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A weak collisionality constrained solution of a steady electron depleted hole

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Electron hole treatments normally match collisionless distribution functions associated with spatially varying electrostatic potential wells containing bound or reflecting electrons to unbound or non-reflecting electrons. When the matching is piecewise continuous at the separatrix between the bound and unbound, the second derivative in velocity space normally diverges, indicating collisions should be retained to remove the singular behaviour. The solubility constraint introduced by the collision operator severely restricts the allowed bound and unbound solutions, as illustrated by solving a model problem for a solitary electron depleted hole. As a result, a narrow collisional boundary layer encloses the separatrix between the bound and barely unbound electrons most sensitive to collisions. The procedure employed does not require resolving the collisional boundary layer, yet self-consistently accounts for the discontinuous behaviour to demonstrate an electron hole can last a long time when collisions are very weak.

Key words: plasma nonlinear phenomena, space plasma physics, plasma flows

1. Introduction

An electron hole is a **spatiotemporal** localised region having a depleted electron density. Steady state electron hole or solitary wave structures involve a piecewise matching of bound and unbound distribution functions. An excellent tutorial (Hutchinson 2017) and a comprehensive review (Hutchinson 2024) give many details and examples of these collisionless Bernstein, Greene & Kruskal (1957) or BGK modes as well as an extensive list of references for collisionless treatments. The piecewise continuous nature of the velocity space matching of the bound portion of the electron distribution function associated with the phase space island, to the unbound portion of the electron distribution function, suggests the possible existence of a narrow collisional boundary layer enclosing the phase space island's separatrix since the second derivative in velocity space can diverge there. This aspect of the allowed solutions to Fokker–Planck equation seems to be largely unexplored. The pioneering collisional treatment of periodic holes by Korn & Schamel (1996*a*) using a Hermite polynomial expansion found rapid relaxation to a Maxwellian always occurred for immobile ions with no dissipative coherent structure recovered. However, using mobile ions, Korn & Schamel (1996*b*) found non-Maxwellian features and evidence

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of phase space structures indicating the existence of bound as well as unbound electrons, with only hints of a collisional boundary layer due to the limited resolution of polynomial expansions. In addition, experimental measurements in low collisionality limits are challenging because of external dissipation (Danielson, Anderegg & Driscoll 2004), but seem to indicate a damping rate sensitivity to amplitude. Changes occurring at large amplitudes and low collisionality when Landau damping (1946) evolves into a weaker damping, nonlinear collisional regime were originally considered by Zakharov & Karpman (1963) and more recently by Catto (2025*a*). Notably, evidence of the weaker damping rate of Zakharov & Karpman (who first proved Landau damping is actually a collisional process) has also been observed at low collisionality in the particle in cell simulations of Valentini (2008) using a Langevin model with an externally driven sinusoidal electric field at the plasma frequency.

The model calculation presented here is for a conventional two component plasma satisfying Poisson's equation in the presence of a small, stable solitary phase space island that allows for the formation of a moving, spatially depleted electron region or hole. The goal of the calculation is the evaluation of the greatly extended lifetime for this nearly steady state solitary electron hole structure generated by a finite amplitude electrostatic potential in the extremely small collisionality limit. The solution procedure employed for a solitary island is related to that of a periodic island chain considered by Zakharov & Karpman (1963) and Catto (2025*a*,*b*). It demonstrates, for the special case of a perturbed background Maxwellian, that the solution is determined by collisional solubility constraints that restrict the bound (or librating) as well as the unbound portions of the electron distribution function near the separatrix (the terminology trapped and passing or untrapped is reserved for motion in a magnetic field to avoid confusion). Collisions constrain the position and fine scale velocity space structure of the bound, skew symmetric vortex or phase mixed electron motion inside and adjacent to the phase space separatrix. The retention of the narrow collisional boundary layer does not remove all discontinuities in the electron distribution function, but rather determines the allowed piecewise discontinuous behaviour in a self-consistent manner satisfying the required collisional solubility constraints. The weak dissipation in the narrow collisional boundary layer due to pitch angle and energy scattering by the background plasma causes the phase space island and, therefore, the spatial hole, to decay at a slow rate that is evaluated in detail.

The electron distribution function for a small isolated phase space island in the presence of weak collisions is considered in the next section. The plasma response and Poisson equation are considered in § 3 to show spatial electron depletion occurs in the hole and to highlight the restrictions on the validity of the solution. The electron power loss to the background is evaluated in § 4 to obtain the slow damping rate of the island. Section 5 gives a brief discussion of the results.

