the portfolio returns of the retired.

After exploring the case where portfolios are optimally chosen, we assume that both the personal and the pension portfolios are exogenously determined, allowing age-constant or age-declining equity shares. This allows us to consider more realistic portfolio compositions, whereas it is in line with evidence that individuals tend to take the "path of least resistance" and avoid taking optimal financial decisions on their plans even when they are allowed to do so. Although qualitatively the welfare effects of our risk-reallocation schemes are the same as before, quantitatively they become much smaller. This is also the case for the variations we consider, including the absence of a first pillar and alternative ways of redistributing accidental bequests.

We also consider extensions to our basic risk-reallocation schemes when portfolio compositions are exogenous. In the first, we compensate the young by giving them a higher expected return on their pension assets financed by a reduction in the expected return on the retirees' pension assets. Hence, this latter group trades a lower expected benefit for a more certain benefit. Compared with the situation without a risk-reallocation schedule, we are able to find Pareto-improving policy parameter settings, although the welfare benefits are very small. Moreover, they disappear once we assume that portfolio compositions are again optimally selected or when the degree of risk aversion is sufficiently reduced. Second, we extend our schedules to combine the shift in asset market risk from retirees to workers with

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a shift in wage risk in the opposite direction. For reasonable parameter ranges, no Pareto improvement is found here. The increase in asset market risk borne by workers overwhelms the effect of the reduction in wage risk.

The introduction of skill differences hardly affects the welfare consequences for the group of workers as a whole or the group of retirees as a whole. However, looking at the consequences for individual skill groups, we see that the more highly skilled is a retiree is, the more he benefits from the introduction of a risk-reallocation scheme. This is not surprising, because higher-skilled groups hold relatively more assets during retirement and, hence, benefit more from a stabilization of the return on those assets. The opposite is the case for higher-skilled workers. They absorb more of the risk that is shifted and lose more in terms of consumption equivalent variation.

This paper relates to an established literature on computer-based simulations of quantitative overlapping generation models to analyze the welfare implications of introducing a pension system [e.g., Imrohoroglu et al. (1995, 2003); Huang et al. (1997); Conesa and Krueger (1999); Huggett and Ventura (1999); Bucciol (2011)]. A substantial number of works also study intergenerational risk reallocation in pension systems. For example, Wagener (2004) and Gottardi and Kubler (2011) study risk reallocation within PAYG systems, whereas Matsen and Thogersen (2004) investigate the optimal division between PAYG and funding from a risk-reallocation perspective. Other works studying intergenerational risk reallocation within funded pension systems are Beetsma and Bovenberg (2009) and Cui et al. (2011). However, we are the first to explore the aforementioned risk-reallocation schedules within individual DC schemes. Further, our framework incorporates some features that are often absent in other contributions. In particular, we allow endogenous labor supply and intragenerational heterogeneity.[2]

The remainder of this paper is structured as follows. Section 2 presents the model. Section 3 discusses the calibration. Section 4 first presents the simulation results for optimally chosen personal and pension portfolios. Then it presents the results for exogenous portfolio compositions. In Section 5 we do some robustness checks on our results, and in Section 6 we investigate potential extensions of our risk-reallocation schemes. Finally, Section 7 concludes the paper.

2. THE MODEL

2.1. General Framework

Time is discrete and a period corresponds to one year. All the variables are expressed in real terms. We allow for three sources of uncertainty, in mortality, wage growth and stock returns. The first is an individual risk, whereas wage growth and stock returns constitute aggregate risks.

Each individual will be identified by two indices, $i = 1, \ldots, I$ and $j = 1, \ldots, J$. The first index denotes the skill group, where a higher value of i corresponds to a higher skill level. Individuals born in a given skill group remain in the

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same skill group during their entire life. The second index denotes age, which is measured as the number of years since entry into the labor force. Each skill–age group consists of a continuum of individuals.

At the turn of each year a fraction of each cohort dies and the oldest generation dies offt completely, whereas a new generation is born that is $1 + n$ times larger than the cohort born one period earlier. In each period, there are J overlapping generations. Under our assumption of a constant growth rate of the newborn cohort, the relative sizes of the various age groups remain constant over time. Denoting by $N_{j,t},\ j = 1, \ldots, J$, the size at time t of the cohort of age j it is easy to see that

$$N_{j,t} = N_{j-1,t} \frac{\pi_j}{1+n}, \tag{1}$$

where $0 < \pi_j < 1$ is the constant probability that a person will be alive at age j conditional on his or her being alive at age $j - 1$. Because J is the maximum age, $\pi_{J+1} = 0$. Notice that, within this setting, the size of each cohort varies over time, but the ratio between the sizes of any two cohorts is constant over time.

Individuals start each period with given levels of personal and retirement savings. Then workers choose their optimal consumption and labor supply. Retirees choose only their optimal consumption level, while they earn an income from government-provided social security and from a private pension savings plan. Moreover, all individuals earn accidental bequests left by those who die. In our initial setting, all individuals also choose the optimal compositions of their personal and pension savings portfolios. Personal and retirement savings are subject to the same market returns, but the returns on the personal and pension portfolios may differ according to their compositions. The two types of savings differ in two other major ways. First, retirement savings are less liquid than personal savings, which are immediately available for deposit and withdrawal at any moment. Retirement savings are instead available for withdrawal only at retirement and for deposit only while working. Second, investing in retirement savings is more rewarding, because contributions to the retirement pension scheme are partly tax-deductible and matched by the employer.

2.2. The Income Process and Retirement Benefits

The pension system consists of two pillars. The first pillar, the social security system, is a pay-as-you-go (PAYG) arrangement. In each period the system receives contributions from workers and pays benefits to retirees. The second pillar is a funded DC arrangement.

Individuals work until the exogenous retirement age $\overline{J} + 1$ and live for at most J years. Income prior to retirement ($j \leq \overline{J}$) is described by

$$y_{i,j,t} = \left(1 - \theta_t^{\mathrm{SS}} - \theta^{\mathrm{DC}}\right) w_{i,j,t} l_{i,j,t}, \quad j \leq \overline{J},$$

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where $l_{i,j,t} \in [0, 1]$ is the amount of time spent working and $w_{i,j,t}$ is the wage rate per unit of labor input, given by

$$w_{i,j,t} = e_i o_j z_t,$$

where e_i $(i = 1, \ldots, I)$ is the efficiency index for skill group i and o_j $(j = 1, \ldots, \overline{J})$ is a seniority index that for a given skill level causes income to vary with age. Moreover, θ_t^{SS} and θ^{DC} are the contribution rates to social security and the private retirement pension plan (in the sequel referred to as the "retirement plan"), respectively. Notice that all working individuals pay the same (mandatory) contribution rates, whereas, moreover, only the contribution rate to social security is time-dependent. Income depends on the exogenous process

$$z_t = (1 + g_t)\, z_{t-1}, \tag{2}$$

where $g_t = \bar{g} + \epsilon_t^g$ is the growth rate of the process, which is the sum of the constant deterministic component $\bar{g}$ and the mean-zero stochastic component ϵ_t^g. We set $z_0 = 1$.

For convenience, we will define $x_{i,j,t}$ as the level of individual contributions to the retirement plan:

$$x_{i,j,t} = \theta^{\mathrm{DC}} w_{i,j,t} l_{i,j,t}. \tag{3}$$

In line with U.S. arrangements and those in many other countries, retirement plan contributions are tax-deductible up to a certain maximum $\nu_t = (1 + g_t)\, \nu_{t-1}$ that grows at the same rate as the wage rate. Hence, $\nu_t = z_t \frac{\nu_0}{z_0}$. The tax deduction $d_{i,j,t}$ is given by

$$d_{i,j,t} = \delta \min\{x_{i,j,t}, \nu_t\},$$

where δ is the fraction of the contribution to the retirement plan (with a maximum) that is given back to the employee by the government. Further, the employer may match contributions to the retirement plan up to a certain maximum. The amount of matching by the employer $m_{i,j,t}$ is given by

$$m_{i,j,t} = \mu \min\{x_{i,j,t}, \nu_t\},$$

where μ is the match rate and ν_t is the match limit. Obviously, although the market returns to personal and retirement savings are the same, investing in the retirement plan is actually more rewarding because contributions are tax-deductible and matched by the employer.

Retirees $(j > \overline{J})$ earn an income given by the benefits from social security (b_t^{SS}) and the retirement plan $(b_{i,j,t}^{\mathrm{DC}})$:

$$y_{i,j,t} = b_t^{\mathrm{SS}} + b_{i,j,t}^{\mathrm{DC}}, \quad j > \overline{J}.$$

The social security benefit is a fixed fraction of the exogenous income process, identical for all retirees:

$$b_t^{\mathrm{SS}} = \rho z_t, \quad 0 < \rho < 1. \tag{4}$$

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Hence, through the social security system, retirees implicitly run a wage risk. The system is progressive, in that less skilled individuals pay lower contributions but receive the same benefits as richer individuals. The rate of contribution θ_t^{SS} to the first pillar is set so that this pillar is balanced on a period-by-period basis; that is,

$$\sum_{j=1}^{\overline{J}} \frac{N_{j,t}}{I} \sum_{i=1}^{I} \theta_t^{\mathrm{SS}} e_i o_j z_t l_{i,j,t} = \sum_{j=\overline{J}+1}^{J} \frac{N_{j,t}}{I} \sum_{i=1}^{I} \rho z_t. \tag{5}$$

Notice that z_t can be eliminated from (5) and, hence, the contribution rate depends on time only through the labor choice $l_{i,j,t}$.

In contrast to the social security benefit, the retirement plan benefit is a fraction of the accumulated plan savings, given by

$$a_{i,j,t} = b_{i,j,t}^{\mathrm{DC}} \sum_{l=0}^{J-j} \frac{\frac{1}{\pi_j} \prod_{s=j}^{j+l} \pi_s}{\left(1 + E\left[r_{j,t}^{p}\right]\right)^{l}} \Rightarrow b_{i,j,t}^{\mathrm{DC}} = a_{i,j,t} \Big/ \sum_{l=0}^{J-j} \frac{\frac{1}{\pi_j} \prod_{s=j}^{j+l} \pi_s}{\left(1 + E\left[r_{j,t}^{p}\right]\right)^{l}}, \tag{6}$$

where $E[r_{j,t}^{p}]$ is the expected rate of return on the pension portfolio during retirement (discussed later). That is, the benefit from the retirement plan is computed in such a way that for given asset holdings $a_{i,j,t}$ the same constant level of benefits $b_{i,j,t}^{\mathrm{DC}}$ can be provided up to death, taking account of mortality risk and assuming a rate of return equal to that on the pension portfolio.[3] However, from period to period this benefit level may fluctuate with the level of the asset $a_{i,j,t}$. Hence, the benefit $b_{i,j,t}^{\mathrm{DC}}$ is a variable annuity of the type considered in Feldstein and Ranguelova (1998, 2001) and differs from an annuity that pays out the same amount in each period. Further, individuals face uncertainty in retirement plan benefits but not in their rate if contribution to the plan.

