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Large-amplitude Alfvén waves in cylindrical magnetic tubes

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Abstract. The aim of this paper is to analyze the propagation of large-amplitude Alfvén waves in a perfectly conducting plasma in the magnetohydrodynamic approximation, for cylindrical magnetic tubes. In a similar way to that done in Sallago and Platzeck, instead of linearizing the magnetohydrodynamic equations and searching monochromatic waves, we impose the conditions that the perturbations propagate without distortion with a group velocity parallel to the background magnetic induction field, in the reference system in which the plasma is locally at rest, the perturbations are incompressible, there exists a relation between the velocity and induction magnetic field perturbations, and the total pressure remains constant. The solutions are similar to the case in which the background fields vary with a rectilinear variable. When the solution is searched using this methodology there is no singularity problem. If the background total pressure is not uniform and constant, it is not possible to find waves of large amplitude that propagate like Alfvén waves in cylindrical magnetic tubes.

1. Introduction

Alfvén waves have been detected in a variety of space plasmas, such as the Solar wind (Denskat and Neubauer 1982; Tsurutani et al. 2001) and the Earth's magnetosphere (Drell et al. 1965; Mallinkrodt and Carlson 1978). The main characteristic of Alfvén waves in the magnetohydrodynamic approximation (MHD) and in uniform plasmas is that they propagate without distortion with a group velocity parallel to the background magnetic induction field. Moreover, Alfvén waves are incompressible perturbations, there exists a relation between the velocity and magnetic induction field disturbances, and the total pressure (plasma plus magnetic) has the same value in the perturbed and the unperturbed regions (Priest 1982).

Alfvén wave propagation has been analyzed by many authors. The most extended part of such analyses was performed for uniform background fields, linearizing the MHD equations for perturbations developed in monochromatic waves. But sometimes one must take into account the spatial dependence of plasmas; in this case, in the linearized limit, the perturbations can no longer be considered as sinusoidal perturbations in all directions. Some authors considered sinusoidal perturbations in the directions perpendicular to the direction of variation of the background fields, and the amplitude a function dependent on the other coordinate (Roberts 1991). When this was done, they faced singularities that lead to the

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existence of resonant Alfvén waves. These waves are used as a tool for explaining some heating mechanisms in plasmas. However, such a singularity is an artificial mathematical fact, an adverse consequence of considering linearized equations and monochromatic waves (Bellan 1996). This is a useful method when the objective is to get the singularities for a further analysis in the context of dissipative MHD, otherwise it leads to contradictory behaviour, since small amplitude perturbations give rise to infinity in the singularity neigbourhood (Roberts 1991). Searching for any possible heating mechanism some authors have studied Alfvén wave propagation in non-uniform plasmas, among others one can list Heyvaerst and Priest (1983), Nocera et al. (1984), Mok and Einaudi (1985), Goosens (1991), Sakurai et al. (1991) and Goossens and Ruderman (1996). On the other hand, it is known that many cylindrical structures exist that can be called magnetic flux tubes; some definitions imply only restrictions on symmetry (Priest 1982) and others require the physical parameters to be homogeneous inside the structure (Roberts 2004). With the more general definition, approximately cylindrical magnetic flux tubes exist in a variety of plasma systems, such as planetary magnetospheres, the Solar wind and some structures in the Sun. In particular, the Solar coronal loops are studied with interest by Solar physicists and astronomers specially with the modern instruments provided by SOHO (Solar and Heliospheric Observatory) and TRACE (Transition Region and Colonial Explorer). Also HOKOH (Japanese spacecraft) has provided important measurements for understanding the behaviour of magnetic flux tubes in the Solar corona (Uchida et al. 2001). Oscillations in coronal loops have been studied by Díaz et al. (2004) and MHD waves in thin tubes in the Solar atmosphere by Roberts (2004). Many authors have developed nonlinear techniques for explaining the phenomena that take place in more general structures in the Sun, some by studying stationary structures with symmetry (Petrie et al. 2002), some by using perturbative methods (Ruderman 2003); Ruderman has studied the propagation of waves in magnetic tubes of the Sun, other than Alfvén waves. Torsional magnetic tube waves in a linear approximation have been analyzed by Noble et al. (2003).