2. Electron response in the presence of a hole

In the frame moving with the electrons, assume there exists a (Sagdeev 1966) soliton or (Schamel 1972) electron hole generating potential moving at speed V of the form

$$\phi = \phi_0 \,\mathrm{sech}^2 x = \phi_0/\cosh^2 x, \tag{2.1}$$

where $x = k(z - Vt)$ is the phase, $k > 0$ is the wavenumber and $\phi_0 > 0$. If the speed of the potential is greater than the electron thermal speed, $V > v_e = (2T/m)^{1/2}$, there is the possibility of a long-lived electron island containing bound or reflecting or librating electrons in a weakly collisional plasma in the presence of a finite electric

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field $E=-\partial\phi/\partial x$, where T and m are the electron temperature and mass. The electron hole requires $\partial^2\phi/\partial x^2<0$ in a small region $x^2\lesssim 1$ about $x=0$, implying there is a deficit of electrons associated with the hole (Hutchinson 2017, 2024). A nearly steady state solution to Poisson's equation, $\partial E/\partial x=4\pi e(Zn_i-n)$, must be found to sustain a hole for a very long time, where n and n_i are the electron and ion densities, e is the proton charge and Z the ion charge number.

As Zakharov & Karpman (1963) and, more recently, Catto (2025*a*,*b*), have shown, there is no collisionless limit for a finite amplitude plasma wave when the Coulomb logarithm is large. As a result, Landau damping is actually a collisional process even though the collision frequency does not appear in the damping rate. The absence of the explicit appearance of the collision frequency in what is a collisional calculation is often referred to as a plateau regime (as in neoclassical theory of a large aspect ratio tokamak), and when there is a well defined resonance, it is often referred to as a resonant plateau process. As Landau damping would be expected to result in rather strong damping of an electron hole and require a collisional velocity space boundary layer wider than a velocity space island width it is not considered in detail here. Instead, the calculation here is performed in the extremely small collisionality limit by assuming a finite amplitude, electron hole is established by some perturbing event and its associated velocity space island width is much larger than any collisional boundary layer width. Once the hole exists the question of how long it continues to exist determines whether it has the desired long lifetime property of an electron hole. The calculation performed here evaluates the damping rate of a phase space island that allows electron hole formation, to demonstrate that an electron hole is able to persist for a very long time in the weak collisionality, finite amplitude limit, and that collisions provide a meaningful estimate of its lifetime.

Assuming the lowest order electron distribution function is Maxwellian

$$f_0=n_0(m/2\pi T)^{3/2}e^{-mv^2/2T}=n_0\pi^{-3/2}v_e^{-3}e^{-v^2/v_e^2}, \tag{2.2}$$

with n_0 the constant unperturbed electron density, and taking $f=f_0+f_1$ with $f_1\ll f_0$, but allowing $\partial f_1/\partial v_{||}\sim\partial f_0/\partial v_{||}$, then f_1 must satisfy the nonlinear kinetic equation

$$\partial f_1/\partial t+v_{||}\partial f_1/\partial z+(e/m)(\partial\phi/\partial z)\partial(f_0+f_1)/\partial v_{||}=C\{f_1\}=\nu v_{\perp z}^2\partial^2 f_1/\partial v_{||}^2, \tag{2.3}$$

where $\nu=\nu_e v_e^3/2v^3$, $\nu_e=3\sqrt{\pi}(Z+1)\nu_{ee}/4$, $\nu_{ee}=4\sqrt{2\pi}e^4n_0\ell n\Lambda/3m^{1/2}T^{3/2}=\nu_{ei}/Z$ and $v_{\perp z}^2=v_\perp^2+v_e^2/(Z+1)$. The velocity $v_{||}$ is in the direction of z, with $v_\perp^2=v^2-v_{||}^2$. The precise form of the diffusive collision operator is unimportant until the dissipation is evaluated. At that point only pitch angle and energy scattering by the ions and electrons as in (2.3) will matter. The weak collisions occur in a very narrow collisional boundary layer and act to oppose any phase space vortex structure formation inside the localised phase space island.

To clarify the limit of interest, notice there are two velocity space scales in a weakly collisional plasma when $f_1(z,v_{||},t)=f_1(x,v_{||})$ is assumed. The first is a linear regime in which nonlinearities are unimportant and collisions resolve the singularity associated with the resonance in the kinetic equation. Balancing $\partial f_1/\partial t+v_{||}\partial f_1/\partial z=k(v_{||}-V)\partial f_1/\partial x\sim kf_1\Delta v_{||}$ with collisions, by letting $(\Delta v_{||})_\nu=v_{||}-V$ and estimating $\partial^2 f_1/\partial v_{||}^2\sim f_1/(\Delta v_{||})_\nu^2$ and $\partial f_1/\partial x\sim f_1$, gives a normalised collisional boundary layer velocity space width of

$$(\Delta v_{||})_\nu/v_e\approx(\nu/kv_e)^{1/3}\ll 1, \tag{2.4}$$

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with $v_{\perp z}^2 \sim v_e^2$. The effective collision frequency is then