Finally, the government finances the tax deductions for the contributions to the retirement plan through a value-added tax on consumption. Consumption is taxed at a rate τ_t. Hence, to enjoy $c_{i,j,t}$ units of consumption, an individual pays $(1 + \tau_t)\, c_{i,j,t}$. We require the government's budget to be balanced on a yearly basis:

$$\sum_{i=1}^{I} \sum_{j=1}^{J} N_{j,t} \tau_t c_{i,j,t} = \sum_{i=1}^{I} \sum_{j=1}^{\overline{J}} N_{j,t} d_{i,j,t}. \tag{7}$$

2.3. Accidental Bequests

The only role of accidental bequests in the model is to ensure that resources do not "disappear" because people die. At the start of period t the government collects the personal assets of the individuals who die at the end of $t - 1$. The sum of the

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personal assets collected by the government is

$$S_t = \sum_{j=2}^{J}(1-\pi_j)N_{j-1,t-1}\frac{1}{I}\sum_{i=1}^{I}s_{i,j,t},$$

where $s_{i,j,t}$ are the personal assets at the end of period $t-1$ of an individual from cohort j and skill class i. This expression sums to $j = J$ rather than to $j = J+1$, because those of age J in period $t-1$ know for sure that they will not survive into period t and, hence, they consume all assets with which they enter period $t-1$. The government redistributes S_t equally over all the living individuals. Hence, each individual receives a transfer h_t that is credited to his personal savings account:

$$h_t = S_t \Big/ \sum_{j=1}^{J} N_{j,t}.$$

Similarly, the pension fund collects the retirement savings of those who die:

$$A_t = \sum_{j=2}^{J}(1-\pi_j)N_{j-1,t-1}\frac{1}{I}\sum_{i=1}^{I}a_{i,j,t}.$$

The fund redistributes A_t equally over all the living individuals. Hence, each individual receives a transfer q_t, which is credited to his retirement savings account:

$$q_t = A_t \Big/ \sum_{j=1}^{J} N_{j,t}.$$

2.4. Savings and Risk Reallocation

The elderly are heavily exposed to financial market shocks, because they hold a relatively large fraction of assets and they have a relatively short remaining lifespan to recover from losses. At the same time, risks associated with wage growth primarily affect working cohorts, although retirees already share some of the wage risk through the first pillar. In this section we describe schemes intended to shift financial market risk from the elderly to younger cohorts and wage risk from the young to the old.

Personal savings $s_{i,j,t} \geq 0$ evolve as

$$s_{i,j,t} = \begin{cases} (1+r_{i,j,t})[s_{i,j-1,t-1} + h_{t-1} + y_{i,j-1,t-1} - (1+\tau_t)\,c_{i,j-1,t-1} + d_{i,j-1,t-1}] & j \leq \overline{J} \\ (1+r_{i,j,t})[s_{i,j-1,t-1} + h_{t-1} + y_{i,j-1,t-1} - (1+\tau_t)\,c_{i,j-1,t-1}] & j > \overline{J}, \end{cases} \tag{8}$$

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where $r_{i,j,t}$ is the return on the individual's personal investment portfolios. It is given by

$$r_{i,j,t} = \xi_{i,j,t} r_t^{\mathrm{s}} + (1 - \xi_{i,j,t}) r^{\mathrm{f}},$$

where r_t^{s} and r^{f} are, respectively, the returns on stocks and risk-free deposits and $\xi_{i,j,t}$ is the share of the portfolio that is invested in stocks. In the sequel we will consider both cases in which $\xi_{i,j,t}$ is endogenous and cases in which it is exogenous.[4] Stock returns are uncertain and follow the process

$$r_t^{\mathrm{s}} = \bar{r}^{\mathrm{s}} + \epsilon_t^{\mathrm{s}},$$

where $\bar{r}^{\mathrm{s}}$ is the average stock return and ϵ_t^{s} is a shock to the stock return. The aggregate shocks on wage growth and stock returns jointly follow a bivariate mean-zero normal distribution with variance–covariance matrix Σ:

$$\begin{pmatrix} \epsilon_t^{\mathrm{g}} \\ \epsilon_t^{\mathrm{s}} \end{pmatrix} \tilde{} N(0, \Sigma). \tag{9}$$

Pension savings evolve as

$$a_{i,j,t}^{\mathrm{b}} = \begin{cases} \left(1 + r_{i,j,t}^{\mathrm{p}}\right)\left(a_{i,j-1,t-1} + q_{t-1} + x_{i,j-1,t-1} + m_{i,j-1,t-1}\right) & j \leq \overline{J} \\ \left(1 + r_{i,j,t}^{\mathrm{p}}\right)\left(a_{i,j-1,t-1} + q_{t-1} - b_{i,j-1,t-1}^{\mathrm{DC}}\right) & j > \overline{J}, \end{cases} \tag{10}$$

where $a_{i,j,t}^{\mathrm{b}}(a_{i,j,t})$ are pension asset holdings before (after) a risk-reallocation scheme has been applied, and $r_{i,j,t}^{\mathrm{p}} = \xi_{i,j,t}^{\mathrm{p}} r_t^{\mathrm{s}} + (1 - \xi_{i,j,t}^{\mathrm{p}}) r^{\mathrm{f}}$ is the return on the pension asset portfolio. The composition of the pension asset portfolio may also be endogenous or exogenous, with a share $\xi_{i,j,t}^{\mathrm{p}}$ invested in stocks.

The DC pension fund redistributes over the population shocks to returns in retirement savings, subject to the condition

$$\sum_{j=1}^{J} \frac{N_{j,t}}{I} \sum_{i=1}^{I} a_{i,j,t} = \sum_{j=1}^{J} \frac{N_{j,t}}{I} \sum_{i=1}^{I} a_{i,j,t}^{\mathrm{b}}. \tag{11}$$

This expression says that the total amount of retirement savings *after* the risk-reallocation scheme has been applied (the left-hand side) must be equal to the total amount of retirement savings *before* it is applied (the right-hand side). Hence, the risk-reallocation schemes that we present in the following are "budget-neutral."

Concretely, risk reallocation takes place as follows. Each individual will get retirement savings, which are the amount accumulated at the end of the previous year, $a_{i,j-1,t-1} + q_{t-1} + x_{i,j-1,t-1} + m_{i,j-1,t-1}$, grossed up by the expected market return $\bar{r}_{i,j,t}^{\mathrm{p}} = \xi_{i,j,t}^{\mathrm{p}} \bar{r}^{\mathrm{s}} + (1 - \xi_{i,j,t}^{\mathrm{p}}) r^{\mathrm{f}}$ rather than the actual market return $r_{i,j,t}^{\mathrm{p}} = \xi_{i,j,t}^{\mathrm{p}} r_t^{\mathrm{s}} + (1 - \xi_{i,j,t}^{\mathrm{p}}) r^{\mathrm{f}} = \bar{r}_{i,j,t}^{\mathrm{p}} + \xi_{i,j,t}^{\mathrm{p}} \epsilon_t^{\mathrm{s}}$, plus a transfer $t_{i,j,t}$. Hence, whereas

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retirement savings before risk reallocation evolve as in equation (10), retirement savings after risk reallocation are given by

$$a_{i,j,t} = \begin{cases} \left(1 + \bar{r}^{\mathrm{p}}_{i,j,t}\right)\left(a_{i,j-1,t-1} + q_{t-1} + x_{i,j-1,t-1} + m_{i,j-1,t-1}\right) \\ \quad + t_{i,j,t}, & j \le \overline{J} \\ \left(1 + \bar{r}^{\mathrm{p}}_{i,j,t}\right)\left(a_{i,j-1,t-1} + q_{t-1} - b^{\mathrm{DC}}_{i,j-1,t-1}\right) + t_{i,j,t}, & j > \overline{J}, \end{cases} \tag{12}$$

where $t_{i,j,t}$ is the transfer resulting from the risk-reallocation scheme and is given by

$$t_{i,j,t} = \begin{cases} \left[f(i,j)\left(r^{\mathrm{p}}_{i,j,t} - \bar{r}^{\mathrm{p}}_{i,j,t}\right) + c^{\mathrm{w}}_t\right] \\ \quad \times \left(a_{i,j-1,t-1} + q_{t-1} + x_{i,j-1,t-1} + m_{i,j-1,t-1}\right) \\ \quad - \alpha^{\mathrm{w}}_t\left(\theta^{\mathrm{DC}}\bar{l}_{i,j,t-1} e_i o_j z_{t-1} \right. \\ \quad \left. + \mu \min\left\{\theta^{\mathrm{DC}}\bar{l}_{i,j,t-1} e_i o_j z_{t-1}, \nu_{t-1}\right\}\right)\epsilon^{\mathrm{g}}_t, & j \le \overline{J}, \\ \left[f(i,j)\left(r^{\mathrm{p}}_{i,j,t} - \bar{r}^{\mathrm{p}}_{i,j,t}\right) - c^{\mathrm{r}}_t\right] \\ \quad \times \left(a_{i,j-1,t-1} + q_{t-1} - b^{\mathrm{DC}}_{i,j-1,t-1}\right) \\ \quad + \alpha^{\mathrm{r}}_t\left(\theta^{\mathrm{DC}}\bar{l}_{t-1} e_i \bar{o} z_{t-1} \right. \\ \quad \left. + \mu \min\left\{\theta^{\mathrm{DC}}\bar{l}_{t-1} e_i \bar{o} z_{t-1}, \nu_{t-1}\right\}\right)\epsilon^{\mathrm{g}}_t, & j > \overline{J}, \end{cases} \tag{13}$$

where $f(i, j)$ is a function of the individual's skill level and age and c^{w}_t, c^{r}_t, α^{w}_t, and α^{r}_t are time-varying parameters. The transfer is a function of the difference between the actual and expected return on retirement savings multiplied by the function $f(i, j)$. This function creates the possibility of shifting financial market risk from retirees to workers and across skill groups. The scheme also makes it possible to compensate workers by giving them a higher average return on their pension assets, which is to be financed by a lower average return on the retirees' pension assets. In other words, we allow for the possibility that $c^{\mathrm{w}}_t > 0$ and $c^{\mathrm{r}}_t > 0$, such that the sum of $c^{\mathrm{w}}_t(a_{i,j-1,t-1} + q_{t-1} + x_{i,j-1,t-1} + m_{i,j-1,t-1})$ over all cohorts of workers equals the sum of $c^{\mathrm{r}}_t(a_{i,j-1,t-1} + q_{t-1} - b^{\mathrm{DC}}_{i,j-1,t-1})$ over all cohorts of retirees. The second line in the expression for $t_{i,j,t}$ captures the shift of wage risk from workers to retirees when $\alpha^{\mathrm{w}}_t > 0$ and $\alpha^{\mathrm{r}}_t > 0$. The term $(\theta^{\mathrm{DC}}\bar{l}_{i,j,t-1} e_i o_j z_{t-1} + \mu \min[\theta^{\mathrm{DC}}\bar{l}_{i,j,t-1} e_i o_j z_{t-1}, \nu_{t-1}])\epsilon^{\mathrm{g}}_t$ is the part of the accumulation of the worker's account (contribution plus employer's match) that is uncertain, because wage growth in period t is uncertain. Here, $\bar{l}_{i,j,t-1}$ is the average labor supply of those of skill i and age j in period $t-1$. It is taken as given by the optimizing worker. Because retirees do not work and hence do not contribute to the pension assets, in

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their transfer we replace labor supply and the seniority index with the respective averages $\bar{l}_{t-1}$ and $\bar{o}$ in the working population. We set α_t^{r} and α_t^{w} at time t suchso that the sum of $\alpha_t^{\mathrm{w}}(\theta^{\mathrm{DC}}\bar{l}_{i,j,t-1}e_i o_j z_{t-1} + \mu \min[\theta^{\mathrm{DC}}\bar{l}_{i,j,t-1}e_i o_j z_{t-1}, \nu_{t-1}])\epsilon_t^{\mathrm{g}}$ over all workers is equal to the sum of $\alpha_t^{\mathrm{r}}(\theta^{\mathrm{DC}}\bar{l}_{t-1}e_i\bar{o}z_{t-1}+\mu \min[\theta^{\mathrm{DC}}\bar{l}_{t-1}e_i\bar{o}z_{t-1}, \nu_{t-1}])\epsilon_t^{\mathrm{g}}$ over all retirees.

