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LARGE DEVIATIONS FOR INFINITE-DIMENSIONAL STOCHASTIC SYSTEMS WITH JUMPS

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Abstract. Uniform large deviation principles for positive functionals of all equivalent types of infinite-dimensional Brownian motions acting together with a Poisson random measure are established. The core of our approach is a variational representation formula, which for an infinite sequence of independent and identically distributed real Brownian motions and a Poisson random measure was shown in [A. Budhiraja, P. Dupuis and V. Maroulas, Variational representations for continuous time processes. *Ann. Inst. H. Poincaré* (to appear)].

§1. *Introduction.* The theory of large deviations is one of the most active research fields in probability, having many applications to areas such as statistical inference, queuing systems, communication networks, information theory, risk sensitive control, partial differential equations and statistical mechanics. We refer the reader to [**7**, **8**, **11**, **20**] for background, motivation, applications, and fundamental results in the area. In this paper we establish a general uniform large deviation for functionals of a Poisson random measure (PRM) and infinite-dimensional Brownian motion. These two types of driving noises are used in a wide range of processes describing various physical and/or financial phenomena, e.g. reaction–diffusion of particles, environmental pollution, stock return, etc. The uniform large deviation result is expected to be fruitful in the study of asymptotics of steady state behavior for such infinite-dimensional stochastic partial differential equations with jumps describing the aforementioned phenomena. The uniformity is with respect to a parameter ζ that takes values in some compact subset of a Polish space $\mathcal{E}_0$. Typically, ζ is the initial condition of the corresponding stochastic partial differential equation (SPDE) whose solution's large deviation estimates are considered. A similar large deviation result for functionals of an infinite-dimensional Brownian motion was established in [**2**] and its uniform analogue in [**3**]. These results were used to study small noise asymptotics for a variety of infinite-dimensional stochastic dynamical models, and a partial list of such studies is [**1**, **3**, **4**, **6**, **9**, **10**, **13**–**17**, **19**, **21**–**23**, **25**]).

Our approach to the large deviation analysis is based on a variational representation for Polish space valued functionals of a PRM and infinite-dimensional Brownian motion. Such a variational result was already established in [**5**] for an infinite sequence of standard real Brownian motions and a PRM. Depending on the application, the infinite nature of the Brownian noise

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may be equivalently expressed as a Brownian sheet, a Hilbert-space valued Brownian motion, or a cylindrical Brownian motion. In this paper the variational representation result for functionals of any tantamount type of infinite-dimensional Brownian motion and a PRM will be presented.

A key ingredient in the variational formulation is the appropriate version of controlled PRM and infinite Brownian motion that will be used for purposes of representation. In the Brownian case, the control shifts the mean. In the PRM case, the control process enters as a censoring/thinning function, which in turn allows for elementary weak convergence arguments in proofs of large deviation results. In [**24**], Zhang has also proved a variational representation for functionals of a PRM. The corresponding control there moves the atoms of the PRM through a rather complex nonlinear transformation. However, the fact that atoms are neither created nor destroyed is partly responsible for the fact that the representation does not cover the standard Poisson process.

The usefulness of the representations is the fact that this approach does not require any exponential probability estimates to be established. Exponential continuity (in probability) and exponential tightness estimates are perhaps the hardest and most technical parts of the usual proofs based on discretization and approximation arguments, and this becomes particularly hard in infinite-dimensional settings where these estimates are needed with metrics on exotic function spaces. Furthermore, what is required for the weak convergence approach, beyond the variational representations, is that basic qualitative properties (existence, uniqueness and law of large number limits) can be demonstrated for certain controlled versions of the original process. Hence, using the above machinery, Theorem 4.4 establishes large deviation estimates for the family of solutions of SPDEs, $\{Z^{\epsilon,\zeta}\}$, as $\epsilon \to 0$, uniformly with respect to the initial solution, ζ. The family of random elements $\{Z^{\epsilon,\zeta}\}$ are expressed via a family of measurable maps $\mathcal{G}^{\epsilon}$ that satisfy a weak form of asymptotic continuity (Condition 4.1). The reader should remark here that although weak solutions (in the probability sense) of SPDEs exist, a strong solution property is used here instead for defining these measurable maps $\mathcal{G}^{\epsilon}$.

We now give an outline of the paper. Section 2 contains some background material on large deviations, infinite-dimensional Brownian motions and a PRM. In §3 we present a variational representation for bounded non-negative functionals of an infinite sequence of real Brownian motions and PRM. This variational representation, originally obtained in [**5**], is the starting point of our study. We also provide analogous representations for other formulations of infinite-dimensional Brownian motions and PRM. Section 4, the main section of this paper, gives a uniform large deviation result for Polish space valued functionals of infinite-dimensional Brownian motions and PRM. Sufficient conditions for the uniform large deviation principle (LDP) for each of the formulations of an infinite-dimensional Brownian motion mentioned above are provided.

Notation and a topology. The following notation will be used.

The Borel sigma-field on $\mathbb{S}$ will be denoted as $\mathcal{B}(\mathbb{S})$. Given $\mathbb{S}$-valued random variables X_n, X, we will write $X_n \Rightarrow X$ to denote the weak convergence of

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$P \circ X_n^{-1}$ to $P \circ X^{-1}$. For a real bounded measurable map h on a measurable space $(V, \mathcal{V})$, we denote $\sup_{v \in V} |h(v)|$ by $\|h\|_\infty$.

For a locally compact Polish space $\mathbb{S}$, we denote by $\mathcal{M}_F(\mathbb{S})$ the space of all measures ν on $(\mathbb{S}, \mathcal{B}(\mathbb{S}))$ satisfying $\nu(K) < \infty$ for every compact $K \subset \mathbb{S}$. We endow $\mathcal{M}_F(\mathbb{S})$ with the weakest topology such that for every $f \in C_c(\mathbb{S})$ the function $\nu \mapsto \langle f, \nu \rangle = \int_{\mathbb{S}} f(u)\nu(du)$, $\nu \in \mathcal{M}_F(\mathbb{S})$ is a continuous function. This topology can be metrized such that $\mathcal{M}_F(\mathbb{S})$ is a Polish space. One metric that is convenient for this purpose is the following. Consider a sequence of open sets $\{O_j, j \in \mathbb{N}\}$ such that $\bar{O}_j \subset O_{j+1}$, each $\bar{O}_j$ is compact, and $\bigcup_{j=1}^{\infty} O_j = \mathbb{S}$ (cf. [**18**, Theorem 9.5.21]). Let $\phi_j(x) = [1 - d(x, O_j)] \vee 0$, where d denotes the metric on $\mathbb{S}$. Given any $\mu \in \mathcal{M}_F(\mathbb{S})$, let $\mu^j \in \mathcal{M}_F(\mathbb{S})$ be defined by $[d\mu^j/d\mu](x) = \phi_j(x)$. Given $\mu, \nu \in \mathcal{M}_F(\mathbb{S})$, let

$$\bar{d}(\mu, \nu) = \sum_{j=1}^{\infty} 2^{-j} \|\mu^j - \nu^j\|_{BL},$$

where $\|\cdot\|_{BL}$ denotes the bounded, Lipschitz norm:

$$\|\mu^j - \nu^j\|_{BL} = \sup\Bigg\{\int_{\mathbb{S}} f\, d\mu^j - \int_{\mathbb{S}} f\, d\nu^j : |f|_\infty \le 1, |f(x) - f(y)| \le d(x, y) \text{ for all } x, y \in \mathbb{S}\Bigg\}.$$

It is straightforward to check that $\bar{d}(\mu, \nu)$ defines a metric under which $\mathcal{M}_F(\mathbb{S})$ is a Polish space, and that convergence in this metric is essentially equivalent to weak convergence on each compact subset of $\mathbb{X}$. Specifically, $\bar{d}(\mu_n, \mu) \to 0$ if and only if for each $j \in \mathbb{N}$, $\mu_n^j \to \mu^j$ in the weak topology as finite non-negative measures; i.e., for all $f \in C_b(\mathbb{X})$,

$$\int_{\mathbb{S}} f\, d\mu_n^j \to \int_{\mathbb{S}} f\, d\mu^j.$$

Throughout, $\mathcal{B}(\mathcal{M}_F(\mathbb{S}))$ will denote the Borel sigma-field on $\mathcal{M}_F(\mathbb{S})$ under this topology.

§2. *Preliminaries.* In this section we recall some basic definitions and the equivalence between a LDP and the Laplace principle for a family of probability measures on some Polish space. We next recall some commonly used formulations for an infinite-dimensional Brownian motion, such as an infinite sequence of independent and identically distributed standard real Brownian motions, a Hilbert space valued Brownian motion, a cylindrical Brownian motion, and a space–time Brownian sheet. Relationships between these various formulations are noted as well. At the end of the section the definition of a PRM is presented.