An earlier paper on torsional Alfvén waves in circular cross section cylindrical magnetic tubes in a linear approximation is due to Spruit (1982) and an analysis of large amplitude torsional Alfvén waves in magnetic loops by using three-dimensional (3D) MHD simulations is given by Miyagoshi et al. (2001). Using an analytical method an exact solution of the MHD equations for large-amplitude Alfvén waves propagating in non-uniform background fields, varying with a rectilinear variable is found by Sallago and Platzeck (2000). This methodology is extended here for large-amplitude Alfvén waves propagating in cylindrical magnetic tubes.

On the other hand, it is known that Alfvén waves generated by a conducting source moving in a magnetized plasma can build up structures, called Alfvén wings. These are regions characterized by perturbed fields and intense electric currents. In a previous work (Sallago and Platzeck 2002) it was proved that Alfvén wings exist in plasmas with spatial dependence on r, φ, or a function $\eta = \eta(r, \varphi)$ where r, φ are cylindrical coordinates.

The aim of this paper is to analyze the propagation of large-amplitude Alfvén waves in a perfectly conducting plasma in the MHD approximation, for background fields ρ_0, p_0, $\mathbf{V}_0$ and $\mathbf{B}_0$, varying with the radius in cylindrical coordinates since, for this field structure, Alfvén wings exist. Instead of linearizing the MHD equations

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and searching monochromatic waves, we impose the conditions that the perturbation propagates without distortion with a group velocity parallel to the background magnetic induction field, in a reference system in which the plasma is locally at rest, the perturbation is incompressible, there exists a relation between the velocity and induction magnetic field perturbations, and the total pressure remains constant. When the solution is found using this methodology there is no singularity problem.

In Sec. 2 we analyse the full set of equations of the ideal MHD written for a cylindrical coordinate system, background fields varying with r and perturbations that are Alfvén waves. Two modes of propagation are found, as in the rectilinear case.

2. An MHD equation system for Alfvén waves

Let us consider a perfectly conducting magnetized plasma with density $\rho_0(r)$, plasma pressure $p_0(r)$, plasma velocity $\mathbf{V}_0 = V_{0\varphi}(r)\breve{e}_\varphi + V_{0Z}(r)\breve{e}_Z$ and a magnetic induction field $\mathbf{B}_0 = B_{0\varphi}(r)\breve{e}_\varphi + B_{0Z}(r)\breve{e}_Z$.

The ideal MHD equations, if viscous and gravitational forces are not taken into account, are (Priest 1982)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0, \tag{1}$$

$$\rho\left[\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V}\right] = -\nabla p + \frac{\mathbf{J}}{c}\times\mathbf{B}, \tag{2}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{3}$$

$$\mathbf{J} = \frac{c}{4\pi}\nabla\times\mathbf{B}, \tag{4}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{V}\times\mathbf{B}). \tag{5}$$

Equations of state and energy must be considered in order to complete the set of equations.

The background fields must fulfill

$$\frac{dP_0}{dr} + \frac{1}{r}\left[\frac{B_{0\varphi}^2}{4\pi} - \rho_0 V_{0\varphi}^2\right] = 0, \tag{6}$$

where $P_0 = p_0 + |\mathbf{B}_0|^2/8\pi$ is the total pressure.

Since we want to search for Alfvén waves in cases in which it has been proved that Alfvén wings can be built up, we impose that P_0 be constant, and so $V_{0\varphi}^2 = B_{0\varphi}^2/4\pi\rho_0$. This last relation is satisfied if the φ-component of the group velocity

$$\mathbf{V}'_{\mathrm{A}} = \mathbf{V}_0 \pm \frac{\mathbf{B}_0}{\sqrt{4\pi\rho_0}} \tag{7}$$

is null.

Taking into account the properties of Alfvén waves in uniform plasmas, we now search for solutions of the MHD equations which satisfy the following conditions.