The function $f(i, j)$ will be chosen to protect the elderly (high j) and (in case) the less-skilled individuals (low i) relatively more against unexpected shocks. The rationale for these choices is that (1) the elderly have relatively little flexibility to respond to shocks, and hence they may benefit from less uncertainty about the benefits they receive, and (2) policymakers want to protect the less skilled, hence poorer, individuals from too large fluctuations in their income. As a result, for the elderly and less skilled individuals, $f(i, j)$ will be smaller in absolute magnitude.

In qualitative terms we would expect this to be the outcome under a utilitarian planner who decides about the allocation of resources over the population. Because the total amount of retirement savings must be unaffected by the ex post transfers, some groups (the younger and more skilled individuals) will actually face retirement savings that are more volatile than in the absence of the risk-reallocation scheme. Notice that when $c_t^{\mathrm{w}} = c_t^{\mathrm{r}} = 0$, there is no ex ante redistribution, because $E_{t-1}[a_{i,j,t}] = E_{t-1}[a_{i,j,t}^{\mathrm{b}}]$ for all i, j and t.

We consider five different formats for the rescaling function $f(i, j)$.

a. Uniform scheme (benchmark). This scheme serves as the benchmark. Under this scheme,

$$f(i, j) = 1, \quad \forall i, j,$$

and, hence, $a_{i,j,t}^{\mathrm{b}} = a_{i,j,t}$ for all i and j. Hence, there is no risk reallocation among individuals. We refer to this case as the *pure* DC scheme.

b. Status-contingent scheme. Retirees face no financial market uncertainty and always receive the expected return on their retirement savings. In contrast, all workers face proportionally more uncertainty than under the uniform scheme:

$$f(i, j) = \begin{cases} \alpha_t^1, & j \le \overline{J} \\ 0, & j > \overline{J}. \end{cases}$$

The idea behind this scheme is that, unlike retirees, workers can react to shocks in their asset returns by changing their labor input. Moreover, they have a relatively large amount of time left to recover from adverse shocks in their asset returns.

c. Age-contingent scheme. Under this scheme the uncertainty about the returns to retirement savings falls linearly with working age to zero at the retirement age $\overline{J} + 1$ and remains zero in all the retirement years:

$$f(i, j) = \begin{cases} \alpha_t^2(1 + \overline{J} - j), & j \le \overline{J} \\ 0, & j > \overline{J}. \end{cases}$$

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This scheme is a refinement of the previous one and its motivation is analogous. Younger workers have more room to vary their labor input and to recover from adverse developments in the financial markets.

d. Asset-contingent scheme I. Uncertainty is zero for retirees and for individuals with the largest amount of retirement savings in their skill groups (i.e., those exactly at retirement age $\overline{J}$). It is different from zero for all the other workers. The rescaling function $f(i, j)$ is now given by[5]

$$f(i, j) = \begin{cases} \alpha_t^3(\max_j\{E[a_{i,j,0}]\} - E[a_{i,j,0}]), & j \leq \overline{J} \\ 0, & j > \overline{J}. \end{cases}$$

Hence, suppose that for a given skill level i, period-0 expected pension assets are strictly increasing in age up to the age $j = \overline{J}$, and then the function $f(i, j)$ is falling in age j ($j \leq \overline{J}$) and, hence, risks associated with the asset portfolio are falling with the age of the worker. In our following simulations, $f(i, j)$ is indeed falling in working age for a given skill level and rising in the skill level for a given working age. The scheme combines the ideas that higher-income and younger workers can manage more risk in their asset income. For $E[a_{i,j,0}]$, $j \leq \overline{J}$, we will use the average (across the simulation runs) amounts of assets at the end of the "shadow" years of each simulation run; the simulation setup is discussed later.

e. Asset-contingent scheme II. Asset-contingent scheme I gives rise to a rescaling function that differs by age and skill group. It leads to ex post redistribution along two dimensions, age and skill. Asset-contingent scheme II closely resembles asset-contingent scheme I, except that it is specifically designed to avoid ex post redistribution across skill groups. In the case where all individuals have identical skills, asset-contingent scheme II coincides with asset-contingent scheme I. The rescaling function for asset-contingent scheme II is

$$f(i, j) = \begin{cases} \alpha_{i,t}^4(\max_j\{E[a_{i,j,0}]\} - E[a_{i,j,0}]), & j \leq \overline{J} \\ 0, & j > \overline{J}, \end{cases}$$

where the parameter $\alpha_{i,t}^4$ is skill-specific.

The parameters $\alpha_t^1, \alpha_t^2, \alpha_t^3$ are larger than zero and chosen so that equation (11) is satisfied. The parameters $\alpha_{i,t}^4$, $i = 1, \ldots, I$, are also larger than zero, but each of them is chosen to satisfy the budget balance equation at the level of the skill group when $c_t^{\mathrm{w}} = c_t^{\mathrm{r}} = \alpha_t^{\mathrm{w}} = \alpha_t^{\mathrm{r}} = 0$:

$$\sum_{j=1}^{J} N_{j,t} a_{i,j,t|c_t^{\mathrm{w}}=c_t^{\mathrm{r}}=\alpha_t^{\mathrm{w}}=\alpha_t^{\mathrm{r}}=0} = \sum_{j=1}^{J} N_{j,t} a_{i,j,t}^{\mathrm{b}}, \tag{14}$$

implying that there are no net ex post transfers across skill groups, but that all the ex post redistribution takes place within the same skill group. Choosing all the $\alpha_{i,t}^4$ parameters this way implies that equation (11) is respected.

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2.5. The Individual Decision Problem

The individual's value function is

$$V_{i,j,t}(s_{i,j,t}, a_{i,j,t}) = \max_{c_{i,j,t}, l_{i,j,t}, \xi_{i,j,t+1}, \xi^{\mathrm{p}}_{i,j,t+1}} \{u(c_{i,j,t}, l_{i,j,t}) + \beta \pi_{j+1} E_t[V_{i,j+1,t+1}(s_{i,j+1,t+1}, a_{i,j+1,t+1})]\}, \tag{15}$$

where period utility u is the following function of consumption $c_{i,j,t} \geq 0$ and labor supply $l_{i,j,t} \in [0, 1]$:

$$u(c_{i,j,t}, l_{i,j,t}) = \frac{1}{1-\gamma}\left[c_{i,j,t}^{1-\phi}(1-l_{i,j,t})^{\phi}\right]^{1-\gamma}.$$

Equation (15) is maximized subject to the dynamics of (2.4) and (12). Further, we assume that individuals are liquidity-constrained. Specifically, this prevents individuals from borrowing against expected higher future income.[6] We also assume that individuals cannot borrow to leverage their equity position or short-sell equity and invest the proceeds in the risk-free asset. In other words, we restrict $\xi_{i,j,t}$ and $\xi^{\mathrm{p}}_{i,j,t}$ to the interval [0, 1].

Maximization of the value function (15) yields the following set of first-order conditions for an internal solution:

$$\begin{cases} \frac{\partial u}{\partial c_{i,j,t}} = \beta \pi_{j+1} E_t\left[(1+r_{i,j+1,t+1})\frac{\partial u}{\partial c_{i,j+1,t+1}}\right], \\ \frac{\partial u}{\partial l_{i,j,t}}\frac{1}{w_{i,j,t}} + \frac{\partial u}{\partial c_{i,j,t}}\frac{1-\theta_t^{\mathrm{SS}}-\theta^{\mathrm{DC}}\left(1-\delta \mathbf{1}_{\{x_{i,j,t}<\nu_t\}}\right)}{1+\tau} \\ = \beta \pi_{j+1} E_t\left[R^{\mathrm{p}}_{i,j,t+1}\frac{1+\mu \mathbf{1}_{\{x_{i,j,t}<\nu_t\}}}{1+\mu \mathbf{1}_{\{x_{i,j+1,t+1}<\nu_{t+1}\}}}\left[+\frac{\partial u}{\partial c_{i,j+1,t+1}}\frac{\frac{\partial u}{\partial l_{i,j+1,t+1}}\frac{1}{w_{i,j+1,t+1}}}{\frac{1-\theta^{\mathrm{SS}}_{t+1}-\theta^{\mathrm{DC}}\left(1-\delta \mathbf{1}_{\{x_{i,j+1,t+1}<\nu_{t+1}\}}\right)}{1+\tau}}\right]\right], \\ E_t\left[\left(r^{\mathrm{s}}_{t+1}-r^{\mathrm{f}}\right)\frac{\partial u}{\partial c_{i,j+1,t+1}}\right] = 0, \\ E_t\left[\frac{\left(\bar{r}^{\mathrm{s}}-r^{\mathrm{f}}\right)+f(i,j)\epsilon^{s}_{t+1}}{1+\mu \mathbf{1}_{\{x_{i,j+1,t+1}<\nu_{t+1}\}}}\left[+\frac{\partial u}{\partial c_{i,j+1,t+1}}\frac{\frac{\partial u}{\partial l_{i,j+1,t+1}}\frac{1}{w_{i,j+1,t+1}}}{\frac{1-\theta^{\mathrm{SS}}_{t+1}-\theta^{\mathrm{DC}}\left(1-\delta \mathbf{1}_{\{x_{i,j+1,t+1}<\nu_{t+1}\}}\right)}{1+\tau}}\right]\right] = 0, \end{cases} \tag{16}$$

where $R^{\mathrm{p}}_{i,j,t+1} = 1 + r^{\mathrm{p}}_{i,j,t+1} + [f\ (i,\ j)\ (r^{\mathrm{p}}_{i,j,t+1} - \bar{r}^{\mathrm{p}}_{i,j,t+1}) + c^{\mathrm{w}}_t]$ and $\mathbf{1}_{\{x_{i,j,t}<\nu_t\}}$ is a dummy variable equal to 1 if $x_{i,j,t} < \nu_t$ and 0 otherwise. These equations have been derived using the envelope conditions associated with personal savings and pension savings. The first equation is the standard Euler equation, and it equates the current utility gain from a marginal increase in consumption to the discounted expected future utility loss, where the latter takes account of the survival probability. The second equation determines the optimal labor choice of an individual of working

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age $j \leq \overline{J}$. It says that the marginal utility change in a given period from an increase in labor supply is the sum of two components: the marginal utility loss from an increase in the labor supply and the marginal utility gain from an increase in wage earnings generated by working more (weighed down by the consumption tax and contributions to the first and second pension pillars and heightened by the employer's matching). At the optimum, the current marginal utility change must be equal to the discounted marginal utility change in the next period. The third and fourth equations are the conditions for optimal personal and pension portfolio choice, respectively. They say that the expected future marginal utility of an additional dollar invested in stocks and the risk-free asset via the personal or pension portfolio should be equal. Obviously, the risk-reallocation schemes we analyze modify the uncertainty on retirement savings, thus altering labor supply and consumption behavior.