2.1. *Large deviation principle and Laplace asymptotics.* Let $\{X^\epsilon, \epsilon > 0\} \equiv \{X^\epsilon\}$ be a family of random variables defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and taking values in a Polish space $\mathcal{E}$. Denote the metric on $\mathcal{E}$ by $d(x, y)$ and expectation with respect to $\mathbb{P}$ by $\mathbb{E}$. The theory of large deviations is concerned

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with events A whose probabilities $\mathbb{P}(X^\epsilon \in A)$ converge to zero exponentially fast as $\epsilon \to 0$. The exponential decay rate of such probabilities is typically expressed in terms of a "rate function" I mapping $\mathcal{E}$ into $[0, \infty]$. If a sequence of random variables satisfies the large deviation principle (LDP) with some rate function, then the rate function is unique. In many problems one is interested in obtaining exponential estimates on functions that are more general than indicator functions of closed or open sets. This leads to the study of the Laplace principle, which is tantamount to the LDP. The reader should refer to [**11**] for all the aforementioned definitions and for equivalence between the LDP and the Laplace principle.

In view of this equivalence, the rest of this work will be concerned with the study of the Laplace principle. In fact we will study a somewhat strengthened notion, namely a *uniform Laplace principle*, as introduced below. The uniformity is critical in certain applications, such as the study of exit time and invariant measure asymptotics for small noise Markov processes [**12**].

Let $\mathcal{E}_0$ and $\mathcal{E}$ be Polish spaces. For each $\epsilon > 0$ and $y \in \mathcal{E}_0$ let $X^{\epsilon,y}$ be $\mathcal{E}$ valued random variables given on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$.

Definition 2.1. A family of rate functions I_y on $\mathcal{E}$, parameterized by $y \in \mathcal{E}_0$, is said to have compact level sets on compacts if for all compact subsets K of $\mathcal{E}_0$ and each $M < \infty$, $\Lambda_{M,K} \doteq \bigcup_{y\in K}\{x \in \mathcal{E} : I_y(x) \le M\}$ is a compact subset of $\mathcal{E}$.

Definition 2.2 (Uniform Laplace principle). Let I_y be a family of rate functions on $\mathcal{E}$ parameterized by y in $\mathcal{E}_0$ and assume that this family has compact level sets on compacts. The family $\{X^{\epsilon,y}\}$ is said to satisfy the Laplace principle on $\mathcal{E}$ with rate function I_y, uniformly on compacts, if for all compact subsets K of $\mathcal{E}_0$ and all bounded continuous functions h mapping $\mathcal{E}$ into $\mathbb{R}$,

$$\lim_{\epsilon\to 0} \sup_{y\in K} \left| \epsilon \log \mathbb{E}_y \left\{ \exp\left[-\frac{1}{\epsilon} h(X^{\epsilon,y}) \right] \right\} + \inf_{x\in\mathcal{E}} \{h(x) + I_y(x)\} \right| = 0.$$

2.2. *Infinite-dimensional Brownian motions and Poisson random measure.* This section revisits basic definitions for infinite-dimensional Brownian motions and a PRM. We first start with a definition of a PRM.

Definition 2.3. Let $(K, \mathcal{K}, \mu)$ be some measure space with σ-finite measure μ. The *Poisson random measure (PRM)* with intensity measure μ is a family of random variables $\{N(A), A \in \mathcal{K}\}$ defined on some probability space $(\Omega, \mathcal{F}, \mathbb{P})$ such that:

(1) for all $\omega \in \Omega$, $N(\cdot, \omega)$ is a measure on $(K, \mathcal{K})$;
(2) for all $A \in \mathcal{K}$, $N(A)$ is a Poisson random variable with rate $\mu(A)$, i.e. $\mathbb{P}(N(A) = n) = (\mathrm{e}^{-\mu(A)}/n!)\mu(A)^n$; and
(3) if $A_1, A_2, \ldots, A_n \in \mathcal{K}$ are disjoint, then $N(A_1), N(A_2), \ldots, N(A_n)$ are mutually independent.

The rest of this section deals with all the equivalent types of an infinite-dimensional nature of the Brownian motion, for example, depending on the application: an infinite sequence of independent and identically distributed standard (one-dimensional) Brownian motions, a Hilbert space valued Brownian

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motion, a cylindrical Brownian motion, and a space–time Brownian sheet. The reader should refer to [3] and references therein for an explanation of the way these infinite-dimensional Brownian motions are related to one another.

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space with an increasing family of right continuous $\mathbb{P}$-complete sigma fields $\{\mathcal{F}_t\}_{t\geq 0}$. We shall refer to $(\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t\})$ as a filtered probability space. Let $\{\beta_i\}_{i=1}^{\infty}$ be an infinite sequence of independent, standard, one-dimensional, $\{\mathcal{F}_t\}$-Brownian motions given on this filtered probability space. We shall frequently consider all of our stochastic processes as being defined on a finite time interval $[0, T]$, where $T \in (0, \infty)$ is a fixed arbitrary terminal time. We denote by $\mathbb{R}^{\infty}$ the product space of countably infinite copies of the real line. Then $\beta = \{\beta_i\}_{i=1}^{\infty}$ is a random variable with values in the Polish space $C([0, T] : \mathbb{R}^{\infty})$ and represents the simplest model for an infinite-dimensional Brownian motion.

Frequently in applications it is convenient to express the Brownian noise, analogous to that in finite-dimensional theory, as a Hilbert space valued stochastic processes. Let $(H, \langle \cdot, \cdot \rangle)$ be a real separable Hilbert space. Let Q be a bounded, strictly positive, trace class operator on H.

Definition 2.4. An H-valued stochastic process $\{W(t), t \geq 0\}$ given on a filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t)\}$ is called a Q-Wiener process with respect to $\{\mathcal{F}_t\}$ if, for every non-zero $h \in H$,

$$\{\langle Qh, h\rangle^{-\frac{1}{2}} \langle W(t), h\rangle, \{\mathcal{F}_t\}\}_{t\geq 0}$$

is a one-dimensional standard Wiener process.

Remark 2.1. Consider the Hilbert space

$$l_2 \doteq \left\{ x \equiv (x_1, x_2, \ldots) : x_i \in \mathbb{R} \text{ and } \sum x_i^2 < \infty \right\}$$

with the inner product $\langle x, y\rangle \doteq \sum x_i y_i$. Let $\{\lambda_i\}_{i=1}^{\infty}$ be a sequence of strictly positive numbers such that $\sum \lambda_i < \infty$. Then the Hilbert space

$$\bar{l}_2 \doteq \left\{ x \equiv (x_1, x_2, \ldots) : x_i \in \mathbb{R} \text{ and } \sum \lambda_i x_i^2 < \infty \right\}$$

with the inner product $\langle x, y\rangle_1 \doteq \sum \lambda_i x_i y_i$ contains l_2 and the embedding map is Hilbert–Schmidt. Furthermore, the infinite sequence of real Brownian motions β takes values in $\bar{l}_2$ almost surely and can be regarded as a $\bar{l}_2$ valued Q-Wiener process with $\langle Qx, y\rangle_1 = \sum_{i=1}^{\infty} \lambda_i^2 x_i y_i$.

The trace class operator Q may be interpreted as injecting a "coloring" to a white noise, namely an independent sequence of standard Brownian motions, in such a manner that the resulting process has better regularity. In some models of interest, such coloring is obtained indirectly in terms of (state-dependent) diffusion coefficients. It is natural in such situations to consider the driving noise as a "cylindrical Brownian motion" rather than a Hilbert space valued Brownian motion. Let $(H, \langle \cdot, \cdot \rangle)$ be a Hilbert space valued Brownian motion and denote the norm on H by $\| \cdot \|$. Fix a filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t\})$.

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Definition 2.5. A family $\{B_t(h) \equiv B(t, h) : t \geq 0, h \in H\}$ of real random variables is said to be an $\{\mathcal{F}_t\}$-cylindrical Brownian motion if:

(1) for every $h \in H$ with $\|h\| = 1$, $\{B(t, h), \mathcal{F}_t\}_{t\geq 0}$ is a standard Wiener process; and
(2) for every $t \geq 0$, $a_1, a_2 \in \mathbb{R}$ and $f_1, f_2 \in H$,

$$B(t, a_1 f_1 + a_2 f_2) = a_1 B(t, f_1) + a_2 B(t, f_2) \text{ almost surely.}$$

In many physical dynamical systems with randomness, the Brownian noise is given as a space–time white noise process, also referred to as a Brownian sheet. Let us fix a bounded open subset $\mathcal{O} \subseteq \mathbb{R}^d$.