$$\nu_{eff} \approx \nu v_e^2/(\Delta v_{||})_v^2 \gg \nu. \tag{2.5}$$

The second velocity space scale is defined by letting $(\Delta v_{||})_{is} = v_{||} - V$ and balancing the nonlinear term, $ek\phi_0 f_1/m(\Delta v_{||})_{is}$, with streaming to obtain the normalised island width

$$(\Delta v_{||})_{is}/v_e \approx (e\phi_0/T)^{1/2} \ll 1. \tag{2.6}$$

To allow a localised island to exist the treatment presented here focuses on the very weakly collisional limit in which the island width is much larger than the collisional boundary layer

$$1 \gg (e\phi_0/T)^{1/2} \gg (\nu/kv_e)^{1/3}. \tag{2.7}$$

In this limit the separatrix enclosing the phase space island is surrounded by a narrow collisional boundary layer and no fine scale velocity space structure is allowed for the bound electrons inside the island separatrix.

Far from the island centred at $v_{||} = V$ the outer unbound response is found from

$$u\partial f_1/\partial x + (e/m)(\partial\phi/\partial x)\partial f_0/\partial v_{||} = 0, \tag{2.8}$$

where $u = v_{||} - V$, with the flow speed larger than the electron thermal speed, $V > v_e$. The $V > v_e$ ordering allows a non-Maxwell Boltzmann electron response (not unlike a plasma wave). Solving using $f_1 = f_1^{out} + f_2^{out} + \cdots$ gives the outer solution away from the island to be

$$\begin{aligned} f_1^{out} &= -\frac{e\phi}{mu}\frac{\partial f_0}{\partial v_{||}} = -\frac{e\phi_0\,\mathrm{sech}^2 x}{m(v_{||}-V)}\frac{\partial f_0}{\partial v_{||}} \\ &= \frac{e\phi v_{||} f_0}{T(v_{||}-V)} \approx -\frac{e\phi v_{||} f_0}{TV}\left[1 + \frac{v_{||}}{V} + \left(\frac{v_{||}}{V}\right)^2 + \left(\frac{v_{||}}{V}\right)^3 + \cdots\right], \end{aligned} \tag{2.9}$$

where here $\partial f_0/\partial v_{||} = -mv_{||}f_0/T$. The next order outer response for $v_e \sim v_{||} < V$ from

$$u\partial f_2^{out}/\partial x + (e/m)(\partial\phi/\partial x)\partial f_1^{out}/\partial v_{||} \approx u\partial/\partial x[f_2^{out} - Vf_0 e^2\phi^2/2mTu^3] \approx 0 \tag{2.10}$$

yields the lowest order result

$$f_2^{out} \approx Vf_0(e\phi)^2/2mTu^3 \approx -f_0(v_e e\phi/TV)^2/4, \tag{2.11}$$

which needs to be retained in Poisson's equation to balance the $\mathrm{sech}^4 x$ term from $(\partial^2/\partial x^2)\,\mathrm{sech}^2 x = -6\,\mathrm{sech}^4 x + 4\,\mathrm{sech}^2 x$.

In the island vicinity the full electron kinetic equation must be solved. To begin let

$$f_1 = g(u, x) - [u - \sigma(e\phi_0/m)^{1/2}\alpha]\partial f_0/\partial v_{||}|_{u=0}, \tag{2.12}$$

where $\sigma = u/|u| = \pm 1$ or 0, α is a constant to be determined and $\partial f_0/\partial v_{||}|_{u=0} \approx -mV\bar{f}_0/T$ with $\bar{f}_0 = f_0(v_{||} = V) = n_0\pi^{-3/2}v_e^{-3}e^{-(v_\perp^2+V^2)/v_e^2}$ a constant to lowest order

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in the highly localised inner velocity space region containing the island having a width much bigger than that of the collisional boundary layer surrounding its separatrix. Substituting (2.12) into (2.3) leads to

$$ku\ \partial g/\partial x + (ek/m)(\partial\phi/\partial x)\partial g/\partial u = \nu v_{\perp z}^2 \partial^2 g/\partial u^2. \tag{2.13}$$

Far from the island f_1 must go to (2.9), implying the inner solution for g must match to

$$g \to \left[u - \sigma\alpha\left(\frac{e\phi_0}{m}\right)^{1/2} - \frac{e\phi_0}{mu}\,\mathrm{sech}^2 x\right]\frac{\partial f_0}{\partial v_{||}}|_{u=0}. \tag{2.14}$$

Defining

$$j = (m/e\phi_0)^{1/2}u \tag{2.15}$$

and

$$\Delta = \nu k^{-1} v_{\perp z}^2 (m/e\phi_0)^{3/2} \approx (\Delta v_{||})_v^3/(\Delta v_{||})_{is}^3 \tag{2.16}$$

then (2.3) becomes a similar equation to the one solved numerically by Hamilton *et al.* (2023) for the motion associated with a galactic bar or spiral arm galaxy, namely

$$j\frac{\partial g}{\partial x} + \frac{\partial\phi}{\partial x}\frac{\partial g}{\partial j} = \Delta\frac{\partial^2 g}{\partial j^2}, \tag{2.17}$$

where $\Delta > 0$ since $v_\perp^2 \approx v^2 - V^2 > 0$ in $v_{\perp z}^2$. Here, however, a localised non-periodic solution is essential as a solitary electron hole potential is being considered.