2.6. Measures of the Welfare Comparisons Between Policies

The welfare consequences of alternative policies are expressed in terms of consumption equivalent variations. A similar approach is taken in Conesa and Krueger (1999), Huggett and Ventura (1999), Imrohoroglu et al. (2003), and Cocco et al. (2005). Specifically, we calculate the constant percent increase in consumption in all periods of life and in all states $\text{CEV}^{\text{A,B}}_{i,j,1}$ that needs to be given to an individual from skill group i of cohort j alive at $t = 1$ under pure DC (scenario A) to bring utility to the same level as under some specific risk-reallocation arrangement (scenario B). That is,

$$
\begin{aligned}
V^{\text{A}}_{i,j,1}\left(\text{CEV}^{\text{A,B}}_{i,j,1}\right) &= \sum_{s=j}^{J} \beta^{s-j} \frac{\pi_s}{\pi_j} u\left(\left(1+\text{CEV}^{\text{A,B}}_{i,j,1}\right) c_{i,s,1+s-j}, l_{i,s,1+s-j}\right) \\
&= \sum_{s=j}^{J} \beta^{s-j} \frac{\pi_s}{\pi_j} \frac{1}{1-\gamma} \left\{\left[\left(1+\text{CEV}^{\text{A,B}}_{i,j,1}\right) c_{i,s,1+s-j}\right]^{1-\phi} (1-l_{i,s,1+s-j})^{\phi}\right\}^{1-\gamma} \\
&= \left(1+\text{CEV}^{\text{A,B}}_{i,j,1}\right)^{(1-\phi)(1-\gamma)} \sum_{s=j}^{J} \beta^{s-j} \frac{\pi_s}{\pi_j} \frac{1}{1-\gamma} \left[c^{1-\phi}_{i,s,1+s-j}(1-l_{i,s,1+s-j})^{\phi}\right]^{1-\gamma} \\
&= \left(1+\text{CEV}^{\text{A,B}}_{i,j,1}\right)^{(1-\phi)(1-\gamma)} V^{\text{A}}_{i,j,1}(0),
\end{aligned}
$$

where $V^{\text{A}}_{i,j,1}$ is utility under scenario A. Because

$$
V^{\text{A}}_{i,j,1}\left(\text{CEV}^{\text{A,B}}_{i,j,1}\right) = V^{\text{B}}_{i,j,1},
$$

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it follows that

$$\mathrm{CEV}^{\mathrm{A,B}}_{i,j,1} = \left(\frac{V^{\mathrm{B}}_{i,j,1}}{V^{\mathrm{A}}_{i,j,1}(0)} \right)^{\frac{1}{(1-\phi)(1-\gamma)}} - 1.$$

For a group of individuals (e.g., all workers), we define analogously the aggregate welfare effect as the constant (across all group members, over all remaining periods of their life, and in all states of the world) percentage increase in consumption applied to the baseline scenario with the uniform scheme such that the sum of all expected utilities over the group members becomes equal to the corresponding sum under the alternative scenario.

We consider as an alternative aggregate measure the percentage of individuals in favor of the alternative policy:

$$\mathrm{PER}^{\mathrm{A,B}} = \frac{1}{\sum_{j=1}^{J} N_{j,1}} \sum_{j=1}^{J} N_{j,1} \frac{1}{I} \sum_{i=1}^{I} \mathbf{1}_{\left\{V^{\mathrm{B}}_{i,j,1} > V^{\mathrm{A}}_{i,j,1}\right\}} \times 100\%,$$

where $1_{\{.\}}$ is an indicator equal to 1, if the condition within the curly brackets holds, and 0 otherwise. We will also report corresponding aggregate measures for the groups of living workers and retirees separately and for individual skill groups.

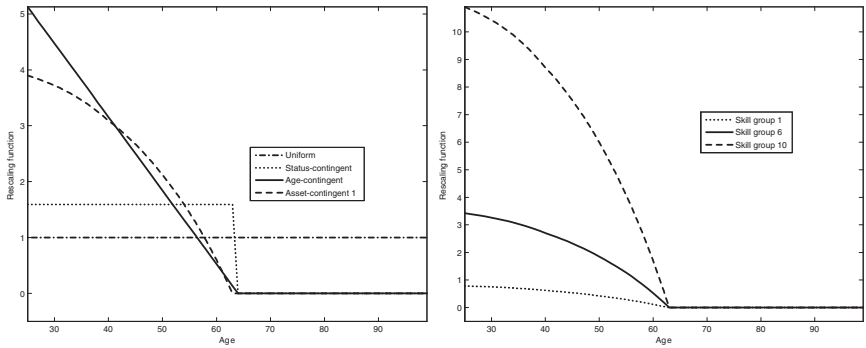
We focus the analysis on the generations that are born at $t = 1$ or earlier. We do not consider later-born generations, because they are very similar to individuals born at $t = 1$. The reason is that all individuals born at $t = 1$ or later start with zero pension and zero personal assets, whereas the shocks are independently and identically distributed. Those born after $t = 1$ only differ from those born at $t = 1$ because they face different shocks and different "institutional" parameters that are endogenous in this framework (the consumption tax rate, the social security contribution rate, and the rescaling parameters). However, the time variation in these institutional parameters is too small to induce differences in behavior between those born at $t = 1$ and those born later. From an ex ante perspective, all generations born at $t = 1$ or later are essentially identical.

3. CALIBRATION AND SIMULATION SET-UP

3.1. Calibration

The model includes exogenous and endogenous parameters. The social security contribution rate θ^{SS}_t, the consumption tax rate τ_t, and the various parameters of the rescaling function $f(i, j)$ are endogenous. The social security contribution rate is derived from equation (5). At the start of each simulation run it equals 13.209%, and then it evolves with the labor supply decisions. Using a trial and error approach we determine the parameters α^1_t, α^2_t, and α^3_t from equation (11) and the parameters $\alpha^4_{i,t}$ from equation (14). The values of the parameters depend on the specific case we consider. For the case of endogenous personal and pension

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(a) (b)

FIGURE 1. Rescaling functions: (a) all; (b) asset-contingent I, different skills.

portfolio compositions we will assume first that the values of α_t^1, α_t^2, and α_t^2 are on the average 1.85, 0.17, and 1.39, respectively, whereas $\alpha_{i,t}^4$ ranges from 9.01 (skill group 1) on the average to 0.33 (skill group 10). The parameters show very little variation over time and induce the rescaling functions shown in Figure 1. Rescaling differs by skill group only under the two asset-contingent schemes. Except when we explicitly assume otherwise, we set $c_t^{\mathrm{w}} = c_t^{\mathrm{r}} = \alpha_t^{\mathrm{w}} = \alpha_t^{\mathrm{r}} = 0$, implying that we analyse purely the shift of financial market risk from retirees to workers.

Table 1 lists the exogenous parameters of the model and their calibration under the benchmark analysis. The parameters are chosen so that they reproduce the main features of the U.S. economy. The economically active life of an individual starts at real-life age 25, which in the model we reset to $j = 1$. He then works for $\overline{J} = 37$ years, which corresponds to a real-life age of 62. At that moment retirement starts. This retirement age is in line with median figures in Munnell et al. (2008) and calibrations in Laibson et al. (2007) and Bucciol (2012). Individuals live for at most $J = 75$ years after entry into the labor force, which corresponds to real-life age 100. Our age-specific survival probabilities π_j are obtained from the Social Security Administration [see Bell and Miller (2005)] and refer to the year 2005. They imply a remaining life expectancy of 53 years at the moment of entry into the labor force. Hence, life expectancy is 78 years. Using the survival probabilities and equation (1), the constant population growth rate n is determined to produce a dependency ratio (the ratio of retirees to working-age individuals) of 33%.

Preference parameters. We set the discount factor at $\beta = 0.96$, a rather common choice for an annual discount factor in the macroeconomic literature [e.g., see Imrohoroglu (1989) or Krebs (2007)], and the coefficient of relative risk aversion at $\gamma = 3$, which accords quite well with the assumed risk aversion in much of the macroeconomic literature [see, e.g., Imrohoroglu et al. (2003)] as well as estimates at the individual level [for example, Gertner (1993) and Beetsma and

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TABLE 1. Benchmark calibration

Symbol	Description	Calibration
$\overline{J}$	Length of working life	37
J	Maximum death age after entry into labour force	75
π_j	Age-specific survival probabilities	Bell–Miller (2005)
n	Population growth rate	0.870%
β	Discount factor	0.96
γ	Relative risk-aversion parameter	3
ϕ	Consumption-leisure parameter	0.3
$\{e_i\}_{i=1}^{\mathrm{I}}$	Efficiency index (10 groups)	WIID (2008)
$\{o_j\}_{j=1}^{\mathrm{I}}$	Seniority index	Hansen (1993)
r^{f}	Risk-free real return	1.327%
$\bar{g}$	Average real-wage growth rate	2.343%
$\bar{r}^{\mathrm{s}}$	Average real stock return	6.120%
ρ	Benefit scale factor	0.132
θ^{DC}	Retirement asset contribution rate	0.05
ν_0	Match limit	0.06
μ	Match rate	0.5
δ	Tax deduction parameter	0.2

Schotman (2001)]. The parameter ϕ, which determines the substitution between consumption and leisure, is chosen to produce an average labor choice of $l = 0.33$. This corresponds roughly to half of the time during nonweekend days being devoted to work and the other half to leisure. This calibration implies setting $\phi = 0.3$.

Income. The efficiency index $\{e_i\}_{i=1}^{\mathrm{I}}$ is based on the income deciles for the United States for the year 2000 reported by the World Income Inequality Database (WIID, version 2.0c, May 2008). We normalize the index so that it has an average value of unity. The seniority index $\{o_j\}_{j=1}^{\mathrm{I}}$ uses the average of Hansen's (1993) estimates of median wage rates by age group. We take the averages between males and females and interpolate the data using the spline method.

Portfolio investment and financial markets. The average risk-free rate is set to $r^{\mathrm{f}} = 1.327\%$, which is the historical yield of three-month U.S. Treasury bills over the sample period 1986–2010, covered at the monthly frequency. This yield is corrected for inflation, measured as the growth rate in the CPI for all urban consumers.

Shocks. The averages, variances, and covariances of wage growth and the stock returns are estimated at the monthly frequency from U.S. historical data covering the period 1986–2010. Specifically, we take series on personal income net of government transfers, rents, and revenues from financial assets (the source is the U.S. Bureau of Economic Analysis) and the S&P U.S. stock price index.

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Wage growth and returns are then converted into real terms by subtracting the growth rate in the CPI for all urban consumers.

We estimate the average real wage growth rate and the stock return at, respectively, $\bar{g} = 2.343\%$, and $\bar{r}^s = 6.120\%$. The standard deviations associated with the corresponding shocks are, respectively, 2.127% and 17.556%; the correlation between the two shocks is 45.444%. With these figures, we fill the variance–covariance matrix of the shocks Σ.

Social security and retirement assets. Social security in the United States provides a replacement rate, i.e., a ratio between the first pension benefit and the final salary payment, of around 40%. We set $\rho = 0.132$, which is obtained as 0.4 times the average labor choice of 0.33 over the working life. We set the rate of contribution to the retirement plan at $\theta^{DC} = 0.05$, a level that is rather realistic. Depending on the case we consider, it implies a benefit from the retirement plan roughly between zero and 20% higher than that from social security. We then set the three parameters describing the features of the retirement plan to match the features of a typical 401(k) plan. Following the calibration in Love (2006), we assume a match rate by the employer of $\mu = 0.5$ and an initial match limit $\nu_0 = 0.06$.[7] That is, the employer matches 50 cents for each dollar contributed by the employee, with the match capped at the level that corresponds to employee contributions equal to 6% of compensation. We also assume a deduction rate of $\delta = 0.2$, in line with the effective federal tax rate [see CBO (2007)]. These assumptions give rise to an effective return on retirement assets above the return on personal assets, as frequently documented in the literature [see Brennan and Subrahmanyam (1996) and Pàstor and Stambaugh (2003)]. Finally, the consumption tax rate τ_t is obtained from equation (7) and is 0.046 on the average in the benchmark case. Of course, compared to reality, this is a very low rate for the value-added tax. However, the value-added tax in the model serves only to finance the deduction of second-pillar contributions, whereas in reality it needs to cover a broad range of different expenditures.