(1) The perturbations ρ_1, p_1, V_{1i}, B_{1i} depend on r and t in the form

$$f(\mathbf{r} - \mathbf{V}'_{\mathrm{A}}t) = f(r, \varphi, z - V'_{\mathrm{A}Z}t) = f(x'_1, x'_2, x'_3). \tag{8}$$

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(2) The disturbance is incompressible:

$$\nabla \cdot \mathbf{V}_1 = 0 \tag{9}$$

and adiabatic

$$\frac{D}{Dt}\left(\frac{p}{\rho^\gamma}\right) = 0, \tag{10}$$

where γ is the ratio between the specific heats.

(3) The total pressure is constant,

$$P = P_0. \tag{11}$$

(4) The perturbations on velocity and magnetic induction fields are related through

$$\mathbf{V}_1 = \mp \frac{\mathbf{B}_1}{\sqrt{4\pi\rho}} + \boldsymbol{\alpha}, \tag{12}$$

where ρ is the total density and $\boldsymbol{\alpha}$, a vector to be determined, must vanish for uniform background fields or null disturbances. The upper (lower) sign in (12) corresponds to the upper (lower) sign in (7).

The first condition leads to the following relations between derivatives:

$$\frac{\partial}{\partial t} = -\left(V'_{AZ}\frac{\partial}{\partial x'_3}\right) = -\mathbf{V}'_A \cdot \nabla', \tag{13}$$

$$\frac{\partial}{\partial r} = \frac{\partial}{\partial x'_1} - t\left(\frac{dV'_{AZ}}{dr}\frac{\partial}{\partial x'_3}\right) = \frac{\partial}{\partial x'_1} - t\left(\frac{d\mathbf{V}'_A}{dr} \cdot \nabla'\right), \tag{14}$$

$$\frac{\partial}{\partial \varphi} = \frac{\partial}{\partial x'_2}, \tag{15}$$

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial x'_3}, \tag{16}$$

where $\nabla' = \breve{e}_i(\partial/\partial x'_i)$. Note that $(d\mathbf{V}'_A/dr \cdot \nabla')$ applied to a background field is zero because background fields depend only on r and the group velocity has no r-component.

Since the background fields are not uniform, the first requirement imposes conditions on the perturbed fields in order to satisfy the MHD equations every time.

From the solenoidal condition on magnetic fields,

$$\nabla \cdot \mathbf{B} = \nabla' \cdot \mathbf{B} - t\left(\frac{d\mathbf{V}'_A}{dr} \cdot \nabla'\right) B_{1r} = 0; \tag{17}$$

the first term is a function only of x'_1, x'_2, x'_3 and not of t; therefore (17) gives

$$\nabla' \cdot \mathbf{B} = 0, \tag{18}$$

$$\left(\frac{d\mathbf{V}'_A}{dr} \cdot \nabla'\right) B_{1r} = 0. \tag{19}$$

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In a similar way, the incompressibility of the perturbation (see (9)) implies

$$\boldsymbol{\nabla}' \cdot \mathbf{V}_1 = 0, \tag{20}$$

$$\left(\frac{d\mathbf{V}'_{\mathrm{A}}}{dr} \cdot \boldsymbol{\nabla}'\right) V_{1r} = 0. \tag{21}$$

From the continuity equation, after using (7), (9) and (13), one obtains

$$(\mathbf{V} - \mathbf{V}'_{\mathrm{A}}) \cdot \boldsymbol{\nabla}' \rho = \left(\mathbf{V}_1 \mp \frac{\mathbf{B}_0}{\sqrt{4\pi\rho_0}}\right) \cdot \boldsymbol{\nabla}' \rho_1 + V_{1r} \frac{d\rho_0}{dr} = 0, \tag{22}$$

$$V_{1r} \left(\frac{d\mathbf{V}'_{\mathrm{A}}}{dr} \cdot \boldsymbol{\nabla}'\right) \rho = 0. \tag{23}$$