The steady state solutions of interest are skew symmetric so satisfy

$$g(j, x) = -g(-j, -x). \tag{2.18}$$

An analytic treatment is possible for weak collisions satisfying $\Delta \ll 1$ by taking advantage of the lowest order collisionless trajectories. Employing the reduced constant of the motion

$$h(j, x) = j^2/2 - \mathrm{sech}^2 x = j^2/2 - 1/\cosh^2 x, \tag{2.19}$$

leads to an island with a separatrix at $h = 0$ in the phase space j and x, when $-1 \le h < 0$ and $h > 0$ are considered. The solution (2.14) far from the island becomes

$$g \to \sigma\left(\sqrt{2h} - \alpha\right)(e\phi_0/m)^{1/2}\,\partial f_0/\partial v_{||}|_{u=0}. \tag{2.20}$$

For the unbound,

$$j = \pm\sqrt{2(h + \mathrm{sech}^2 x)}, \tag{2.21}$$

with $h + \mathrm{sech}^2 x > h > 0$, while for the bound, a real j requires $-1 \le h < 0$. The separatrix at $h = 0$ is enclosed by a narrow collisional boundary layer separating the bound ($-1 < h < 0$) and the unbound ($h > 0$) electrons.

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Using the constant of the motion h and x as variables, the kinetic equation becomes

$$\left.\frac{\partial g}{\partial x}\right|_h = \Delta \left.\frac{\partial}{\partial h}\right|_x \left(j \left.\frac{\partial g}{\partial h}\right|_x \right). \tag{2.22}$$

The weak collisionality $\Delta \ll 1$ limit allows a solution of the form $g = g_1(h) + g_2(h, x) + \cdots$ to be found. To lowest order $\partial g_1/\partial x|_h = 0$, while to next order

$$\left.\frac{\partial g_2}{\partial x}\right|_h = \Delta \left.\frac{\partial}{\partial h}\right|_x \left(j \left.\frac{\partial g_1}{\partial h}\right|_x \right). \tag{2.23}$$

Further progress is made by defining the x integral of j at fixed h as

$$\langle j \rangle_h = \oint_h \mathrm{d}x j, \tag{2.24}$$

where for the unbound x is from $-\infty$ to ∞, but for the bound it is over a complete bounce.

For both the bound and unbound, integration over x leads to the solubility constraint

$$\left.\frac{\partial}{\partial h}\right|_x \left(\langle j \rangle_h \left.\frac{\partial g_1}{\partial h}\right|_x \right) = 0, \tag{2.25}$$

since

$$\int_{-\infty}^{\infty} dx \left.\frac{\partial g_2}{\partial x}\right|_h = 0. \tag{2.26}$$

For the bound, the preceding is clearly satisfied since an integration over a full bounce returns to the same g_2. For the unbound, skew symmetry is employed to find

$$\begin{aligned} g_2\big(j = \sigma\sqrt{2h}, \infty\big) - g_2\big(j = \sigma\sqrt{2h}, -\infty\big) &= g_2\big(j = \sigma\sqrt{2h}, \infty\big) \\ + g_2\big(-j = -\sigma\sqrt{2h}, \infty\big) &= 0. \end{aligned} \tag{2.27}$$

By requiring (2.27) to vanish, the unbound solution odd in j avoids a jump in g_2 as $|x| \to \infty$.

Of course, the inner solution g_1 must have the desired skew symmetry to match the outer solution far from the island where $f_1 \propto 1/u \to 0$. The solution for g_1 unconstrained by solubility would just be a version of a BGK mode (Bernstein *et al.* 1957). But the constrained solution found next is no longer a typical piecewise continuous BGK mode because its form is determined by a collisional solubility constraint, whereas Bernstein *et al.* (1957) were careful to note that their treatment ignored collisions.

Integrating the solubility constraint once more gives

$$\langle j \rangle_h \left.\frac{\partial g_1}{\partial h}\right. = \text{constant}, \tag{2.28}$$

where the constant must be independent of h. Using integrals no. 2.261 and no. 2.266 from Gradshteyn & Ryzhik (2000) the integrals in $\langle j \rangle_h$ are performed by letting $y = \mathrm{sech}^2 x$.