Definition 2.6. A Gaussian family of real-valued random variables $\{B(t, x), (t, x) \in \mathbb{R}_+ \times \mathcal{O}\}$ on the above filtered probability space is called a *Brownian sheet* if:

(1) $\mathbb{E}B(t, x) = 0$, for all $(t, x) \in \mathbb{R}_+ \times \mathcal{O}$;
(2) $B(t, x) - B(s, x)$ is independent of $\{\mathcal{F}_s\}$, for all $0 \leq s \leq t$ and $x \in \mathcal{O}$;
(3) $\mathrm{Cov}(B(t, x), B(s, y)) = \lambda(A_{t,x} \cap A_{s,y})$, where λ is the Lebesgue measure on $\mathbb{R}_+ \times \mathcal{O}$ and $A_{t,x} \doteq \{(s, y) \in \mathbb{R}_+ \times \mathcal{O} \mid 0 \leq s \leq t \text{ and } y_j \leq x_j \ j = 1, \ldots, d\}$; and
(4) the map $(t, u) \mapsto B(t, u)$ from $[0, \infty) \times \mathcal{O}$ to $\mathbb{R}$ is continuous almost surely.

§3. *Variational representations for functionals of Poisson random measure and Brownian motions.* In this section we state the representation for functionals of both a PRM and infinite-dimensional Brownian motions.

Fix $T \in (0, \infty)$. Let $\mathbb{X}$ be a locally compact Polish space and $\mathbb{X}_T = [0, T] \times \mathbb{X}$. Fix a measure $\nu \in \mathcal{M}_F(\mathbb{X})$ and let $\nu_T = \lambda_T \otimes \nu$, where λ_T is the Lebesgue measure on $[0, T]$. Let $\mathbb{M} = \mathcal{M}_F(\mathbb{X}_T)$ and denote by $\mathbb{P}$ the unique probability measure on $(\mathbb{M}, \mathcal{B}(\mathbb{M}))$ under which the canonical map, $N : \mathbb{M} \to \mathbb{M}$, $N(m) \doteq m$, is a PRM with intensity measure ν_T. The corresponding expectation operator will be denoted by $\mathbb{E}$.

Furthermore, denote the product space of countable infinite copies of the real line by $\mathbb{R}^\infty$. Endowed with the topology of coordinate-wise convergence, $\mathbb{R}^\infty$ is a Polish space. Also let us write the Polish space $C([0, T] : \mathbb{R}^\infty)$ as $\mathbb{W}$, and consider the product space $\mathbb{V} \doteq \mathbb{W} \times \mathbb{M}$. Abusing the above notation, let $N : \mathbb{V} \to \mathbb{M}$ be defined by $N(w, m) = m$, and for the coordinate maps, $\beta = \{\beta_i\}_{i=1}^\infty$, on $\mathbb{V}$ let $\beta_i(w, m) = w_i$, for any $(w, m) \in \mathbb{V}$. Define

$$\mathcal{G}_t \doteq \sigma\{N((0, s] \times A), \beta_i(s) : 0 \leq s \leq t, A \in \mathcal{B}(\mathbb{X}), i \geq 1\}. \tag{3.1}$$

With applications to large deviations in mind, for $\theta > 0$, denote by $\mathbb{P}_\theta$ the unique probability measure on $(\mathbb{V}, \mathcal{B}(\mathbb{V}))$ such that, under $\mathbb{P}_\theta$:

(1) $\{\beta_i\}_{i=1}^\infty$ is an independent and identically distributed family of standard Brownian motions;
(2) N is a PRM with intensity measure $\theta\nu_T$; and
(3) $\{\beta_i(t), t \in [0, T]\}$, $\{N([0, t] \times A), t \in [0, T]\}$ are $\mathcal{G}_t$-martingales for every $i \geq 1$, $A \in \mathcal{B}(\mathbb{X})$.

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Let $\mathbb{Y} = \mathbb{X} \times [0, \infty)$ and $\mathbb{Y}_T = [0, T] \times \mathbb{Y}$. Let $\bar{\mathbb{M}} = \mathcal{M}_F(\mathbb{Y}_T)$ and let $\bar{\mathbb{P}}$ be the unique probability measure on $(\bar{\mathbb{M}}, \mathcal{B}(\bar{\mathbb{M}}))$ such that the canonical map, $\bar{N} : \bar{\mathbb{M}} \to \bar{\mathbb{M}}$, $\bar{N}(m) \doteq m$, is a PRM with intensity measure $\bar{\nu}_T = \lambda_T \otimes \nu \otimes \lambda_\infty$, where λ_∞ is Lebesgue measure on $[0, \infty)$. The corresponding expectation operator will be denoted by $\bar{\mathbb{E}}$.

Analogously, let us define $\bar{\mathbb{V}} \doteq \mathbb{W} \times \bar{\mathbb{M}}$. Furthermore, abusing notation, let $\bar{N} : \bar{\mathbb{V}} \to \bar{\mathbb{M}}$ be $N(w, \bar{m}) = \bar{m}$ and let the coordinate maps on $\bar{\mathbb{V}}$ be denoted again as $\beta = \{\beta_i\}_{i=1}^\infty$. The control will act through this additional component of the underlying point space. Let

$$\mathcal{G}_t \doteq \sigma\{\bar{N}((0, s] \times A), \beta_i(s) : 0 \le s \le t, A \in \mathcal{B}(\mathbb{Y}), i \ge 1\},$$

and to facilitate the use of a martingale representation theorem let $\bar{\mathcal{F}}_t$ denote the completion under $\bar{\mathbb{P}}$. We denote by $\bar{\mathcal{P}}$ the predictable σ-field on $[0, T] \times \bar{\mathbb{V}}$ with the filtration $\{\bar{\mathcal{F}}_t : 0 \le t \le T\}$ on $(\bar{\mathbb{V}}, \mathcal{B}(\bar{\mathbb{V}}))$. Let $\bar{\mathcal{A}}$ be the class of all $(\bar{\mathcal{P}} \otimes \mathcal{B}(\mathbb{X}))\backslash\mathcal{B}[0, \infty)$ measurable maps $\varphi : \mathbb{X}_T \times \bar{\mathbb{V}} \to [0, \infty)$. For $\varphi \in \bar{\mathcal{A}}$, define a counting process N^φ on $\mathbb{X}_T$ by

$$N^\varphi((0, t] \times U) = \int_{(0,t]\times U} \int_{(0,\infty)} 1_{[0,\varphi(s,x)]}(r) \bar{N}(ds\, dx\, dr), \tag{3.2}$$

where $t \in [0, T]$, $U \in \mathcal{B}(\mathbb{X})$. N^φ is to be thought of as a controlled random measure, with φ selecting the intensity for the points at location x and time s, in a possibly random but non-anticipating way. Obviously N^θ has the same distribution on $\bar{\mathbb{V}}$ with respect to $\bar{\mathbb{P}}$ as N has on $\mathbb{V}$ with respect to $\mathbb{P}_\theta$. N^θ therefore plays the role of N on $\bar{\mathbb{V}}$. Define $\ell : [0, \infty) \to [0, \infty)$ by

$$\ell(r) = r \log r - r + 1, \quad r \in [0, \infty).$$

For any $\varphi \in \bar{\mathcal{A}}$ the quantity

$$L_T(\varphi) = \int_{\mathbb{X}_T} \ell(\varphi(t, x, \omega)) \nu_T(dt\, dx) \tag{3.3}$$

is well-defined as a $[0, \infty]$-valued random variable.

Consider the ℓ_2 Hilbert space as defined in Remark 2.1, denote

$$\mathcal{P}_2 = \Big\{ \psi = \{\psi_i\}_{i=1}^\infty : \psi_i \text{ is } \bar{\mathcal{P}}\backslash\mathcal{B}(\mathbb{R}) \text{ measurable and } \int_0^T \|\psi(s)\|^2\, ds < \infty, \text{ almost surely } \bar{\mathbb{P}} \Big\}, \tag{3.4}$$

and set $\mathcal{U} = \mathcal{P}_2 \times \bar{\mathcal{A}}$. For $\psi \in \mathcal{P}_2$ define $\tilde{L}_T(\psi) = \frac{1}{2} \int_0^T \|\psi(s)\|^2\, ds$ and, for $u = (\psi, \varphi) \in \mathcal{U}$, set $\bar{L}_T(u) = L_T(\varphi) + \tilde{L}_T(\psi)$. For $\psi \in \mathcal{P}_2$, let $\beta^\psi = (\beta_i^\psi)$ be defined as $\beta_i^\psi(t) = \beta_i(t) + \int_0^t \psi_i(s)\, ds$, $t \in [0, T]$, $i \in \mathbb{N}$. The following variational representation theorem was established in [5].