3.2. Simulation Setup

We simulate the model for $Y = 2J - 1 = 149$ years in total. This period is split into two subperiods. The first $J - 1$ years of the simulation are done in the absence of any risk-reallocation scheme but in the presence of our full shock menu. We treat these years as "shadow" years. This first part of the simulation period serves to produce a distribution of asset holdings over the various cohort–skill combinations that forms a starting point for the second part of the simulation run, consisting of J years, which is done in the presence of both shock combinations drawn from (9) and one of the proposed risk-reallocation schemes. For this second part of the run, we calculate the statistics for the various groups in the population that we report in the following. In the first year of this second part, which for convenience we relabel as $t = 1$, agents start with the levels of personal and retirement assets obtained in

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the latest "shadow" year. The length of the second part of the simulation run ensures that we calculate statistics for each possible age category $(0, \ldots, J-1)$ at the start of this run. As explained aboveearlierlengthening the second part of the simulation run and hence calculating statistics for future-born generations is pointless.

We base our calculations on 1,000 simulation runs. We draw an original set of 1,000 shock combinations on wage growth and stock returns. In each year of the simulation run, the set of shocks is a bootstrap draw from the original set without replacement. To allow the cleanest possible comparison among our risk-reallocation schedules, for all policy scenarios we use the same shock series for our 1,000 simulation runs. This also implies that we start the second part of each simulation run with identical (across the policy scenarios) asset holdings by the various groups of individuals.

It is worthwhile to note that individuals take uncertainty into account when making decisions. As a result, they engage in precautionary saving. In addition, the policy change (the introduction of the risk-reallocation scheme) after the completion of the shadow years comes unexpectedly and, hence, is not anticipated during the shadow years. However, from the introduction of the scheme on, individuals take the new policy regime into account when making decisions. Individuals of age $j > 1$ at $t = 1$ experience both the old and the new policy regime and represent the transition phase, whereas those who enter the labor market at $t = 1$ represent the new status-quo situation.

4. BASELINE RESULTS

We start our analysis assuming identical skill levels. We omit the case of the asset-contingent scheme II, because in the absence of skill differences it coincides with the asset-contingent scheme I. First, we consider the situation in which agents choose the composition of both their personal and pension savings portfolios optimally, whereas in the ensuing part we focus on the case in which the composition of these portfolios is exogenous.[8] The reason we present our results in this way is that we want to show that our main findings are qualitatively independent of whether the composition of the portfolios is endogenously or exogenously determined. However, in the latter case we can impose more realistic portfolio compositions, as endogenously determined portfolios lead to unrealistically large fractions invested in equity. Moreover, in reality, investors tend to follow the "path of least resistance" and not make optimal financial decisions about their plans. As regards the exogenous composition of the portfolios, we will consider two cases. In one, the fractions invested in equity are age-independent. In the other, they are a stepwise declining function of age, as widely recommended in theory.

4.1. Endogenous Portfolio Compositions

In this subsection, we assume that individuals select the optimal composition of both their personal and pension asset portfolios, under the restriction that

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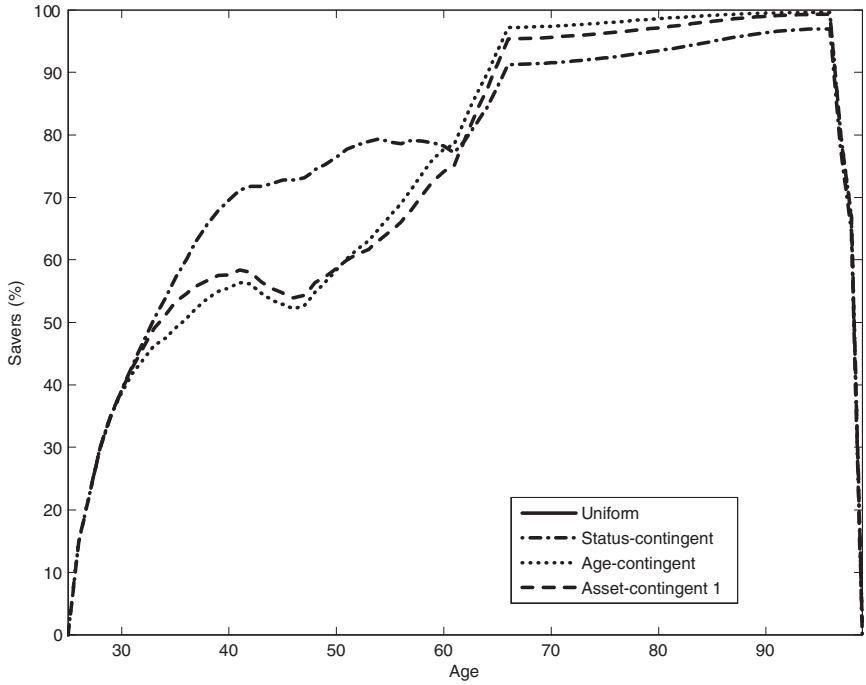


FIGURE 2. Likelihood of having personal savings for an agent born at $t = 1$.

$0 \leq \xi_{i,j,t} \leq 1$. Figure 2 plots the frequency of an agent born at time $t = 1$ having nonzero personal savings. Not surprisingly, this frequency increases as the agent becomes older, because he wants to smooth consumption and his income increases during his working life. Whenever an agent has positive personal savings, he invests all of them in equity: the higher expected return on equity dominates the effect of the higher risk in choosing the optimal composition of the portfolio of personal assets. For the age range 35–65, the propensity to have personal savings differs markedly across the various schemes. It is highest under the uniform scheme. Under a risk-reallocation scheme, personal savings become less attractive, because the equity investments carry a larger (identical) risk with their pension portfolio. This happens once individuals have accumulated a certain amount of pension assets. Hence, as confirmed by Figure 3, at retirement the average "normalized" (i.e., divided by the trend $(1 + \bar{g})^{t-1}$) amount of personal savings is highest under the uniform scheme and lower under the other schemes.

Figure 4 shows the share of the pension portfolio invested in equity. Not surprisingly, under the uniform scheme, the equity share is falling with age, in line with the idea that as one gets older the possibility of making up for investment losses shrinks and it may be preferable to take less risk. Under the other schemes, individuals start with low equity shares because of the high risk they already run by guaranteeing the return on the retirees' stakes in the fund. Individuals invest

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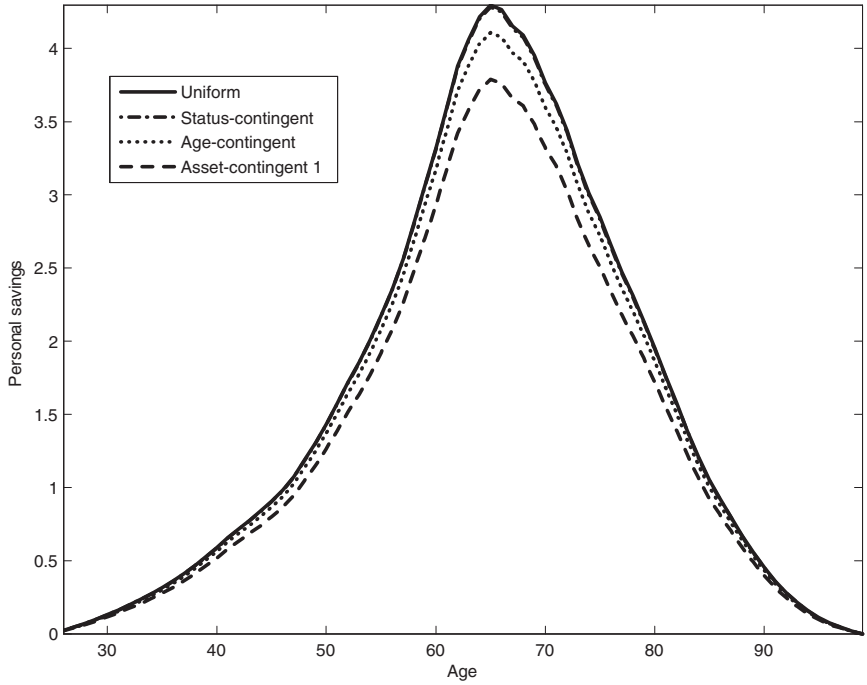


FIGURE 3. Normalized personal savings of an agent born at $t = 1$.

progressively more in equity, as the burden of risk reallocation falls with age. As of the age of 65, 100% of the portfolio is invested in equity, because all the return risk is absorbed by the workers and retirees thus earn the equity premium for free.

The labor supply is also affected by the specific risk-reallocation scheme that is used. Figure 5 plots the age profile of the labor supply for an agent born at time $t = 1$. It is higher at early ages, when income is lower and agents choose to work more to satisfy consumption needs and avoid hitting their credit constraint. The labor supply falls progressively as age rises and personal wealth increases. Around age 30, the labor supply is lowest under the uniform scheme and highest under the age-contingent scheme and asset-contingent scheme I. Under these schemes, individuals face greater risks in their pension assets and, hence, the chances of facing a bad realization in the accumulation of pension assets are relatively large. Bad realizations in pension wealth accumulation force individuals to work more.[9] Figure 6 plots normalized consumption over the life cycle of an agent born in period $t = 1$. Normalized consumption tracks income, which increases with age. However, around age60 it starts falling in order to smooth it and accommodate future lower (pension) income.

Having discussed the life-cycle patterns of the propensity to save, the labor supply, and the consumption of an individual born at $t = 1$, we now turn to discussing the aggregate statistics for the economy in any year of the simulation

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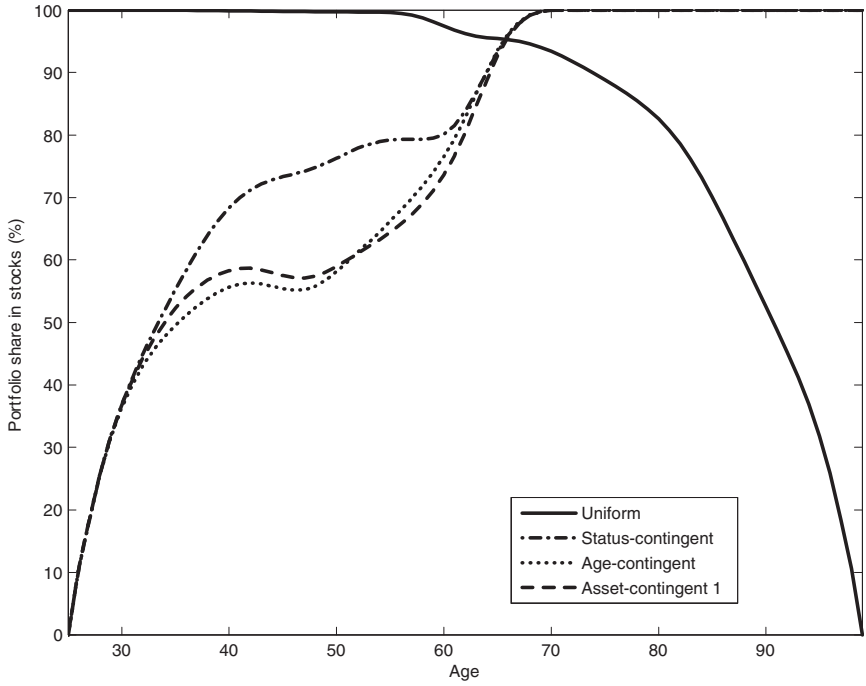


FIGURE 4. Pension portfolio composition for an agent born at $t = 1$.