Similarly, from the equation of motion, using the third condition, the following equations give

$$\breve{e}_i \left\{ \rho \left[\left(\mathbf{V}_1 \mp \frac{\mathbf{B}_0}{\sqrt{4\pi\rho_0}}\right) \cdot \boldsymbol{\nabla}' \right] V_i - (\mathbf{B} \cdot \boldsymbol{\nabla}') \frac{B_i}{4\pi} \right\}$$

$$= -\frac{\breve{e}_r}{r} \left(\frac{B_\varphi^2}{4\pi} - \rho V_\varphi^2\right) + \frac{\breve{e}_\varphi}{r} \left(\frac{B_{1r} B_\varphi}{4\pi} - \rho V_{1r} V_\varphi\right) \tag{24}$$

and

$$\rho V_{1r} \left(\frac{d\mathbf{V}'_{\mathrm{A}}}{dr} \cdot \boldsymbol{\nabla}'\right) \mathbf{V}_1 - B_{1r} \left(\frac{d\mathbf{V}'_{\mathrm{A}}}{dr} \cdot \boldsymbol{\nabla}'\right) \frac{\mathbf{B}_1}{4\pi} = 0, \tag{25}$$

and from the induction equation

$$\breve{e}_i \left\{ \left[\left(\mathbf{V}_1 \mp \frac{\mathbf{B}_0}{\sqrt{4\pi\rho_0}}\right) \cdot \boldsymbol{\nabla}' \right] B_i - (\mathbf{B} \cdot \boldsymbol{\nabla}') V_i \right\} = -\frac{\breve{e}_\varphi}{r} (B_{1r} V_\varphi - V_{1r} B_\varphi), \tag{26}$$

$$V_{1r} \left(\frac{d\mathbf{V}'_{\mathrm{A}}}{dr} \cdot \boldsymbol{\nabla}'\right) \mathbf{B}_1 - B_{1r} \left(\frac{d\mathbf{V}'_{\mathrm{A}}}{dr} \cdot \boldsymbol{\nabla}'\right) \mathbf{V}_1 = 0. \tag{27}$$

Taking into account the adiabaticity condition, we find that

$$\left(\mathbf{V}_1 \mp \frac{\mathbf{B}_0}{\sqrt{4\pi\rho_0}}\right) \cdot \boldsymbol{\nabla}' p = 0, \tag{28}$$

$$V_{1r} \left(\frac{d\mathbf{V}'_{\mathrm{A}}}{dr} \cdot \boldsymbol{\nabla}'\right) p = 0. \tag{29}$$

Since P is constant

$$\left(\mathbf{V}_1 \mp \frac{\mathbf{B}_0}{\sqrt{4\pi\rho_0}}\right) \cdot \boldsymbol{\nabla}' |\mathbf{B}|^2 = 0, \tag{30}$$

$$V_{1r} \left(\frac{d\mathbf{V}'_{\mathrm{A}}}{dr} \cdot \boldsymbol{\nabla}'\right) |\mathbf{B}|^2 = 0. \tag{31}$$

We note that (22), (23) and (28)–(31) imply that the material derivatives of ρ, p and $|\mathbf{B}|^2$ are null. Note that (24) and (26) are similar to the motion and induction equations that result if the background fields vary with a rectilinear variable (Sallago and Platzeck 2000), except in the terms of the right-hand side. However, there is no problem with these terms since P_0 is constant, as we show later in this paper.

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Equations (25) and (27) constitute a homogeneous system for $[(d\mathbf{V}'_{\mathrm{A}}/dr \cdot \nabla')]\mathbf{V}_1$ and $[(d\mathbf{V}'_{\mathrm{A}}/dr \cdot \nabla')]\mathbf{B}_1$. Its determinant is zero if

$$V_{1r} = \mp \frac{B_{1r}}{\sqrt{4\pi\rho}}, \tag{32}$$

and then, α_r is null (cf. (12)); after replacing this last relation in (25) and taking into account the relation between the perturbations in velocity and the perturbations in the magnetic induction field given by (12), one obtains

$$B_{1r}\left(\frac{d\mathbf{V}'_{\mathrm{A}}}{dr} \cdot \nabla'\right)\boldsymbol{\alpha} = 0. \tag{33}$$

The last equation has two possible solutions:

$$B_{1r} = 0, \tag{34}$$

$$\left(\frac{d\mathbf{V}'_{\mathrm{A}}}{dr} \cdot \nabla'\right)\boldsymbol{\alpha} = 0. \tag{35}$$

From (22) it is evident that when ρ_0 is not uniform, the perturbation in density is not null, except in the case where the perturbation in velocity, hence in magnetic induction, has no component in r, the direction of variation of the background fields. Therefore, it is convenient to separately analyze the cases $B_{1r} = 0$ and $B_{1r} \neq 0$.