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THEOREM 3.1. *Let $F \in M_b(\mathbb{V})$. Then, for $\theta > 0$,*

$$-\log \mathbb{E}_\theta(\mathrm{e}^{-F(\beta,N)}) = -\log \bar{\mathbb{E}}(\mathrm{e}^{-F(\beta,N^\theta)}) = \inf_{u=(\psi,\varphi)\in\mathcal{U}} \bar{\mathbb{E}}[\theta \bar{L}_T(u) + F(\beta^{\sqrt{\theta}\psi}, N^{\theta\varphi})].$$

As mentioned in the introduction, depending on the application the infinite nature of the Brownian noise may be written in several other equivalent forms. First, let us establish an analogous variational representation for a functional of Hilbert space valued Brownian motion and a PRM. Let $(H, \langle\cdot,\cdot\rangle)$ be a Hilbert space and let W be an H valued Q-Wiener process, where Q is a bounded, strictly positive, trace class operator on the Hilbert space H. Let $H_0 = Q^{1/2}H$; then H_0 is a Hilbert space with the inner product $\langle h, k\rangle_0 \doteq \langle Q^{-1/2}h, Q^{-1/2}h\rangle$, $h, k \in H_0$. Also, the embedding map $i : H_0 \mapsto H$ is a Hilbert–Schmidt operator and $ii^* = Q$. Let $\|\cdot\|_0$ denote the norm in the Hilbert space H_0.

Furthermore, denote the Polish space $C([0, T] : H)$ by $\mathbb{W}(H)$ and denote by $\mathbb{V}(H)$ the product space $\mathbb{W}(H) \times \mathbb{M}$, where $\mathbb{M}$ is as defined at the beginning of the current section. Let $\bar{\mathbb{V}}(H) = \mathbb{W}(H) \times \bar{\mathbb{M}}$. Abusing notation, let $N : \mathbb{V}(H) \to \mathbb{M}$ be defined by $N(w, m) = m$, for $(w, m) \in \mathbb{V}(H)$. The map $\bar{N} : \bar{\mathbb{V}}(H) \to \bar{\mathbb{M}}$ is defined analogously. Let W be defined on $\mathbb{V}(H)$ as $W(w, m) = w(t)$. Analogous maps on $\bar{\mathbb{V}}(H)$ are denoted again as W. Define $\mathcal{G}_t \doteq \sigma\{N((0, s] \times A), W(s) : 0 \le s \le t, A \in \mathcal{B}(\mathbb{X})\}$. For $\theta > 0$, denote by $\mathbb{P}_\theta$ the unique probability measure on $(\mathbb{V}(H), \mathcal{B}(\mathbb{V}(H)))$ such that, under $\mathbb{P}_\theta$:

(1) $W(t)$ is an H-valued Q-Wiener process;
(2) N is a PRM with intensity measure $\theta\nu_T$; and
(3) $\{W(t), t \in [0, T]\}$, $\{N([0, t] \times A), t \in [0, T]\}$ are $\mathcal{G}_t$-martingales for every $A \in \mathcal{B}(\mathbb{X})$.

Define $(\bar{\mathbb{P}}, \{\bar{\mathcal{G}}_t\})$ on $(\bar{\mathbb{V}}(H), \mathcal{B}(\bar{\mathbb{V}}(H)))$ analogous to $(\mathbb{P}_\theta, \{\mathcal{G}_t\})$ by replacing $(N, \theta\nu_T)$ with $(\bar{N}, \bar{\nu}_T)$. Now, let us consider the $\bar{\mathbb{P}}$-completion of the filtration $\{\bar{\mathcal{G}}_t\}$ and denote it by $\{\bar{\mathcal{F}}_t\}$. We denote by $\bar{\mathcal{P}}$ the predictable σ-field on $[0, T] \times \bar{\mathbb{V}}(H)$ with the filtration $\{\bar{\mathcal{F}}_t : 0 \le t \le T\}$ on $(\bar{\mathbb{V}}(H), \mathcal{B}(\bar{\mathbb{V}}(H)))$. Let $\bar{\mathcal{A}}$ be the class of all $(\bar{\mathcal{P}} \otimes \mathcal{B}(\mathbb{X}))\backslash\mathcal{B}[0, \infty)$ measurable maps $\varphi : \mathbb{X}_T \times \bar{\mathbb{V}} \to [0, \infty)$. For $\varphi \in \bar{\mathcal{A}}$, define $L_T(\varphi)$ and the counting process N^φ on $\mathbb{X}_T$ as in (3.3) and (3.2) respectively.

Define

$$\mathcal{P}_2 \equiv \mathcal{P}_2(H) = \Big\{\psi : \psi \text{ is } \bar{\mathcal{P}}\backslash\mathcal{B}(\mathbb{R}) \text{ measurable and } \int_0^T \|\psi(s)\|_0^2\, ds < \infty, \text{ almost surely } \bar{\mathbb{P}}\Big\} \tag{3.5}$$

and set $\mathcal{U}(H) = \mathcal{P}_2(H) \times \bar{\mathcal{A}}$. For $\psi \in \mathcal{P}_2$ define

$$\tilde{L}_T \equiv \tilde{L}_T^H(\psi) = \frac{1}{2}\int_0^T \|\psi(s)\|_0^2\, ds$$

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and, for $u = (\psi, \varphi) \in \mathcal{U}$, set $\bar{L}_T(u) = L_T(\varphi) + \tilde{L}_T(\psi)$. For $\psi \in \mathcal{P}_2$, let W^ψ be defined as $W^\psi(t) = W(t) + \int_0^t \psi(s)\, ds$, $t \in [0, T]$. The following representation follows from Theorem 3.1 and [**3**, Proposition 1].

THEOREM 3.2. *Let $F \in M_b(\mathbb{V}(H))$. Then for $\theta > 0$,*

$$-\log \mathbb{E}_\theta(e^{-F(W,N)}) = -\log \bar{\mathbb{E}}(e^{-F(W,N^\theta)}) = \inf_{u=(\psi,\varphi)\in\mathcal{U}} \bar{\mathbb{E}}[\theta \bar{L}_T(u) + F(W^{\sqrt{\theta}\psi}, N^{\theta\varphi})].$$

Finally, we provide the representation theorem for a Brownian sheet acting together with a PRM. Let us denote the Polish space $C([0, T] \times \mathcal{O} : \mathbb{R})$ by $\mathbb{W}_{BS}$ and denote by $\mathbb{V}_{BS}$ the product space $\mathbb{W}_{BS} \times \mathbb{M}$. Let $\bar{\mathbb{V}}_{BS} = \mathbb{W}_{BS} \times \bar{\mathbb{M}}$. Abusing notation, let $N : \mathbb{V}_{BS} \to \mathbb{M}$ be defined by $N(w, m) = m$, for $(w, m) \in \mathbb{V}_{BS}$. The map $\bar{N} : \bar{\mathbb{V}}_{\mathbb{BS}} \to \bar{\mathbb{M}}$ is defined analogously. Let $B(w, m) = w(t, x)$ on $\mathbb{V}_{BS}$, and analogously on $\bar{\mathbb{V}}_{BS}$ this is denoted again as $B(t, x)$. Define $\mathcal{G}_t \doteq \sigma\{N((0, s] \times A),\ B(s, x) : 0 \le s \le t,\ x \in \mathcal{O},\ A \in \mathcal{B}(\mathbb{X}),\ i \ge 1\}$. For $\theta > 0$, denote by $\mathbb{P}_\theta$ the unique probability measure on $(\mathbb{V}_{BS}, \mathcal{B}(\mathbb{V}_{BS}))$ such that, under $\mathbb{P}_\theta$:

(1) B is a Brownian sheet;
(2) N is a PRM with intensity measure $\theta \nu_T$; and
(3) $\{B(t, x), t \in [0, T]\}$, $\{N([0, t] \times A), t \in [0, T]\}$ are $\mathcal{G}_t$-martingales for every $i \ge 1$, $A \in \mathcal{B}(\mathbb{X})$.

Define $(\bar{\mathbb{P}}, \{\bar{\mathcal{G}}_t\})$ on $(\bar{\mathbb{V}}_{BS}, \mathcal{B}(\bar{\mathbb{V}}_{BS}))$ analogously to $(\mathbb{P}_\theta, \{\mathcal{G}_t\})$ by replacing $(N, \theta\nu_T)$ with $(\bar{N}, \bar{\nu}_T)$. Define the $\bar{\mathbb{P}}$-completion of the filtration $\{\bar{\mathcal{G}}_t\}$ and denote it by $\{\bar{\mathcal{F}}_t\}$. We denote by $\bar{\mathcal{P}}$ the predictable σ-field on $[0, T] \times \mathcal{O} \times \bar{\mathbb{V}}_{BS}$ with the filtration $\{\bar{\mathcal{F}}_t : 0 \le t \le T\}$ on $(\bar{\mathbb{V}}_{BS}, \mathcal{B}(\bar{\mathbb{V}}_{BS}))$. Let $\bar{\mathcal{A}}$ be the class of all $(\bar{\mathcal{P}} \otimes \mathcal{B}(\mathbb{X}))\backslash\mathcal{B}[0, \infty)$ measurable maps $\varphi : \mathbb{X}_T \times \bar{\mathbb{V}}_{\mathbb{BS}} \to [0, \infty)$. For $\varphi \in \bar{\mathcal{A}}$, define $L_T(\varphi)$ and the counting process N^φ on $\mathbb{X}_T$ as in (3.3) and (3.2) respectively.