(see Figure 7). The fraction of personal savers is very similar across the schemes and rather constant over time at approximately 65% of the entire population. Also, the average labor supply is very similar across the schemes and almost flat over time, at a level of roughly 25% of total available time. Normalized average consumption is also rather similar across the schemes and varies very little over time—it fluctuates mostly between 1.02 and 1.03. Finally, the average normalized retirement benefits from the first and second pillar (not depicted) hover between 0.374 and 0.414, respectively 0.413 and 0.563.

Figure 8 depicts the first-pillar contribution rate (θ_t^{SS})—the second-pillar contribution rate is always fixed at $\theta^{\mathrm{DC}} = 0.05$—and the consumption tax rate (τ_t). The two rates are also relatively stable and similar across the risk-reallocation schemes. The (hardly visible) differences across the schemes are due to labor supply decisions [see equations (5) and (7)]: the larger the labor supply, the lower the first-pillar contribution rate and the higher the consumption tax rate.

Table 2 reports the aggregate welfare effects of the introduction of a risk-reallocation scheme. We report both the percentages of individuals in favor of the change and aggregate equivalent variations in consumption. We do this for the workers as a group, the retirees as a group, and the entire population at $t = 1$. Although virtually all retirees are in favor of some form of risk reallocation,[10] none or only a few of the workers are in favor. In terms of consumption equivalent

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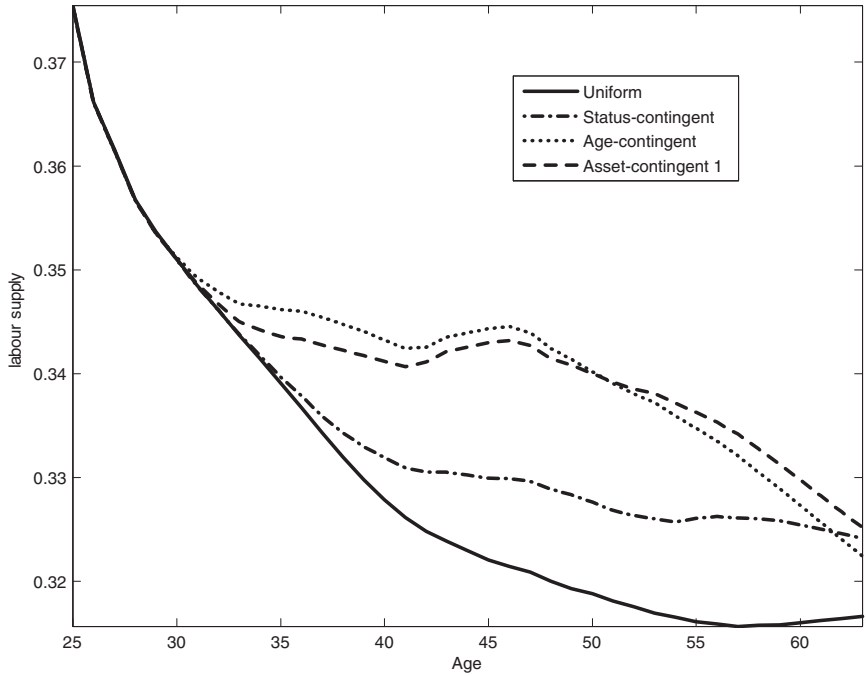


FIGURE 5. Labor supply of an agent born at $t = 1$.

variation, the retirees as a group experience a modest gain, because they see an elimination of the risk in the return on their pension portfolios. For instance, they would need a 0.16% increase in consumption under the uniform scheme to be as well off as under the status-contingent scheme. This effect is small for two reasons. First, retirees have already accumulated a reasonable amount of personal wealth that they can use as a buffer in case of need. Second, they also earn a

TABLE 2. Aggregate welfare effects for endogenous portfolio compositions

Scheme	Workers	Retired	All
% in favor of risk reallocation			
Status	0	99.914	24.791
Age	16.555	99.914	36.238
Asset I	10.069	99.914	32.361
Consumption equivalent variation (CEV)			
Status	−1.523	0.157	−1.106
Age	−2.471	0.168	−1.816
Asset I	−2.875	0.168	−2.120

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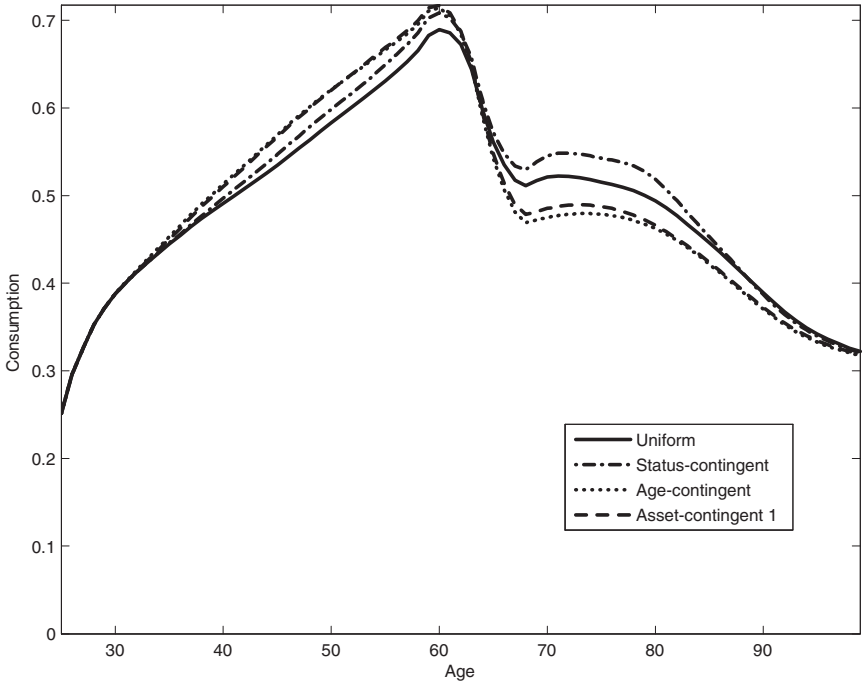


FIGURE 6. Normalized consumption of an agent born at $t = 1$.

first pillar pension benefit that is of the same order of magnitude as the second pillar benefit, but is less uncertain because it depends on labor growth realizations, which are more stable than stock returns. In contrast, workers lose under a risk-reallocation scheme because they have to bear the additional risk shifted away from the retirees. They lose despite the fact that, once retired themselves, they will make use of such a scheme. Under all schemes the negative consequences for workers are larger than the positive consequences for retirees. The workers have accumulated far less pension wealth than the retirees; hence a given reduction in the uncertainty of the retirees' pension wealth produces a larger relative increase in the uncertainty of the workers' pension wealth. Compounding the asset returns until retirement results in a substantially increased spread in pension wealth at the start of retirement. Finally, we see that the consumption equivalent variation for the overall population alive at $t = 1$ is negative.

Table 3 reports pension assets at retirement date for an agent born at $t = 1$. Not surprisingly, uncertainty about pension asset accumulation is substantially larger under the alternatives to the uniform scheme, as the alternatives transfer all the financial market risk to the workers. Figure 9 plots the CEV separately for each age cohort at $t = 1$ and for each scheme. Individuals born at $t = 1$ (or later—these are not depicted) are indifferent across the various schemes: all the additional pension assets risk they experience during their working life is compensated for by the

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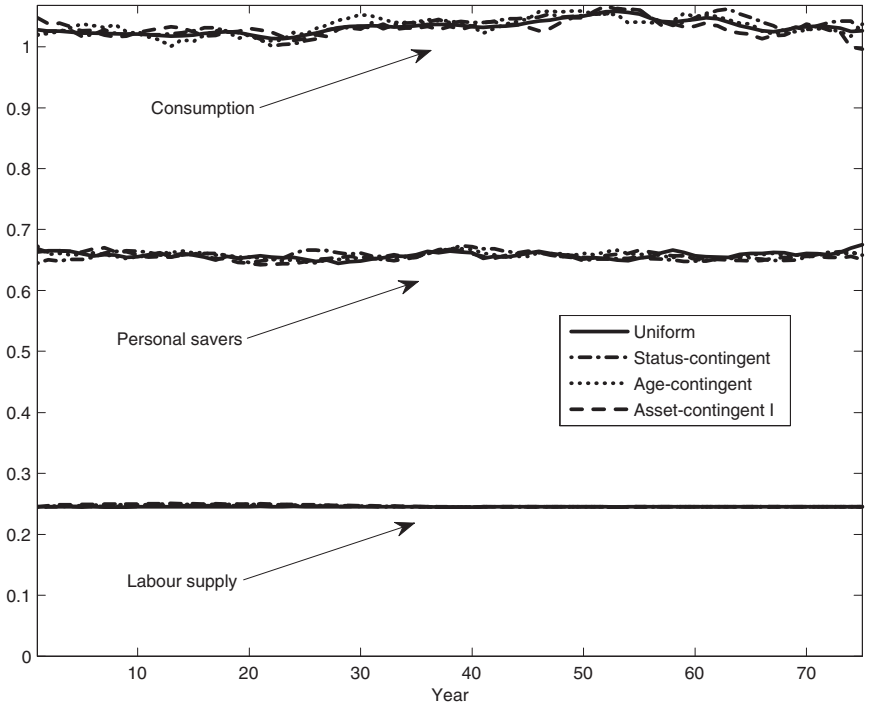


FIGURE 7. Aggregate statistics for the economy.

absence of risk during retirement. Those who suffer most are the cohorts around 40 at $t = 1$ under the age-contingent scheme and asset-contingent scheme I and the cohort around 50 at $t = 1$ under the status-contingent scheme. The status-contingent scheme performs relatively better for younger workers, because this scheme carries less risk at a lower ages (recall Figure 1a). This effect dominates the higher risk that the scheme carries when workers are relatively close to retirement. In contrast, older workers are relatively better off under the age-contingent and asset-contingent scheme I, as these schemes generate less uncertainty in pension assets when workers are relatively close to retirement.