2.1. B_{1r} null

Here V_{1r} is zero (cf. (32)) and then (19) and (21) are automatically satisfied. A solution to (22) and (23) is given by

$$\rho_1 = 0. \tag{36}$$

Taking into account the fact that the background fields depend only on r, the motion and induction equations (24) and (26) are satisfied if

$$\mathbf{V}_1 = \mp \frac{\mathbf{B}_1}{\sqrt{4\pi\rho_0}}. \tag{37}$$

So, in this case, $\boldsymbol{\alpha} = 0$.

From the adiabaticity condition, one finds that

$$p_1 = 0. \tag{38}$$

Then, since P is constant for Alfvénic perturbations:

$$|\mathbf{B}|^2 = |\mathbf{B}_0|^2. \tag{39}$$

Therefore, if the perturbation in the magnetic induction field does not have a component in the direction of the variation of the background fields, Alfvén waves are propagated in cylindrical magnetic tubes with the same characteristics as in a uniform plasma.

2.2. B_{1r} Non-null

When B_{1r} is different from zero, we see from (19), that it must be invariant in the direction of $d\mathbf{V}'_{\mathrm{A}}/dr$, the z direction. The solution of (33) that corresponds to this case is given by (35), so $\boldsymbol{\alpha}$ is also invariant in this direction. As it was explained

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before, ρ_1 is not zero and then, from (23), it is invariant in the z direction. From (32) and (21) V_{1r} is also non-null and invariant in the z direction.

Taking into account that α_r is always zero (cf. (32)) and using (12) and (22), the r component of the induction equation (26) gives

$$\left[\boldsymbol{\alpha} \pm \mathbf{B}_0\left(\frac{1}{\sqrt{4\pi\rho}} - \frac{1}{\sqrt{4\pi\rho_0}}\right)\right] \cdot \boldsymbol{\nabla}' B_{1r} = 0. \tag{40}$$

Let us define a new vector,

$$\mathbf{u} = \boldsymbol{\alpha} \pm \mathbf{B}_0\left(\frac{1}{\sqrt{4\pi\rho}} - \frac{1}{\sqrt{4\pi\rho_0}}\right); \tag{41}$$

from (40) B_{1r} is invariant in the $\mathbf{u}$ direction, but from (19) B_{1r} is also invariant in the direction of $d\mathbf{V}'_{\mathrm{A}}/dr$. Thus

$$\mathbf{u} \parallel d\mathbf{V}'_{\mathrm{A}}/dr. \tag{42}$$

As a consequence u_φ is null and from (41), we find

$$\alpha_\varphi = \mp B_{0\varphi}\left(\frac{1}{\sqrt{4\pi\rho}} - \frac{1}{\sqrt{4\pi\rho_0}}\right).$$

Moreover, from (23) and (35) it is immediately evident

$$\left(\frac{d\mathbf{V}'_{\mathrm{A}}}{dr} \cdot \boldsymbol{\nabla}'\right) u_Z = 0. \tag{43}$$

The equations that remain to be analyzed are those that come from the motion and induction equations (24) and (26). After replacing (12) and (41) and using (42), (22) and (23), these results are obtained:

$$(\mathbf{u} \cdot \boldsymbol{\nabla}')\mathbf{B} + (\mathbf{B} \cdot \boldsymbol{\nabla}')(\mathbf{u} + \mathbf{V}'_{\mathrm{A}}) = 0, \tag{44}$$

$$-(\mathbf{u} \cdot \boldsymbol{\nabla}')\mathbf{B} + (\mathbf{B} \cdot \boldsymbol{\nabla}')(\mathbf{u} + \mathbf{V}'_{\mathrm{A}}) = 0. \tag{45}$$

From them, it is evident that $\mathbf{B}$ is invariant in the direction of $\mathbf{u}$ and therefore of $d\mathbf{V}'_{\mathrm{A}}/dr$, the z direction.