Define

$$\mathcal{P}_2 \equiv \mathcal{P}_2^{BS} = \left\{ \psi : \psi \text{ is } \bar{\mathcal{P}}\backslash\mathcal{B}(\mathbb{R}) \text{ measurable and } \int_0^T \int_{\mathcal{O}} \psi^2(s, x)\, ds\, dx < \infty, \text{ almost surely } \bar{\mathbb{P}} \right\} \tag{3.6}$$

and set $\mathcal{U}^{BS} = \mathcal{P}_2^{BS} \times \bar{\mathcal{A}}$. For $\psi \in \mathcal{P}_2$ define

$$\tilde{L}_T \equiv \tilde{L}_T^{BS}(\psi) = \frac{1}{2} \int_0^T \int_{\mathcal{O}} \psi(s, r)^2\, dr\, ds$$

and, for $u = (\psi, \varphi) \in \mathcal{U}$, set $\bar{L}_T(u) \equiv \bar{L}_T^{BS}(u) = L_T(\varphi) + \tilde{L}_T(\psi)$. For $\psi \in \mathcal{P}_2$, let B^ψ be defined as $B^\psi(t, x) = B(t, x) + \int_0^t \int_{\mathcal{O}\cap(-\infty,x]} \psi^2(s, y)\, dy\, ds$, $t \in [0, T]$, $i \in \mathbb{N}$. We finally remark that the following representation for a Brownian sheet and a PRM follows from Theorem 3.1, [**3**, Proposition 3] and an application of Girsanov's theorem.

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THEOREM 3.3. *Let $F \in M_b(\mathbb{V}_{BS})$. Then, for $\theta > 0$,*

$$-\log \mathbb{E}_\theta(\mathrm{e}^{-F(B,N)}) = -\log \bar{\mathbb{E}}(\mathrm{e}^{-F(B,N^\theta)}) = \inf_{u=(\psi,\varphi)\in\mathcal{U}} \bar{\mathbb{E}}[\theta \bar{L}_T(u) + F(B^{\sqrt{\theta}\psi}, N^{\theta\varphi})].$$

§4. *Uniform large deviations estimates.* This is the central section of this paper where the uniform Laplace principle for functionals of a PRM and an infinite-dimensional Brownian motion of any type are verified. The uniformity is with respect to a parameter ζ (typically an initial condition), which takes values in some compact subset of a Polish space $\mathcal{E}_0$.

Let first consider the case of a Hilbert space valued Wiener process and then use this case to deduce analogous Laplace principle results for functionals of a cylindrical Brownian motion and a Brownian sheet acting independently together with a PRM. Let $(\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t\})$, $(H, \langle\cdot,\cdot\rangle)$, Q be as in §2 and let W be an H-valued Wiener process with trace class covariance Q given on this filtered probability space. Let $\mathcal{E}$ be a Polish space, and for each $\epsilon > 0$, let $\mathcal{G}^\epsilon : \mathcal{E}_0 \times \mathbb{V}(H) \to \mathcal{E}$ be a measurable map. We next discuss a set of sufficient conditions for a uniform LDP to hold for the family

$$\{Z^{\epsilon,\zeta} \doteq \mathcal{G}^\epsilon(\zeta, \sqrt{\epsilon}W, \epsilon N^{\epsilon^{-1}})\} \quad \text{as } \epsilon \to 0. \tag{4.7}$$

Let H_0 be as introduced before, and define for $N \in \mathbb{N}$

$$\tilde{S}^N(H_0) \doteq \{u \in L^2([0,T] : H_0) : \tilde{L}_T(u) \le N\}. \tag{4.8}$$

We will endow $\tilde{S}^N(H_0)$ with the topology obtained from the metric

$$d_1(x, y) = \sum_{i=1}^{\infty} \frac{1}{2^i} \left| \int_0^T \langle x(s) - y(s), e_i(s)\rangle_0 \, ds \right|$$

and refer to this as the weak topology on $S^N(H_0)$.

Also, let

$$S^N = \{g : X_T \to [0,\infty) : L_T(g) \le N\}. \tag{4.9}$$

A function $g \in S^N$ can be identified with a measure $\nu_T^g \in \mathbb{M}$, defined by $\nu_T^g(A) = \int_A g(s,x)\, \nu_T(ds\, dx)$, $A \in \mathcal{B}(\mathbb{X}_T)$. Recalling from the introduction that convergence in $\mathbb{M}$ is essentially equivalent to weak convergence on compact subsets, the superlinear growth of ℓ implies that $\{\nu_T^g : g \in S^N\}$ is a compact subset of $\mathbb{M}$. Throughout we consider the topology on S^N obtained through this identification, which makes S^N a compact space. We let $\bar{S}^N = \tilde{S}^N(H_0) \times S^N$ with the usual product topology. Recall the product space $\mathcal{U} = \mathcal{P}_2(H) \times \bar{\mathcal{A}}$; let $\mathbb{S} = \bigcup_{N\ge1} \bar{S}^N$ and let $\mathcal{U}^N$ be the space of $\bar{S}^N$-valued controls:

$$\mathcal{U}^N = \{u = (\psi, \varphi) \in \mathcal{U} : u(\omega) \in \bar{S}^N, \bar{\mathbb{P}} \text{ a.e. } \omega\}. \tag{4.10}$$

Condition 4.1. There exists a measurable map $\mathcal{G}^0 : \mathcal{E}_0 \times \mathbb{V}(H) \to \mathcal{E}$ such that the following hold.

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(1) For $N \in \mathbb{N}$ let $(f_n, g_n), (f, g) \in \bar{S}^N$ be such that $(\zeta_n, f_n, g_n) \to (\zeta, f, g)$. Then

$$\mathcal{G}^0\left(\zeta_n, \int_0^\cdot f_n(s)\,ds, \nu_T^{g_n}\right) \to \mathcal{G}^0\left(\zeta, \int_0^\cdot f(s)\,ds, \nu_T^{g}\right).$$

(2) For $N \in \mathbb{N}$ let $u_\epsilon = (\psi_\epsilon, \varphi_\epsilon)$, $u = (\psi, \varphi) \in \mathcal{U}^N$ be such that, as $\epsilon \to 0$, u_ϵ converges in distribution to u and $\{\zeta^\epsilon\} \subset \mathcal{E}_0$, $\zeta^\epsilon \to \zeta$, as $\epsilon \to 0$. Then

$$\mathcal{G}^\epsilon\left(\zeta^\epsilon, \sqrt{\epsilon}W(\cdot) + \int_0^\cdot \psi_\epsilon(s)\,ds, \epsilon N^{\epsilon^{-1}\varphi_\epsilon}\right) \Rightarrow \mathcal{G}^0\left(\zeta, \int_0^\cdot \psi(s)\,ds, \nu_T^{\varphi}\right).$$

For $\phi \in \mathcal{E}$, define

$$\mathbb{S}_\phi = \left\{(f, g) \in \mathbb{S} : \phi = \mathcal{G}^0\left(\zeta, \int_0^\cdot f(s)\,ds, \nu_T^g\right)\right\}.$$

Let $I_\zeta : \mathcal{E} \to [0, \infty]$ be defined by

$$I_\zeta(\phi) = \inf_{q=(f,g)\in\mathbb{S}_\phi} \{\bar{L}_T(q)\}. \tag{4.11}$$

THEOREM 4.4. *Let $Z^{\epsilon,\zeta}$ be defined as in (4.7) and suppose that Condition 4.1 holds. Suppose that for all $f \in \mathcal{E}$, $\zeta \mapsto I_\zeta(f)$ is a lower semicontinuous (l.s.c.) map from $\mathcal{E}_0$ to $[0, \infty]$. Then, for all $\zeta \in \mathcal{E}_0$, $f \mapsto I_\zeta(f)$ is a rate function on $\mathcal{E}$ and the family $\{I_\zeta(\cdot), \zeta \in \mathcal{E}_0\}$ of rate functions has compact level sets on compacts. Furthermore, the family $\{Z^{\epsilon,\zeta}\}$ satisfies the Laplace principle on $\mathcal{E}$, with rate function I_ζ, uniformly on compact subsets of $\mathcal{E}_0$.*

Proof. In order to show that I_ζ is a rate function and that it has compact level sets on compacts, it is enough to demonstrate that, for all compact subsets K of $\mathcal{E}_0$ and each $M < \infty$,

$$\Lambda_{M,K} \doteq \bigcup_{\zeta\in K} \{\phi \in \mathcal{E} : I_\zeta(\phi) \le M\}$$

is a compact subset of $\mathcal{E}$. To establish this we will show that $\Lambda_{M,K} = \bigcap_{n\ge1} \Gamma_{M+1/n,K}$ is compact, where

$$\Gamma_{M,K} = \left\{\mathcal{G}^0\left(\zeta, \int_0^\cdot f(s)\,ds, \nu_t^g\right) : x \in \mathcal{E}_0, (f, g) \in \bar{S}^M\right\}.$$