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Integrating from $y=-h$ to 0 to find $\langle j\rangle_{h<0}=4\pi(\sqrt{2}-\sqrt{-2h})$ for the bound, leads to

$$4\pi\left(\sqrt{2}-\sqrt{-2h}\right)\frac{\partial g_1}{\partial h}\bigg|_{h<0}=\text{constant}, \tag{2.29}$$

requiring the constant to vanish to avoid a singularity at $h=-1$, and implying g_1 is a constant at most. But at $x=0$, $g(j,x=0)=-g(-j,-x=0)$ and a constant g_1 is not odd in j. Hence,

$$g_1(h<0)=0, \tag{2.30}$$

along with $\partial g_1/\partial h|_{h<0}=0$. In the steady state the skew symmetric vortex structure of the bound solution is dissipated by collisions. Phase mixing is not required to regularise the moments of f_1 by integrating over velocity space. The full bound solution is

$$f_1\,(h<0)=-u\partial f_0/\partial v_{||}|_{u=0}=mVu\,\bar{f}_0/T. \tag{2.31}$$

For the unbound, skew symmetry implies only $x\geq 0$ and $\sigma=+1$ need be considered. Again letting $y=\text{sec}h^2x$ to perform the integral, but this time the range of interest is from $x=0$ to a large $x=X\gg 1$ at which $y=Y\approx 4e^{-2X}\ll 1$. Again, using no. 2.261 and no. 2.266 from Gradshteyn & Ryzhik (2000)

$$\begin{aligned}\langle j\rangle_{h\geq 0}&=\sqrt{2h}\left[2X+\ell n\left(\frac{h}{1+h}\right)\right]+\sqrt{2}\sin^{-1}\left(\frac{1-h}{1+h}\right)\\&\quad+\frac{\pi}{\sqrt{2}}\rightarrow\begin{cases}\pi\sqrt{2} & h=0,\\ 2X\sqrt{2h}+2/\sqrt{2h} & h\gg 1.\end{cases}\end{aligned} \tag{2.32}$$

Recall, far from the island the unbound solution is $g_1\rightarrow -mV(e\phi_0/m)^{1/2}(\sqrt{2h}-\alpha)\bar{f}_0/T$, thereby giving $\partial g_1/\partial h|_{h\gg 1}=-mV\,(e\phi_0/m)^{1/2}\,\bar{f}_0/T\sqrt{2h}$. Therefore,

$$\langle j\rangle_h\,\frac{\partial g_1}{\partial h}\bigg|_{h>0}=\pi\sqrt{2}\,\frac{\partial g_1}{\partial h}\bigg|_{x,h=0}=-2X\frac{mV}{T}\left(\frac{e\phi_0}{m}\right)^{1/2}\bar{f}_0. \tag{2.33}$$

As a result, for $h\geq 0$, integrating from $h=0$, where $g_1(h=0)=0$ to match the bound, gives

$$g_1(h)=-\sigma\frac{mV\bar{f}_0}{T}\left(\frac{e\phi_0}{m}\right)^{1/2}\left[2X\int_0^h\frac{\mathrm{d}h}{\langle j\rangle_{h\geq 0}}\right]\underset{h\gg 1}{\rightarrow}-\sigma\frac{mV\bar{f}_0}{T}\left(\frac{e\phi_0}{m}\right)^{1/2}[\sqrt{2h}-\alpha(X)], \tag{2.34}$$

where σ is restored to display the proper skew symmetric behaviour of the solution, and

$$\begin{aligned}\alpha(X)&=\int_0^\infty\mathrm{d}h\left(\frac{1}{\sqrt{2h}}-\frac{2X}{\langle j\rangle_{h\geq 0}}\right)\\&=\int_0^\infty\mathrm{d}h\frac{\sqrt{2h}\ln[h/(1+h)]+\sqrt{2}\sin^{-1}[(1-h)/(1+h)]+(\pi/\sqrt{2})}{\sqrt{2h}\,\langle j\rangle_{h\geq 0}}\underset{X\rightarrow\infty}{\longrightarrow}0.\end{aligned} \tag{2.35}$$

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The constant α is a function of X since as $x = X \to \infty$, α must vanish to allow g to become continuous. It is interesting to realise that the value of $\alpha(X)$ is unimportant since it is only required to insure the proper behaviour at $h \gg 1$. The dependence of α on X is an artefact of the hole potential $\phi \propto \mathrm{sech}^2 x$ extending to infinity. An analytic solution allows $X \to \infty$ and $\alpha \to 0$. However, a numerical solution has to pick an $X \gg 1$ and therefore has a finite $\alpha(X)$.