Furthermore, $\mathbf{u}$ must satisfy

$$(\mathbf{B} \cdot \boldsymbol{\nabla}')(\mathbf{u} + \mathbf{V}'_{\mathrm{A}}) = 0. \tag{46}$$

On the other hand, from (30) and (31), we find that $\boldsymbol{\nabla}'|\mathbf{B}|^2$ is parallel to

$$\frac{d\mathbf{V}'_{\mathrm{A}}}{dr} \times \left(\mathbf{V}_1 \mp \frac{\mathbf{B}_0}{\sqrt{4\pi\rho_0}}\right). \tag{47}$$

We arrive at a similar conclusion from (22) and (23) for $\boldsymbol{\nabla}'\rho$; therefore $\boldsymbol{\nabla}'\rho$ is parallel to $\boldsymbol{\nabla}'|\mathbf{B}|^2$, so density is a function of $|\mathbf{B}|^2$,

$$\rho = \rho(|\mathbf{B}|^2). \tag{48}$$

From the fact that P is constant, the same result is obtained for the plasma pressure

$$p = p(|\mathbf{B}|^2). \tag{49}$$

Making a similar analysis to that performed on ρ, we can conclude that $\mathbf{u} + \mathbf{V}'_{\mathrm{A}}$ is a function of $|\mathbf{B}|^2$ (cf. (43) and (46)). From this, and since $\mathbf{u}$ is zero for null disturbance, the value of u_Z is the difference between $V'_{\mathrm{A}Z}$ at the original position

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of a given plasma element and V'_{AZ} at its present position. Consequently, the relation $\mathbf{V} \pm \mathbf{B}/\sqrt{4\pi\rho} = \mathbf{V}'_A$ remains true, as in the uniform plasma case, provided that $\mathbf{V}'_A$ takes the value that corresponds to the original position of the plasma element.

We can conclude that the propagation of Alfvénic perturbations, with a non-null component of the magnetic induction field in the direction of variation of the background fields, is possible in cylindrical magnetic tubes. The perturbation in the magnetic induction determines all the other perturbed fields. The values of $|\mathbf{B}|^2$, ρ and p for a given plasma element, at any time, are the same as those before being reached by the perturbation, since their material derivative are null. Furthermore, all the perturbed fields $(\rho, p, \mathbf{V}, \mathbf{B})$ must be invariant in the direction of $d\mathbf{V}'_A/dr$; this is necessary in order to avoid the failure of the MHD equations to be satisfied, due to the variation of $\mathbf{V}'_A$, since V_{1r} is not null.

Note that the results given in the present paper are similar to the results obtained when the background fields vary with a Cartesian variable (Sallago and Platzeck 2000) but, in the cylindrical case, the group velocity can be in the z direction only.

Spruit (1982) solved the problem of MHD wave propagation in magnetic flux tubes, in unstratified fluids. The spatial dependence of the background fields was restricted to different constant values inside and outside a proposed coronal magnetic tube radius. After linearizing the equations, he concluded that Alfvén waves propagate under the characteristics of torsional waves. From the results given in the present paper, applied to this field structure, it can also be shown that large amplitude Alfvén waves can propagate, as in the uniform plasma case, in the inner region of the tube if the perturbation on velocity and the perturbation on the magnetic induction field is null on the surface of the tube.

3. Conclusions

In the present paper we have shown that large-amplitude Alfvén waves, exact solutions of the MHD equations, can propagate in cylindrical magnetic tubes when ρ_0, p_0, $\mathbf{V}_0$ and $\mathbf{B}_0$ vary in the r direction, and the group velocity is in the z direction. If the background plasma flow is null, the background magnetic field must be in the direction of the tube axis. When the plasma is flowing in the tube, the background magnetic field can also have an azimuthal component, but the plasma background velocity must balance the azimuthal component in such a way that the group velocity results in the z direction.

In both cases, when the perturbed field has no component in the direction of the variation of the background fields, large amplitude Alfvén waves propagate with the same characteristics as in the uniform plasma case. Whereas, if this component is not null, the perturbation must be invariant in the z direction and the relation between the velocity and the induction magnetic field perturbations must be modified. These results are very similar to those obtained when the background fields vary with a Cartesian variable.