There exists $\zeta \in K$ such that $I_\zeta(f) \le M$. We can find, for each $n \ge 1$, $(f_n, g_n) \in \mathbb{S}$ such that, for $\phi \in \Lambda_{M,K}$, $\phi = \mathcal{G}^0(\zeta, \int_0^\cdot f(s)\,ds, \nu_t^g)$ and $\tilde{L}_T(f_n) \le M + 1/n$ and $L_T(g_n) \le M + 1/n$. In particular, $(f_n, g_n) \in \bar{S}^{M+1/n}$, and thus $\phi \in \bar{S}^{M+1/n}$. Since $n \ge 1$ is arbitrary, we have $\Lambda_{M,K} \subseteq \bigcap_{n\ge1} \Gamma_{M+1/n,K}$. Conversely, suppose $\phi \in \bar{S}^{M+1/n}$, such that $\phi = \mathcal{G}^0(\zeta_n, \int_0^\cdot f_n(s)\,ds, \nu_t^g)$. In particular, we have $I_{\zeta_n}(\phi) \le M + 1/n$. The map $\zeta \mapsto I_\zeta(\phi)$ is lower semicontinuous and K is compact, and thus sending $n \to \infty$ for some $\zeta \in K$, $I_\zeta(\phi) \le M$. Thus $\phi \in \Lambda_{M,K}$, and, in turn, $\bigcap_{n\ge1} \Gamma_{M+1/n,K} \subseteq \Lambda_{M,K}$ follows. This proves the first part. For the second part of the theorem consider $\zeta \in \mathcal{E}_0$

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and let $\{\zeta^\epsilon, \epsilon > 0\} \subseteq \mathcal{E}_0$ such that $\zeta^\epsilon \to \zeta$, as $\epsilon \to 0$. Fix a bounded and continuous $F : \mathcal{E} \to \mathbb{R}$. It suffices to show the Laplace Principle's upper and lower bounds, [**11**, §1.2], in terms of I_ζ for the family $Z^{\epsilon,\zeta^\epsilon}$. For notational convenience we will write $\tilde{S}^N(H_0)$, defined in (4.8), as $\tilde{S}^N$, and the reader should recall $\mathcal{P}_2$, S^N, $\mathcal{U}^N$ as in equations (3.5), (4.9) and (4.10) respectively.

Lower bound. From Theorem 3.2, we have

$$-\epsilon \log \bar{\mathbb{E}}\left(\exp\left(-\frac{1}{\epsilon}F(Z^{\epsilon,\zeta^\epsilon})\right)\right)$$
$$= \inf_{u=(\psi,\phi)\in\mathcal{U}} \mathbb{E}\left[\bar{L}_T(u) + F \circ \mathcal{G}^\epsilon\left(\zeta^\epsilon, \sqrt{\epsilon}W + \int_0^\cdot \psi(s)\,ds, \epsilon N^{\epsilon^{-1}\phi}\right)\right], \quad (4.12)$$

since $Z^{\epsilon,\zeta^\epsilon} = \mathcal{G}^\epsilon(\zeta^\epsilon, \sqrt{\epsilon}W, \epsilon N^{\epsilon^{-1}})$ and $N^{\epsilon^{-1}}$ is a PRM with intensity $\epsilon^{-1}\nu_T$. Fix $\delta \in (0, 1)$. Then for every $\epsilon > 0$ there exist $u_\epsilon = (\psi_\epsilon, \varphi_\epsilon) \in \mathcal{U}$ such that the right-hand side of equation (4.12) is bounded below by

$$\bar{\mathbb{E}}\left[\bar{L}_T(u_\epsilon) + F \circ \mathcal{G}^\epsilon\left(\zeta^\epsilon, \sqrt{\epsilon}W + \int_0^\cdot \psi_\epsilon(s)\,ds, \epsilon N^{\epsilon^{-1}\varphi_\epsilon}\right)\right] - \delta. \quad (4.13)$$

Clearly $\bar{\mathbb{E}}(\bar{L}_T(u_\epsilon)) \le 2\|F\|_\infty + 1$. For $t \in [0, T]$, let

$$L_t(u_\epsilon) = \int_{[0,t]} \left(\|\psi_\epsilon(s)\|^2 + \int_{\mathbb{X}} \ell(\varphi_\epsilon(s, x))\,\nu(dx)\right) ds$$

and define the following sequence of stopping times:

$$\tau_M^\epsilon = \inf\{t \in [0, T] : \bar{L}_t(u_\epsilon) \ge M\} \wedge T.$$

Now for the pair of processes $u_{\epsilon,M} = (\psi_{\epsilon,M}, \varphi_{\epsilon,M}) \in \mathcal{U}^M$, where

$$\varphi_{\epsilon,M}(t, x) = 1 + [\varphi_\epsilon(t, x) - 1]1_{[0,\tau_M^\epsilon]}(t),$$
$$\psi_{\epsilon,M}(t) = \psi_\epsilon(t)1_{[0,\tau_M^\epsilon]}(t), \quad t \in [0, T], x \in \mathbb{X},$$

note that

$$\bar{\mathbb{P}}(u_\epsilon \ne u_{\epsilon,M}) \le \bar{\mathbb{P}}(\bar{L}_T(u_\epsilon) \ge M) \le \frac{2\|F\|_\infty + 1}{M}.$$

Choose M large enough so that the right-hand side above is bounded by $\delta/(2\|F\|_\infty)$. Thus (4.13) is bounded below by

$$\bar{\mathbb{E}}\left[\bar{L}_T(u_{\epsilon,M}) + F \circ \mathcal{G}^\epsilon\left(\zeta^\epsilon, \sqrt{\epsilon}W + \int_0^\cdot \psi_{\epsilon,M}(s)\,ds, \epsilon N^{\epsilon^{-1}\varphi_{\epsilon,M}}\right)\right] - 2\delta.$$

Note that $\{u_{\epsilon,M}\}_{\epsilon>0}$ is a family of $\bar{S}^M$-valued random variables. Recalling that $\bar{S}^M$ is compact, choose a weakly convergent subsequence and denote by $u = (\psi, \varphi)$ the weak limit point. From part (2) of Condition 4.1 we have that along this subsequence $\mathcal{G}^\epsilon(\zeta^\epsilon, \sqrt{\epsilon}W + \int_0^\cdot \psi_{\epsilon,M}(s)\,ds, \epsilon N^{\epsilon^{-1}\varphi_{\epsilon,M}})$ converges weakly

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to $\mathcal{G}^0(\zeta, \int_0^\cdot \psi(s)\,ds, \nu_T^\varphi)$. Thus, using Fatou's lemma and lower semicontinuity properties of the relative entropy function,

$$\begin{aligned}
&\liminf_{\epsilon\to 0} -\epsilon \log \bar{\mathbb{E}}\left[\exp -\frac{1}{\epsilon}F(Z^{\epsilon,\zeta^\epsilon})\right] \\
&\quad \geq \liminf_{\epsilon\to 0} \bar{\mathbb{E}}\left[\bar{L}_T(u_\epsilon) + F\circ\mathcal{G}^\epsilon\left(\zeta^\epsilon, \sqrt{\epsilon}W \right.\right. \\
&\qquad\qquad \left.\left. + \int_0^\cdot \psi_{\epsilon,M}(s)\,ds, \epsilon N^{\epsilon^{-1}}\varphi_{\epsilon,M}\right)\right] - 2\delta \\
&\quad \geq \bar{\mathbb{E}}\left[\bar{L}_T(u) + F\circ\mathcal{G}^0\left(\zeta, \int_0^\cdot \psi(s)\,ds, \nu_T^\varphi\right)\right] - 2\delta \\
&\quad \geq \inf_{\phi\in\mathbb{U}} \inf_{q\in\mathbb{S}_\phi} (\bar{L}_T(q) + F(\phi)) - 2\delta \\
&\quad = \inf_{\phi\in\mathbb{U}} (I_\zeta(\phi) + F(\phi)) - 2\delta.
\end{aligned}$$

Since $\delta \in (0, 1)$ is arbitrary, this completes the proof of the lower bound.