Using $g_1(h)$ from (2.35), the full unbound solution for f_1 about the island is

$$f_1(h \geq 0) = \frac{mV\bar{f}_0}{T}\left\{u - \sigma\left(\frac{e\phi_0}{m}\right)^{1/2}\left[\alpha(X) + 2X\int_0^h \frac{\mathrm{d}h}{\langle j\rangle_{h\geq 0}}\right]\right\} \underset{h\gg 1}{\longrightarrow}$$
$$\frac{mV\bar{f}_0}{T}\left[u - \sigma\left(\frac{e\phi_0}{m}\right)^{1/2}\sqrt{2h}\right] = \frac{mV\bar{f}_0}{T}\left[u - u\sqrt{1 - \left(2\,\mathrm{sech}^2 x/j^2\right)}\right] \approx \frac{e\phi V\bar{f}_0}{Tu}. \tag{2.36}$$

Steps in f_1 (for finite X) and $\partial g_1/\partial h$ across the separatrix are allowed because it is enclosed by a thin collisional boundary layer that provides smooth matching. Piecewise continuous matching is adequate for the purposes here because the collisional solubility constraints (2.29) and (2.33) are satisfied – just as it is in the banana regime of neoclassical transport in a tokamak for which there are magnetically trapped and passing particles.

When the collisionality constraint is not imposed only $\partial g/\partial x|_h = 0$ need be satisfied. Then, only unconstrained solutions for g depending on h need to be considered. Taking $\alpha = 0$ in the relation between f_1 and g reduces (2.12) to

$$f_1 = g(h) - u\partial f_0/\partial v_{||}|_{u=0} = g(h) + MuV\bar{f}_0/T, \tag{2.37}$$

with $\bar{f}_0 = f_0(v_{||} = V)$. As a result, an allowed unconstrained piecewise continuous at the separatrix at $h = 0$ that matches to the inner limit of the outer solution is

$$g = g(h) = \begin{cases} 0 & \text{bound } (-1 < h < 0), \\ \sigma\sqrt{2h}\left(\dfrac{e\phi_0}{m}\right)^{1/2}\dfrac{\partial f_0}{\partial v_{||}}|_{u=0} & \text{unbound } (h > 0)\,. \end{cases} \tag{2.38}$$

This leads to the unconstrained unbound and bound solutions

$$f_1 = -u\frac{\partial f_0}{\partial v_{||}}|_{u=0}\begin{cases} 1 & \text{bound}, \\ 1 - \left(1 - 2\,\mathrm{sech}^2 x/j^2\right)^{1/2} & \text{unbound}. \end{cases} \tag{2.39}$$

The unbound unconstrained solution recovers the $h \gg 1$ limit of the collisionally constrained solution as in (2.36), but differs from the collisionally constrained solution as $h \to 0$, that is, in the vicinity of the separatrix for the barely unbound. Its singular behaviour, $\partial^2\sqrt{2h}/\partial h^2 = -1/(2h)^{3/2}$, means collisions can never be neglected when the Coulomb logarithm is large.

3. Electron and ion response and Poisson's equation

To determine the conditions under which the electron response self-consistently satisfies Poisson's equation, both the electron and ion density are required. The skew symmetry of the inner solution means it does not perturb the density to all orders for

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both the correct, collisionally constrained solution, and the unphysical unconstrained solution (2.37)–(2.39). The electron density is only perturbed by the outer or away from the island electron response (that is, in the outer region, including the matching region, where the inner limit of the outer solution equals the outer limit of the inner solution). As the inner or island solution does not perturb the density, the electron density is simply

$$n = \int d^3v \left(f_0 + f_1^{out} + f_2^{out}\right) = n_0 \left[1 - \frac{e\phi}{T}\left(\frac{v_e}{V}\right)^2 \left(1 + \frac{3v_e^2}{2V^2} + \cdots\right) + \frac{1}{4}\left(\frac{v_e e\phi}{VT}\right)^2 + \cdots\right], \tag{3.1}$$

where the outer solution $f_1^{out} + f_2^{out}$ is valid away from the island and ϕ^2 terms are needed, unlike the standard sinusoidal island chain case considered by Zakharov & Karpman (1963) and Catto (2025). The Poisson equation solution allows $e\phi_0/T \ll 1$ for a parameter range, thereby implying the self-consistency of the velocity space localised island model considered. The deficit of electrons needed to form a hole is not directly due to the island, but rather is from the matching region and beyond. The presence of the island allows a non-Maxwell Boltzmann electron response permitting an electron deficit, and thereby an electron hole, as will be shown in the remainder of this section. Numerical simulations (see figure 2 in Hutchinson 2017) form finite islands having a depleted electron population, but are unlikely to properly resolve the very narrow collisional boundary layer surrounding the separatrix to obtain the correct solubility constraint (2.25) and associated damping rate.