References

Bellan, P. M. 1996 *J. Geophys. Res.* **101**, 24 887.

Denskat, K. U. and Neubauer, F. M. 1982 *J. Geophys. Res.* **87**, 2215.

Díaz, A. J., Oliver, R., Ballester, J. L. and Roberts, B. 2004 *Astron. Astrophy.* **424**, 1055.

https://doi.org/10.1017/S0022377806004363 Published online by Cambridge University Press

Drell, S. D., Forley, H. M. and Ruderman, M. A. 1965 *J. Geophys. Res.* **70**, 3131.

Goossens, M. 1991 Magnetohydrodynamic waves and wave heating in non-uniform plasmas. In: *Advances in Solar System Magnetohydrodynamics*, vol. 137 (ed. E. R. Priest and A. W. Hood). New York: Cambridge University Press.

Goossens, M. and Ruderman, M. 1996 MHD waves in magnetic flux tubes. In: *Solar and Astrophysical Magnetohydrodynamic Flows*, vol. 61 (ed. K. C. Tsinganos). Dordrecht: Kluwer Academic Publishers.

Heyvaerts, J. and Priest, E. R. 1983 *Astron. Astrophys.* **117**, 220.

Mallinckrodt, A. J. and Carlson, C. W. 1978 *J. Geophys. Res.* **83**, 1426.

Miyagoshi, T., Uchida, Y., Yabiku, T., Hirose, S. and Cable, S. 2001 *Publ. Astron. Soc. Japan* **53**, 341.

Mok, Y. and Einaudi, G. 1985 *J. Plasma Phys.* **33**, 199.

Noble, M. W., Musielak, Z. E. and Ulmschneider, P. 2003 *Astron. Astrophys.* **409**, 1085.

Nocera, L., Leroy, B. and Priest, E. R. 1984 *Astron. Astrophys.* **133**, 387.

Petrie, G. J. D., Gotikakis, C., Dara, H. and Tsinganos, K. 2002 Exact 2D MHD modelling of coronal loops observed with TRACE and CDS including compressible flow and heating. *Proc. 10th European Solar Physics Meeting. Solar Variability: from Core to Outer Frontiers, Prague 9–14 September 2002 (ESA SP-506)*, p. 713–716.

Priest, E. R. 1982 *Solar Magnetohydrodynamics*. Norwell, MA: Reidel.

Roberts, B. 1991 Magnetohydrodynamic waves in the Sun. In. *Advances in Solar System Magnetohydrodynamics*, vol. 105 (ed. E. R. Priest and A. W. Hood). New York: Cambridge University Press.

Roberts, B. 2004 MHD waves in the solar atmosphere, *Proc. of SOHO 13, Waves, Oscillations and Small-scale Transient Events in the Solar Atmosphere: Joint View from SOHO and TRACE, Palma de Mallorca, 29 September–3 October, 2003 (ESA SP-547)*, p. 1.

Ruderman, M. S. 2003 Nonlinear waves in magnetically structured solar atmosphere, in *Turbulence, Waves and Instabilities in the Solar Plasma: Proc. NATO Advanced Research Workshop*, Hungary, 16–20 September 2002 (Nato Science Series II: Mathematics, Physics, Chemistry) (ed. B. Roberts, M. J. Aschwanden, R. Erdelyi and K. Petrovay) Kluwer.

Sakurai, T., Goossens, M. and Hollweg, J. V. 1991 *Solar Phys.* **133**, 227.

Sallago, P. A. and Platzeck, A. M. 2000 *J. Geophys. Res.* **105**, 27 393.

Sallago, P. A. and Platzeck, A. M. 2002 *J. Plasma Phys.* **67**, 321.

Spruit, H. C. 1982 *Solar Phys.* **75**(3).

Tsurutani, B. T., Buti, B., Arballo, J. K., Hada, T., Smith, E. J. and Balogh, A. 2001 *Adv. Space Res.* **28**, 765.

Uchida, Y., Miyagoshi, T., Yabiku, T., Cable, S. and Hirose, S. 2001 *Publ. Astron. Soc. Japan* **53**, 331.

https://doi.org/10.1017/S0022377806004363 Published online by Cambridge University Press