Upper bound. We need to establish that

$$-\epsilon \log \bar{\mathbb{E}}\left(\exp\left(-\frac{1}{\epsilon}F(Z^{\epsilon,\zeta^\epsilon})\right)\right) \leq \inf_{\phi\in\mathbb{U}} (I_\zeta(\phi) + F(\phi)).$$

Let $\delta \in (0, 1)$ be arbitrary and $\phi_0 \in \mathbb{U}$ such that

$$I_\zeta(\phi_0) + F(\phi_0) \leq \inf_{\phi\in\mathbb{U}} (I_\zeta(\phi) + F(\phi)) + \delta.$$

Choose $q = (f, g) \in \mathbb{S}_{\phi_0}$ such that $\bar{L}_T(q) \leq I_\zeta(\phi_0) + \delta$ and

$$\phi_0 = \mathcal{G}^0\left(\zeta, \int_0^\cdot f(s)\,ds, \nu_T^g\right).$$

But according to equation (4.12) we have that

$$\begin{aligned}
&\limsup_{\epsilon\to 0} -\epsilon \log \bar{\mathbb{E}}\left[\exp\left(-\frac{1}{\epsilon}F(Z^{\epsilon,\zeta^\epsilon})\right)\right] \\
&\quad \leq \bar{L}_T(q) + \limsup_{\epsilon\to 0} \bar{\mathbb{E}}\left[F\circ\mathcal{G}^\epsilon\left(\zeta^\epsilon, \sqrt{\epsilon}W + \int_0^\cdot f(s)\,ds, \epsilon N^{\epsilon^{-1}}g\right)\right] \\
&\quad \leq I_\zeta(\phi_0) + \delta + F\circ\mathcal{G}^0\left(\zeta, \int_0^\cdot f(s)\,ds, \nu_T^g\right) \\
&\quad = I_\zeta(\phi_0) + F(\phi_0) + \delta \\
&\quad \leq \inf_{\phi\in\mathbb{U}} (I_\zeta(\phi) + F(\phi)) + 2\delta,
\end{aligned}$$

and since $\delta \in (0, 1)$ is arbitrary the proof of the theorem is complete. □

Next let $\beta \equiv \{\beta_i\}$ be a sequence of independent standard real Brownian motions on $(\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t\})$. Recall that β is a $(C([0, T] : \mathbb{R}^\infty), \mathcal{B}(C([0, T] : \mathbb{R}^\infty))) \equiv (S, \mathcal{S})$ valued random variable. For each $\varepsilon > 0$ let $\mathcal{G}^\varepsilon : \mathcal{E}_0 \times S \to \mathbb{V}$

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be a measurable map and define

$$Z^{\varepsilon,\zeta} \doteq \mathcal{G}^{\varepsilon}(\zeta, \sqrt{\epsilon}\beta, \epsilon N^{\epsilon^{-1}}). \tag{4.14}$$

We now consider the Laplace principle for the family $\{Z^{\varepsilon,\zeta}\}$, as in (4.14), and introduce the analog of Condition 4.1 for this setting. Define $\tilde{S}^N(l_2)$ as in (4.8), with H_0 there replaced by the Hilbert space l_2. Consider $\bar{S}^N \equiv \bar{S}^N(l_2) = \tilde{S}^N(l_2) \times S^N$ with the usual product topology. Let $\mathbb{S} = \bigcup_{N\geq 1} \bar{S}^N$, and let $\mathcal{U}^N$ be as defined in equation (4.10).

Condition 4.2. There exists a measurable map $\mathcal{G}^0 : \mathcal{E}_0 \times \mathbb{V} \to \mathcal{E}$ such that the following hold.

(1) For $N \in \mathbb{N}$ let $(f_n, g_n), (f, g) \in \bar{S}^N$ be such that $(\zeta_n, f_n, g_n) \to (\zeta, f, g)$. Then

$$\mathcal{G}^0\left(\zeta_n, \int_0^{\cdot} f_n(s)\, ds, \nu_T^{g_n}\right) \to \mathcal{G}^0\left(\zeta, \int_0^{\cdot} f(s)\, ds, \nu_T^{g}\right).$$

(2) For $N \in \mathbb{N}$ let $u_\epsilon = (\psi_\epsilon, \varphi_\epsilon)$, $u = (\psi, \varphi) \in \mathcal{U}^N$ be such that, as $\epsilon \to 0$, u_ϵ converges in distribution to u and $\{\zeta^\epsilon\} \subset \mathcal{E}_0$, $\zeta^\epsilon \to \zeta$, as $\epsilon \to 0$. Then

$$\mathcal{G}^{\epsilon}\left(\zeta^{\epsilon}, \sqrt{\epsilon}\beta + \int_0^{\cdot} \psi_\epsilon(s)\, ds, \epsilon N^{\epsilon^{-1}\varphi_\epsilon}\right) \Rightarrow \mathcal{G}^0\left(\zeta, \int_0^{\cdot} \psi(s)\, ds, \nu_T^{\varphi}\right).$$

For $\phi \in \mathcal{E}$, define

$$\mathbb{S}_\phi = \left\{(f, g) \in \mathbb{S} : \phi = \mathcal{G}^0\left(\zeta, \int_0^{\cdot} f(s)\, ds, \nu_T^{g}\right)\right\}.$$

Let $I_\zeta : \mathcal{E} \to [0, \infty]$ be defined by

$$I_\zeta(\phi) = \inf_{q=(f,g)\in\mathbb{S}_\phi} \{\bar{L}_T(q)\}. \tag{4.15}$$

THEOREM 4.5. *Let $Z^{\epsilon,\zeta}$ be defined as in (4.14) and suppose that Condition 4.2 holds. Suppose that for all $f \in \mathcal{E}$, $\zeta \mapsto I_\zeta(f)$ is a lower semi-continuous (l.s.c.) map from $\mathcal{E}_0$ to $[0, \infty]$, where I_ζ is as in (4.15). Then, for all $\zeta \in \mathcal{E}_0$, $f \mapsto I_\zeta(f)$ is a rate function on $\mathcal{E}$ and the family $\{I_\zeta(\cdot), \zeta \in \mathcal{E}_0\}$ of rate functions has compact level sets on compacts. Furthermore, the family $\{Z^{\epsilon,\zeta}\}$ satisfies the Laplace principle on $\mathcal{E}$, with rate function I_ζ, uniformly on compact subsets of $\mathcal{E}_0$.*

Proof. From Remark 2.1 we can regard β as an H valued Q-Wiener process, where $H = \bar{l}_2$ and Q is a trace class operator, as defined in Remark 2.1. Also, one can check that $H_0 \doteq Q^{1/2}H = l_2$. Since the embedding map $i : C([0, T] : \bar{l}_2) \to C([0, T] : \mathbb{R}^\infty)$ is continuous, $\hat{\mathcal{G}}^\varepsilon : \mathcal{E}_0 \times \mathbb{V}(\bar{l}_2) \to \mathcal{E}$ defined as $\hat{\mathcal{G}}^\varepsilon(\zeta, \sqrt{\varepsilon}v, \epsilon N^{\epsilon^{-1}}) \doteq \mathcal{G}^\varepsilon(\zeta, \sqrt{\varepsilon}i(v), \epsilon N^{\epsilon^{-1}})$, $(\zeta, v) \in \mathcal{E}_0 \times C([0, T] : \bar{l}_2)$ is a measurable map for every $\varepsilon \geq 0$. Note also that for $\varepsilon > 0$, $Z^{\varepsilon,\zeta^\epsilon} = \hat{\mathcal{G}}^\varepsilon(\zeta, \sqrt{\varepsilon}\beta, \varepsilon N^{\varepsilon^{-1}})$ almost surely. Since Condition 4.2 holds, we have that both

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parts of Condition 4.1 are satisfied with $\mathcal{G}^\varepsilon$ there replaced by $\hat{\mathcal{G}}^\varepsilon$ for $\varepsilon \geq 0$ and W replaced with β. Define $\hat{I}_\zeta(\phi)$ by the right-hand side of equation (4.11) with $\mathcal{G}^0$ replaced by $\hat{\mathcal{G}}^0$. Clearly $I_\zeta(\phi) = \hat{I}_\zeta(\phi)$ for all $(x, f) \in \mathcal{E}_0 \times \mathcal{E}$. The result is now an immediate consequence of Theorem 4.4. □

Finally, we consider the uniform Laplace principle for functionals of a Brownian sheet and a PRM. Let B be a Brownian sheet as in Definition 2.6. Let $\mathcal{G}^\varepsilon : \mathcal{E}_0 \times \mathbb{V}_{BS} \to \mathcal{E}$, $\varepsilon > 0$ be a family of measurable maps. Define

$$Z^{\varepsilon,\zeta} \doteq \mathcal{G}^\varepsilon(\zeta, \sqrt{\epsilon}B, \epsilon N^{\epsilon^{-1}}). \tag{4.16}$$

We now provide sufficient conditions for the Laplace principle to hold for the family $\{Z^{\varepsilon,\zeta}\}$.

Analogous to the classes defined in (4.8), we introduce

$$\tilde{S}^N \doteq \left\{\phi \in L^2([0, T] \times \mathcal{O}) : \int_{[0,T]\times\mathcal{O}} \phi^2(s, r)\, ds\, dr \leq N\right\}.$$

Once more, $\tilde{S}^N$ is endowed with the weak topology on $L^2([0, T] \times \mathcal{O})$, under which it is a compact metric space. For $u \in L^2([0, T] \times \mathcal{O})$, define $\mathrm{Int}(u) \in C([0, T] \times \mathcal{O} : \mathbb{R})$ by

$$\mathrm{Int}(u)(t, x) \doteq \int_{[0,t]\times(\mathcal{O}\cap(-\infty,x])} u(s, y)\, ds\, dy, \tag{4.17}$$

where $(-\infty, x] = \{y : y_i \leq x_i \text{ for all } i = 1, \ldots, d\}$. Consider $\bar{S}^N \equiv \bar{S}^N_{BS} = \tilde{S}^N \times S^N$ with the usual product topology. Let $\mathbb{S} = \bigcup_{N\geq 1} \bar{S}^N$ and let $\mathcal{U}^N$ be as defined in (4.10).