TABLE 3. Pension assets at retirement date for an agent born at $t = 1$

Scheme	Mean	Standard deviation	Coeff. of variation
Uniform	4.258	3.356	0.788
Status	4.204	3.897	0.927
Age	3.739	3.604	0.964
Asset I	3.746	4.368	1.166

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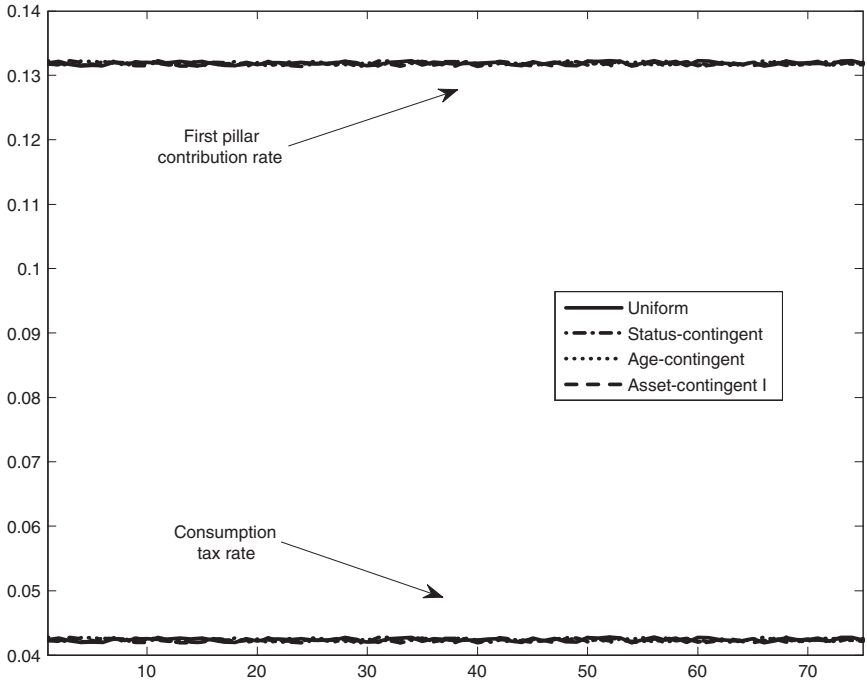


FIGURE 8. Contribution and tax rates.

4.2. Exogenous Portfolio Compositions

In this subsection, we assume that the compositions of both asset portfolios are exogenous. Hence, the third and fourth equations in our system of first-order conditions (2.5) drop out. The reason for studying the case of an exogenous pension portfolio is that there is empirical evidence that, even when individuals can choose the pension portfolio composition they wish, they seldom do so. For example, the work of Choi et al. (2002, 2004, 2006) on U.S. DC plans shows that investors prefer to follow "the path of least resistance" and tend not to make optimal financial decisions on their plans when they are allowed to do so; rather, they participate more frequently in plans with preselected contribution rates and asset allocation [Beshears et al. (in press)]. Further, assuming that the composition of the personal asset portfolio is exogenous allows us to avoid working with unrealistically high portfolio shares invested in equity. As we shall see later, the results are qualitatively unaffected when the asset portfolios are determined exogenously.

For the pension portfolio we choose two age profiles, a flat one in which 50% is invested in equity at any age and one in which the equity share is declining with age, in line with the idea that as one gets older the possibility of making up for investment losses shrinks, and it may be preferable to take less risk. Specifically,

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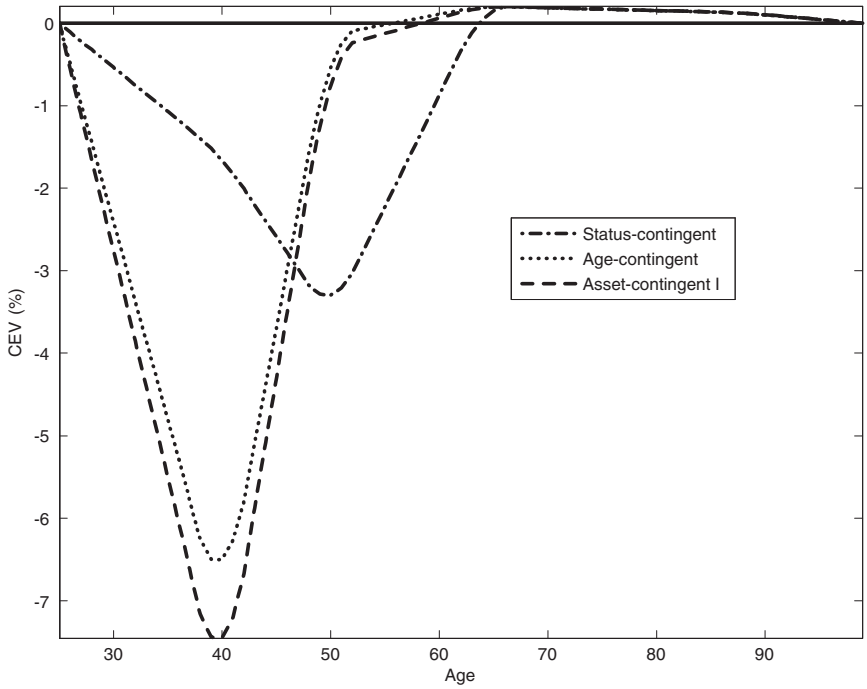


FIGURE 9. Consumption equivalent variation (CEV).

in the latter case we set the equity share of the pension portfolio at 75% up to age 40, at 50% between ages 41 and 60, and at 25% for ages of 61 and higher.

In view of the fact that there is no conclusive empirical evidence on the life-cycle pattern of the composition of personal asset portfolios, we consider here both a flat age profile for the equity share and a decreasing one. Under the flat profile 50% of all personal savings are invested in equity, whereas the other schedule assumes a share that starts at 75% at age 25 and falls linearly to zero at age 100. Compared to actual figures for the United States, the fractions invested in equity may be on the high side—see, for example, Bricker et al. (2012), who report from SCF survey data an average (direct or indirect) holding of around 40%. However, given that the nonequity investments in our model are risk-free, it seems more realistic if we consider our equity investments as a stand-in for all the risky financial investments in reality. We combine these two age profiles for the equity share in the personal portfolio with the two age profiles for the equity share in the pension portfolio, which produces a total of four combinations.

For all the combinations and under all the regimes, (normalized) consumption and labor supply are very stable over time. Normalized consumption fluctuates between 0.94 and 0.95, whereas the labor supply fluctuates between 0.24 and 0.25. Table 4 reports the welfare effects of shifting to a risk-reallocation scheme. Again, introducing such a scheme is beneficial for retirees as a group, but disadvantageous

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TABLE 4. Aggregate welfare effects for exogenous portfolio compositions

	Workers		Retired		All	
	I	D	I	D	I	D
			(a) Age-independent personal portfolio			
			% in favor of risk reallocation			
Status	4.911	10.962	99.914	99.914	28.245	32.251
Age	1.934	1.934	99.914	99.914	26.248	26.245
Asset-I	3.909	1.934	99.914	99.914	27.730	26.245
			Consumption equivalent variation (CEV)			
Status	−0.296	−0.133	0.044	0.040	−0.211	−0.090
Age	−0.456	−0.388	0.045	0.040	−0.331	−0.282
Asset-I	−0.471	−0.394	0.045	0.040	−0.343	−0.287
			(b) Age-decreasing personal portfolio			
			% in favor of risk reallocation			
Status	7.962	21.051	99.914	99.914	30.295	40.618
Age	1.934	1.934	99.914	99.914	26.245	26.245
Asset-I	3.909	0	99.914	99.914	27.730	24.791
			Consumption equivalent variation (CEV)			
Status	−0.298	−0.173	0.043	0.041	−0.213	−0.120
Age	−0.449	−0.378	0.043	0.039	−0.327	−0.275
Asset-I	−0.464	−0.375	0.044	0.039	−0.338	−0.272

Note: "I" stands for age-independent; "D" stands for age-decreasing.

for workers as a group. The results are qualitatively similar to those found before. However, quantitatively the effects are substantially smaller than in the case of endogenously chosen portfolios. The reason is that now the pension portfolios are less risky, implying that the amount of risk-shifting from the elderly to the young has become smaller. Hence, the welfare effects for both groups are also smaller now. Further, compared to the case of an age-independent exogenous equity share, we see that age-declining equity fractions in the pension asset portfolio reduce the losses to the workers from a risk-reallocation scheme. The reason is that the additional uncertainty about the equity returns that workers bear is relatively large at a young age when the amount of accumulated pension rights is still relatively small. Again, losses for workers are smallest under the status-contingent scheme. In fact, the reduction in the loss from shifting to an age-declining equity fraction in the pension asset portfolio is larger under the status-contingent scheme than under the other two regimes, the reason being that the other two regimes shift relatively more of the return uncertainty to young workers, which is more harmful under an age-declining equity fraction.

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TABLE 5. Aggregate welfare effects: No first-pillar pension

Scheme	Workers	Retired	All
% in favor of risk reallocation			
Status	5.925	99.914	29.245
Age	0	99.914	24.791
Asset I	0	99.914	24.791
Consumption equivalent variation (CEV)			
Status	−0.734	0.059	−0.537
Age	−0.432	0.056	−0.311
Asset-I	−0.433	0.056	−0.312

5. ROBUSTNESS

In this section we investigate the robustness of our results for some variations on the setup employed so far. Specifically, we consider the absence of first-pillar pensions and alternative methods of redistributing accidental bequests. In both cases, we continue to assume exogenous portfolio compositions with age-declining equity fractions in both portfolios.

5.1. No First Pillar

Like the second-pillar benefit, the first-pillar benefit is subject to uncertainty, as it is related to wage growth. However, wage growth varies much less than the stock returns that affect the second-pillar benefit. To investigate the importance of the presence of first-pillar benefits for our results, this subsection assumes the complete absence of the first pillar. That is, we impose $\rho = 0$, and hence $\theta_t^{SS} = 0$. We expect that, because the entire income during retirement is now subject to equity risk, the welfare consequences of introducing a risk-reallocation scheme should be larger.[11] This is borne out by the numbers reported in Table 5. Retirees as a group benefit more from the introduction of such a scheme, whereas workers as a group lose more than before.

5.2. Alternative Ways to Redistribute Accidental Bequests

This subsection considers cases in which bequests are no longer distributed uniformly across the population, but are distributed either among workers only or among retirees only. Within these groups the distribution is uniform, though. Table 6 reports the aggregate welfare effects by group and for the entire population under the two alternatives. The effects of introducing a risk-reallocation scheme are virtually unchanged.

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TABLE 6. Consumption equivalent variation (CEV): Different bequest redistribution

Scheme	Workers	Retired	All
	All bequests to workers		
Status	−0.175	0.041	−0.122
Age	−0.378	0.039	−0.275
Asset-I	−0.374	0.039	−0.271
	All bequests to retirees		
Status	−0.166	0.041	−0.114
Age	−0.380	0.039	−0.276
Asset-I	−0.377	0.039	−0.274

6. EXTENSIONS OF THE BASELINE SETUP

In this section we consider a number of extensions of the baseline setup. First, we extend the risk-reallocation schemes by compensating workers with a higher average return on their pension assets, financed by giving retirees a lower average return. Then, we extend the schemes by introducing wage-risk reallocation in addition to financial-risk reallocation. Finally, we consider skill differences. We continue to assume exogenous portfolio compositions with age-declining equity fractions in both portfolios.

6.1. Compensating Workers with a Higher Return on Financial Assets

We now consider the case in which workers are compensated for the additional financial market risk they bear through a higher average rate of return on their pension assets. We still keep $\alpha_t^{\mathrm{w}} = \alpha_t^{\mathrm{r}} = 0$ and consider the welfare effects for three cases. The first is when we lower the rate of return on the retirees' assets to coincide with the risk-free rate of return. This implies that on the average $c_t^{\mathrm{r}} = 1.20\%$ and, to ensure that the additional transfers sum to zero over all cohorts, on the average $c_t^{\mathrm{w}} = 0.71\%$. The values for c_t^{w} and c_t^{r} are very stable over time. Panel (a) of Table 7 reports the results. They indicate that retirees still benefit from removing the risk in their pension assets, whereas compared to the benchmark with $c_t^{\mathrm{w}} = c_t^{\mathrm{r}} = 0$ workers gain only rather marginally from the increase in the return on their pension assets (compare with Table 4b). Panel (c) of Table 7 considers the extreme situation in which the retirees would no longer get a return; i.e., on the average $c_t^{\mathrm{w}} = 1.49\%$ and $c_t^{\mathrm{r}} = 2.53\%$. The welfare effects are now mirrored, with workers gaining and the retirees losing. This raises the question of whether it is possible to find an arrangement that makes both population groups better off. This is indeed the case. Assuming that assets during retirement grow at only three-fourths the risk-free rate, which implies that on the average $c_t^{\mathrm{w}} = 0.90\%$ and $c_t^{\mathrm{r}} = 1.53\%$, makes all individuals better off [Panel (b) of Table 7], although the welfare effects in terms of consumption equivalent variation are very small.