The ions are assumed to be stationary in the electron frame. Their lowest order distribution function is also assumed to be Maxwellian

$$f_{0i} = n_{i0}(M/2\pi T_i)^{3/2} e^{-Mv^2/2T_i}, \tag{3.2}$$

to make the ion–ion collision operator vanish to lowest order, and with no significant resonant interactions occurring at higher order because $V > v_e \gg v_i = (2T_i/M)^{1/2}$. The constants n_{i0}, T_i and M are the ion density, temperature and mass. In the absence of a resonance, ion collisions are unimportant when calculating the corrections to f_{0i}. Therefore, the lowest order perturbed ion response is the solution to

$$v_{||}[\partial f_{1i}/\partial x + (Ze/T_i)(\partial\phi/\partial x) f_{0i}] = 0. \tag{3.3}$$

As $v_{||} \neq 0$, the ion response is Maxwell Boltzmann

$$f_{1i} = -(Ze\phi/T_i) f_{0i} = -(Ze\phi_0/T_i) f_{0i}\, \mathrm{sech}^2 x. \tag{3.4}$$

Solving to next order by noticing $f_{1i} \propto Ze\phi/T \ll 1$, then

$$v_{||}[\partial f_{2i}/\partial x + (Ze/T_i)(\partial\phi/\partial x) f_{1i}] = 0 \tag{3.5}$$

gives the next order Maxwell Boltzmann response

$$f_{2i} = -(Ze\phi/2T_i) f_{1i} = (Ze\phi/T_i)^2 f_{0i}/2 = (Ze\phi_0/T_i)^2 f_{0i}\, \mathrm{sech}^4 x/2. \tag{3.6}$$

Consequently, the ions only experience the Maxwell Boltzmann response

$$n_i(x) = \int d^3v (f_{0i} + f_{1i} + f_{2i}) = n_{i0}\left[1 - \frac{Ze\phi}{T_i} + \frac{1}{2}\left(\frac{Ze\phi}{T_i}\right)^2\right]. \tag{3.7}$$

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As a check, notice that the ion distribution function can only depend on the constant of the motion $Mv^2/2 + Ze\phi$, requiring the Maxwell Boltzmann response $n_i = n_{i0}e^{-Ze\phi/T_i}$. Recall that keeping the $(Ze\phi/T_i)^2$ term is necessary to obtain the assumed potential (2.1) because the Laplacian acting on $\mathrm{sech}^2 x$ gives a $\mathrm{sech}^4 x$ term.

Taking the background plasma as quasineutral ($Zn_{i0} = n_0$), the lowest order Poisson equation only requires the non-resonant electron and ion responses (and is valid for both the collisionally constrained and unconstrained cases), and becomes

$$\frac{\partial^2\phi}{\partial z^2} = -4\pi\rho = 4\pi e\,(n - Zn_i) \approx -\frac{\phi}{\lambda_D^2}\left[\frac{v_e^2}{V^2}\left(1 + \frac{3v_e^2}{2V^2}\right) - \frac{ZT}{T_i}\right] - \frac{e\phi^2}{2T\lambda_D^2}\left(\frac{Z^2T^2}{T_i^2} + \frac{v_e^2}{2V^2}\right), \tag{3.8}$$

with the Debye length defined by $\lambda_D^2 = T/4\pi e^2 n_0$. The last term need not be smaller than the first term on the right side. Inserting ϕ and using $(\partial^2/\partial x^2)\,\mathrm{sech}^2 x = -6\,\mathrm{sech}^4 x + 4\,\mathrm{sech}^2 x = -2\,\mathrm{sech}^2 x(1 - 3\tanh^2 x) \underset{x\to 0}{\longrightarrow} -2$, requires satisfying two conditions from

$$\left[6k^2\lambda_D^2 - \frac{e\phi_0}{2T}\left(\frac{Z^2T^2}{T_i^2} + \frac{v_e^2}{2V^2}\right)\right]\mathrm{sech}^4 x = \left[4k^2\lambda_D^2 + \frac{v_e^2}{V^2}\left(1 + \frac{3v_e^2}{2V^2}\right) - \frac{ZT}{T_i}\right]\mathrm{sech}^2 x. \tag{3.9}$$

Self-consistency requires both sides vanish separately giving

$$4k^2\lambda_D^2 = ZT/T_i - \left(v_e^2/V^2\right)\left(1 + 3v_e^2/2V^2\right) > 0 \tag{3.10}$$

and

$$1 \gg \frac{e\phi_0}{T} = \frac{12k^2\lambda_D^2}{Z^2T^2/T_i^2 + v_e^2/2V^2}. \tag{3.11}$$

If $ZT/T_i = 1$ and $v_e^2/V^2 = 1/2$, then $e\phi_0/T \approx 3/10$ and $k^2\lambda_D^2 \approx 1/32$. But for $ZT/T_i \gg 1$, the $Ze\phi_0/T_i \ll 1$ assumption fails, as does $e\phi_0/T \ll 1$ if $1 \gg ZT/T_i \gg (v_e^2/V^2)(1 + 3v_e^2/2V^2)$. In any case, if $ZT/T_i \approx (v_e^2/V^2)(1 + 3v_e^2/2V^2)$, small $k^2\lambda_D^2$ allows $(e\phi_0/T)^2$ terms to matter.