Condition 4.3. There exists a measurable map $\mathcal{G}^0 : \mathcal{E}_0 \times \mathbb{V}_{BS} \to \mathbb{U}$ such that the following hold.

(1) For $N \in \mathbb{N}$ let $(f_n, g_n), (f, g) \in \bar{S}^N$ be such that $(\zeta_n, f_n, g_n) \to (\zeta, f, g)$. Then
$$\mathcal{G}^0(\zeta_n, \mathrm{Int}(f_n), \nu_T^{g_n}) \to \mathcal{G}^0(\zeta, \mathrm{Int}(f(s)), \nu_T^g).$$

(2) For $N \in \mathbb{N}$ let $u_\epsilon = (\psi_\epsilon, \varphi_\epsilon)$, $u = (\psi, \varphi) \in \mathcal{U}^N$ be such that, as $\epsilon \to 0$, u_ϵ converges in distribution to u and $\{\zeta^\epsilon\} \subset \mathcal{E}_0$, $\zeta^\epsilon \to \zeta$, as $\epsilon \to 0$. Then
$$\mathcal{G}^\epsilon(\zeta^\epsilon, \sqrt{\epsilon}B + \mathrm{Int}(\psi_\epsilon), \epsilon N^{\epsilon^{-1}\varphi_\epsilon}) \Rightarrow \mathcal{G}^0(\zeta, \mathrm{Int}(\psi), \nu_T^\varphi).$$

For $\phi \in \mathcal{E}$, define

$$\mathbb{S}_\phi = \left\{(f, g) \in \mathbb{S} : \phi = \mathcal{G}^0\left(\zeta, \int_{[0,t]\times(\mathcal{O}\cap(-\infty,x])} f(s)\, ds, \nu_T^g\right)\right\}.$$

Let $I_\zeta : \mathcal{E} \to [0, \infty]$ be defined by

$$I_\zeta(\phi) = \inf_{q=(f,g)\in\mathbb{S}_\phi} \{\bar{L}_T(q)\}. \tag{4.18}$$

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THEOREM 4.6. *Let $Z^{\epsilon,\zeta}$ be defined as in (4.16) and suppose that Condition 4.3 holds. Suppose that for all $f \in \mathcal{E}$, $\zeta \mapsto I_\zeta(f)$ is a lower semicontinuous (l.s.c.) map from $\mathcal{E}_0$ to $[0, \infty]$, where I_ζ is as in equation (4.18). Then, for all $\zeta \in \mathcal{E}_0$, $f \mapsto I_\zeta(f)$ is a rate function on $\mathcal{E}$ and the family $\{I_\zeta(\cdot), \zeta \in \mathcal{E}_0\}$ of rate functions has compact level sets on compacts. Furthermore, the family $\{Z^{\epsilon,\zeta}\}$ satisfies the Laplace principle on $\mathcal{E}$, with rate function I_ζ, uniformly on compact subsets of $\mathcal{E}_0$.*

Proof. Let $\{e_i\}_{i=1}^\infty$ be a complete orthonormal system in $L^2(\mathcal{O})$ and let

$$\beta_i(t) \doteq \int_{[0,t]\times\mathcal{O}} e_i(x) B(ds\, dx), \quad t \in [0, T], i = 1, 2, \ldots.$$

Then $\beta \equiv \{\beta_i\}$ is a sequence of independent standard real Brownian motions and can be regarded as an $(S, \mathcal{S})$-valued random variable. Now, from [3, Proposition 3], there is a measurable map $h : C([0, T] : \mathbb{R}^\infty) \to C([0, T] \times \mathcal{O} : \mathbb{R})$ such that $h(\beta) = B$ almost surely. Define, for $\varepsilon > 0$, $\hat{\mathcal{G}}^\varepsilon : \mathcal{E}_0 \times \mathbb{V} \to \mathcal{E}$ as $\hat{\mathcal{G}}^\varepsilon(\zeta, \sqrt{\varepsilon}v, \varepsilon N^{\varepsilon^{-1}}) \doteq \mathcal{G}^\varepsilon(\zeta, \sqrt{\varepsilon}h(v), \varepsilon N^{\varepsilon^{-1}})$, $(\zeta, v, \phi) \in \mathcal{E}_0 \times \tilde{S}^N(l_2)$. Clearly $\hat{\mathcal{G}}^\varepsilon$ is a measurable map and

$$\hat{\mathcal{G}}^\varepsilon(\zeta, \sqrt{\varepsilon}\beta, \varepsilon N^{\varepsilon^{-1}}) = Z^{\varepsilon,\zeta} \text{ almost surely.}$$

Next, note that

$$S_{ac} \doteq \left\{ v \in C([0, T] : \mathbb{R}^\infty) : v(t) = \int_0^t \hat{u}(s)\, ds, t \in [0, T], \right.$$
$$\left. \text{for some } \hat{u} \in L^2([0, T] : l_2) \right\}$$

is a measurable subset of S. For $\hat{u} \in L^2([0, T] : l_2)$, define $u_{\hat{u}} \in L^2([0, T] \times \mathcal{O})$ as

$$u_{\hat{u}}(t, x) = \sum_{i=1}^\infty \hat{u}_i(t) e_i(x), \ (t, x) \in [0, T] \times \mathcal{O}.$$

Define $\hat{\mathcal{G}}^0 : \mathcal{E}_0 \times \mathbb{V} \to \mathcal{E}$ as

$$\hat{\mathcal{G}}^0\left(\zeta, \int_0^\cdot \hat{u}(s)\, ds, \nu_T^g\right) \doteq \mathcal{G}^0(\zeta, \mathrm{Int}(u_{\hat{u}}), \nu_T^g)$$

and note that

$$\left\{ \hat{\mathcal{G}}^0\left(\zeta, \int_0^\cdot \hat{u}(s)\, ds, \nu_T^g\right) : \hat{u} \in S^M(l_2), \zeta \in K, K \subset \mathcal{E}_0 \right\}$$
$$= \{\mathcal{G}^0(\zeta, \mathrm{Int}(u), \nu_T^g) : u \in S^M, \zeta \in K \subset \mathcal{E}_0\}.$$

Since Condition 4.1 holds, we have that its first part holds with $\mathcal{G}^0$ there replaced by $\hat{\mathcal{G}}^0$. Next, an application of Girsanov's theorem gives that, for every $\hat{u}^\varepsilon \in \tilde{S}^M(l_2)$,

$$h\left(\beta + \frac{1}{\sqrt{\epsilon}} \int_0^\cdot \hat{u}^\epsilon(s)\, ds\right) = B + \frac{1}{\sqrt{\epsilon}} \mathrm{Int}(u_{\hat{u}^\varepsilon}), \text{ almost surely.}$$

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In particular for every $M < \infty$ and families $\{\hat{u}^\epsilon, \phi^\epsilon\} \subset \bar{S}^M(l_2)$ and $\{\zeta^\varepsilon\} \subset \mathcal{E}_0$, such that $\{\hat{u}^\epsilon, \phi^\epsilon\}$ converges in distribution to $\{\hat{u}, \phi\}$ and $\zeta^\varepsilon \to \zeta$, we have, as $\varepsilon \to 0$,

$$\begin{aligned}\hat{\mathcal{G}}^\epsilon\left(\zeta^\varepsilon, \sqrt{\epsilon}\beta + \int_0^\cdot \hat{u}^\epsilon(s)\,ds, \epsilon N^{\epsilon^{-1}}\phi_\epsilon\right) &= \mathcal{G}^\epsilon(\zeta^\varepsilon, \sqrt{\epsilon}B + \mathrm{Int}(u_{\hat{u}^\epsilon}), \epsilon N^{\epsilon^{-1}}\phi_\epsilon)\\ &\Rightarrow \mathcal{G}^0(\zeta, \mathrm{Int}(u_{\hat{u}}), \nu_T^\phi)\\ &= \hat{\mathcal{G}}^0\left(\zeta, \int_0^\cdot \hat{u}(s)\,ds, \nu_T^\phi\right).\end{aligned}$$

Thus the second part of Condition 4.1 is satisfied with $\mathcal{G}^\varepsilon$ replaced by $\hat{\mathcal{G}}^\varepsilon$, $\varepsilon \geq 0$. The result now follows on noting that if $\hat{I}_\zeta(f)$ is defined by the right-hand side of equation (4.15) on replacing $\mathcal{G}^0$ there by $\hat{\mathcal{G}}^0$, then $\hat{I}_\zeta(f) = I_\zeta(f)$ for all $(\zeta, f) \in \mathcal{E}_0 \times \mathcal{E}$. □

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