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TABLE 7. Aggregate welfare effects: Giving workers a higher return

	(a) $c_t^w = 0.71\%$, $c_t^r = 1.20\%$			(b) $c_t^w = 0.90\%$, $c_t^r = 1.53\%$			(c) $c_t^w = 1.49\%$, $c_t^r = 2.53\%$		
Scheme	Workers	Retired	All	Workers	Retired	All	Workers	Retired	All
	% in favor of risk reallocation								
Status	22.555	99.914	41.638	52.347	99.914	64.260	80.313	0	60.386
Age	3.931	99.914	27.845	58.912	99.914	69.184	94.934	0	71.379
Asset-I	1.934	99.914	26.245	59.410	99.914	69.558	98.066	0	73.734
	Consumption equivalent variation (CEV)								
Status	−0.166	0.040	−0.107	0.024	0.008	0.019	0.221	−0.042	0.156
Age	−0.348	0.038	−0.252	0.094	0.006	0.071	0.316	−0.040	0.228
Asset-I	−0.336	0.038	−0.243	0.096	0.006	0.073	0.294	−.040	0.212

However, this result is sensitive to the assumptions that we make. The Pareto improvement vanishes when we reduce the relative risk-aversion parameter from 3 to 2. It also vanishes when we allow individuals to select their personal and pension portfolios again.[12] In this case, in the absence of the risk-reallocation scheme, the retiree invests more heavily in equity and, hence, earns a higher return on his pension portfolio. Introduction of the risk-reallocation scheme requires a substantially larger expected return shift of $c_t^r = 3.30\%$ (implying that $c_t^w = 1.95\%$), to bring the retiree's expected return to the risk-free rate. The retiree would now be worse off under such a risk-reallocation schedule. Obviously, he would be even worse off if his pension portfolio return were brought to an even lower level.[13]

6.2. Shifting Wage Risk from Workers to Retirees

We return to the case $c_t^w = c_t^r = 0$ and assume that wage risk is shifted as well. In particular, we set $\alpha_t^r = 1$, so that retirees face full wage risk in their pension assets. The requirement that all ensuing additional transfers must add up to zero then implies that on the average $\alpha_t^w = 0.33$. (The value for α_t^w is very stable over time.) That is, workers shed one-third of their wage risk. Hence, in this setting, we remove financial market risk for retirees in exchange for a shift in wage risk from workers. Previous literature has shown that intergenerational sharing of wage shocks may increase welfare [e.g., Miyazaki et al. (2010)]. Table 8 shows that the losses (gains) associated with a risk-reallocation scheme fall for workers (retirees)—compare with Table 4b. However, the consumption equivalent variations are very small. Variation of the risk-reallocation parameter α_t^r, and hence also α_t^w, over a reasonable range fails to produce a Pareto improvement by making both groups better off, the reason being that asset market risk substantially dominates wage risk.[14]

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TABLE 8. Aggregate welfare effects: Shifting wage risk from workers to retirees

Scheme	Workers	Retired	All
% in favor of risk reallocation			
Status	21.551	99.914	40.793
Age	2.234	99.914	26.353
Asset-I	0.781	99.914	24.935
Consumption equivalent variation (CEV)			
Status	−0.171	0.035	−0.117
Age	−0.376	0.033	−0.272
Asset-I	−0.373	0.032	−0.268

6.3. Differences in Skills

This subsection relaxes the assumption of identical skills across the population. Specifically, we assume ten different and equal-sized skill groups ($I = 10$). For the group of workers as a whole, the introduction of skill differences has no qualitative impact. Quantitatively, the losses to workers as a group are smaller than before. However, those with the highest skill levels absorb a relatively large part of the burden (see Figure 10), because they accumulate relatively large amounts of wealth. This is in its most extreme form the case for the highest-skilled group under the asset-contingent scheme I. This extreme effect vanishes under the asset-contingent scheme II, which avoids ex post redistribution among the skill groups. Although all retirees (except those at the greatest possible age, who are indifferent again) benefit from adopting one of the risk-reallocation schemes, the highest-skilled retirees benefit most. This is not surprising, because higher-skilled groups hold, relative to their consumption, larger stocks of assets during retirement and, hence, they benefit more from taking away the financial market risk associated with these assets.[15]

7. CONCLUSION

In this paper we have explored the introduction of collective risk-reallocation elements in individual DC pension arrangements. The motivation for this investigation is that participants in individual DC arrangements run substantial risk during retirement, because they cannot share their risks with other groups. We measure the consequences of the introduction of risk-reallocation schemes in terms of consumption equivalent variations and find that qualitatively the various schedules we consider work out in similar ways in terms of overall welfare and welfare of workers and retirees as groups. Retirees as a group benefit under all schedules from a reduction in their exposure to financial market risk, whereas workers as a group lose, despite the fact that they benefit later from the same arrangement

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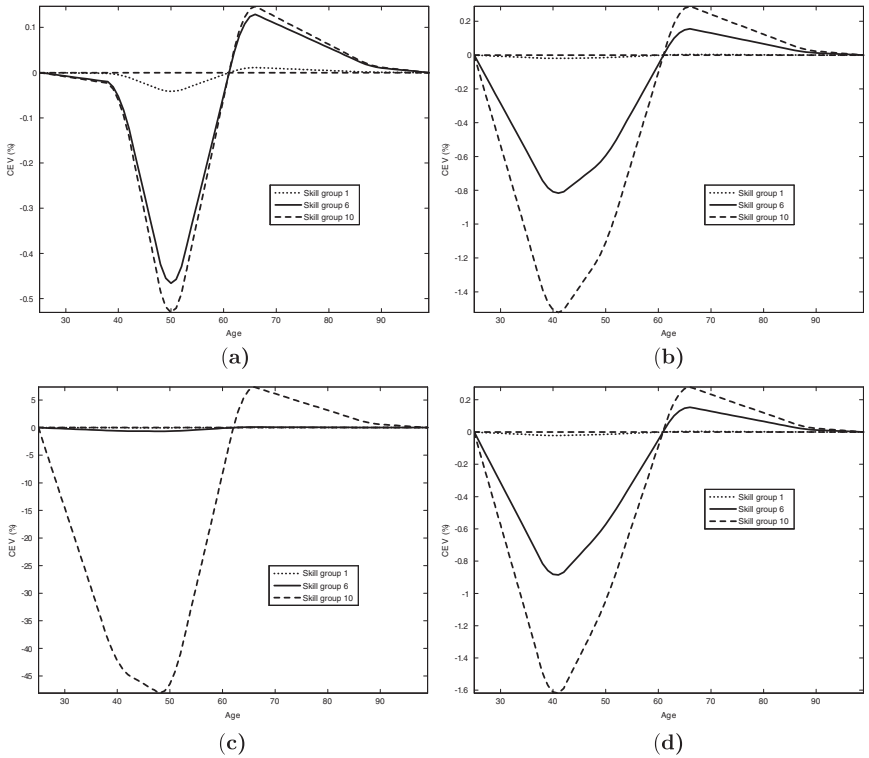


FIGURE 10. Consumption equivalent variation: (a) status-contingent; (b) age-contingent; (c) asset-contingent I; (d) asset-contingent II.

when they are themselves retired. Overall welfare falls, however, indicating that risk reallocation between generations in order to create better DC pensions is not very effective. The welfare effects, and the associated welfare fall for society at large, are substantially larger when the personal and pension portfolios are optimally chosen than when they are exogenous with empirically plausible equity shares. Allowing for intragenerational heterogeneity, the highest-skilled retirees benefit most, whereas the highest-skilled workers lose most. Qualitatively, our main findings are robust against a number of model variations and extensions. For the case with exogenous portfolios and absence of intragenerational heterogeneity, we also tried to find Pareto-improving risk-allocation schedules by extending them to allow for shifts in wage risk from workers to retirees and by compensating the young with a higher expected return on their pension assets financed through a reduced expected return on the retirees' pension portfolio. Only in the latter case did we find some scope for Pareto improvement. However, the improvement vanished when portfolio compositions were again chosen optimally or the degree of risk aversion was reduced sufficiently.

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NOTES

1. See the United Kingdom's Workplace Retirement Income Commission (2011). The Commission was installed out of dissatisfaction with the large share of the working population not covered or insufficiently covered by pension savings.

2. However, some other papers do allow endogenous labor supply—for example, Bonenkamp and Westerhout (2010), Mehlkopf (2010), and Beetsma et al. (2013).

3. Because each age–skill group consists of a continuum of individuals, the fractions of the group that are alive in future periods are known with certainty. Hence, individual longevity risks are shared and the pension fund that is responsible for executing the retirement plans is always able to meet its obligations.

4. We implicitly assume that the subportfolio of stocks has a constant composition. In reality, given that different stocks may have different exposures to wage risks, some characteristics of the outcomes where risk is shared between the elderly and younger workers might be achieved by varying the composition of the subportfolio of stocks with age.

5. We take the time $t = 0$ distribution of retirement savings to avoid the circularity problem of having retirement savings that depend on the rescaling function that in turn depends on retirement savings. How the $t = 0$ distribution is generated is discussed later.

6. Imposing liquidity constraints prevents individuals from freely allocating their savings over the life cycle, making their consumption—and thus welfare—more sensitive to the scenario under investigation.

7. With our calibration, contributions to the funded pension scheme are on average equal to $0.83\nu_t$. This means that the employer usually matches most contributions.

8. For the sake of space, we do not report the results of the case in which the composition of the personal portfolio is endogenous, but that of the pension portfolio is exogenous. This case does not alter our conclusions, and results may be obtained from the authors upon request.

9. Labor supply is 0.306 on the average after a positive shock to the equity return and 0.354 on the average after a negative shock to the equity return.

10. The oldest cohort of retirees is indifferent and, therefore, not counted as "in favor," because it knows that its final pension benefit is the same with or without the scheme.

11. As expected, we find that the propensity to engage in private saving rises substantially in the absence of a first pillar.

12. For the sake of space, the results of these further experiments are not reported, but they can be obtained from the authors upon request.

13. To enhance our understanding of the potential Pareto improvement under exogenous portfolios further, we also ran an experiment in which we assumed the personal portfolio exogenous and only the pension portfolio endogenous (results available upon request). In this case, individuals reach their retirement age with a substantially smaller amount of personal assets than when the personal portfolio is endogenous. The lower amount of personal assets makes them less willing to invest their pension portfolio in equity in the absence of a risk-reallocation arrangement: now they would choose to invest it entirely in the risk-free asset. Hence, given the assumed exogenous composition of the personal portfolio and the assumed risk aversion, retirees attach substantial value to the risk elimination in their pension portfolio via our risk-reallocation schemes.

14. We also simulated the case in which financial market risk was set to zero and α_t^w was set to 1. Workers are indeed better off in this case, whereas retirees are worse off. Aggregate welfare improves in this case.

15. The ratio of average pension assets to average consumption in the year of retirement is 4.750 for skill group 1, 9.132 for skill group 6, and 9.951 for skill group 10.

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