Consequently, the temperatures of interest are expected to be in the range

$$1 \geq \frac{ZT}{T_i} \approx \frac{v_e^2}{V^2}\left(1 + \frac{3v_e^2}{2V^2}\right). \tag{3.12}$$

For example, there are small $e\phi_0/T$ and $k^2\lambda_D^2$ solutions for $ZT/T_i = 1/2$ if $v_e^2/V^2 < 1/3$, for $ZT/T_i = 11/32$ if $v_e^2/V^2 < 1/4$, etc. The temperature restriction is not as rigid as it seems at first glance since there are exponentially few electrons at very large V^2/v_e^2. Finally, the hole potential is only significant for $x^2 \lesssim 1$, implying the spatial extent of an electron hole normalised to a Debye length is $\Delta z/\lambda_D \sim 1/k\lambda_D \gg 1$ or many Debye lengths. The collisional velocity space boundary layer width as well as $e\phi_0/T$ are small since

$$1 \gg (\Delta v_{||})_{is}/v_e \approx (e\phi_0/T)^{1/2} \sim 12k^2\lambda_D^2/\left(Z^2T^2/T_i^2 + v_e^2/2V^2\right). \tag{3.13}$$

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4. Collisional dissipation in the boundary layer

To higher order there is some dissipation associated with the electron hole because the collisional electrons in the separatrix boundary layer heat the background plasma as the island slowly decays away, as will now be demonstrated. The power lost to the background plasma by the slowly decaying hole potential due to the dissipation in the collisional boundary layer surrounding the separatrix in a region of phase space extent $2X \gg 1$ is

$$\langle P\rangle_h = -e\left\langle \frac{\partial\phi}{\partial z}\int d^3v v_{||} f\right\rangle_h = -ek\left\langle \frac{\partial\phi}{\partial z}\int d^3v v_{||} f\right\rangle_h, \tag{4.1}$$

where the subscript h on $\langle\ldots\rangle_h$ denotes it is held fixed to perform the x integration. The skew symmetry of the inner unbound region gives $\sigma\int \mathrm{d}u(\sqrt{2h}-\alpha)=0$ and $\langle\int \mathrm{d}u\ g\rangle_h = 0$, leaving

$$\langle P\rangle_h = -ek\left\langle \frac{\partial\phi}{\partial x}\int d^3v u\ g\right\rangle_h = ek\left\langle \phi\int d^3v u\ \frac{\partial g}{\partial x}\bigg|_h\right\rangle_h. \tag{4.2}$$

Inserting the reduced kinetic equation yields

$$\begin{aligned}\langle P\rangle_h &= ek\left(\frac{e\phi_0}{m}\right)^{1/2}\left\langle \phi\int d^3v j\Delta\ \frac{\partial}{\partial h}\bigg|_x\left(j\ \frac{\partial g}{\partial h}\bigg|_x\right)\right\rangle_h\\ &= 2\pi ek\frac{e\phi_0}{m}\left\langle \phi\int_0^\infty \mathrm{d}v_\perp v_\perp\Delta\int_{-\infty}^\infty \mathrm{d}j j\ \frac{\partial}{\partial h}\bigg|_x\left(j\ \frac{\partial g}{\partial h}\bigg|_x\right)\right\rangle_h\\ &= 4\pi k\frac{e^2\phi_0^2}{m}\int_0^\infty \mathrm{d}v_\perp v_\perp\Delta\int_0^\infty \mathrm{d}h\ \frac{\partial}{\partial h}\bigg|_x\langle j\ \mathrm{sech}^2x\rangle_h\ \frac{\partial g_1}{\partial h}.\end{aligned} \tag{4.3}$$

Then using

$$\int_0^\infty \mathrm{d}h\frac{\partial}{\partial h}\bigg|_x(\langle j\ \mathrm{sech}^2x\rangle_h\frac{\partial g_1}{\partial h} = \langle j\ \mathrm{sech}^2x\rangle_h\frac{\partial g_1}{\partial h}\bigg|_{h=0}^{h\gg1} \tag{4.4}$$

and integrals no. 2.261 and no. 2.264 from Gradshteyn & Ryzhik (2000) to find

$$\langle j\ \mathrm{sech}^2x\rangle_h = \begin{cases}2\sqrt{2h} & h\gg1,\\ \pi/\sqrt{2} & h=0,\end{cases} \tag{4.5}$$

then

$$\langle j\ \mathrm{sech}^2x\rangle_h\ \frac{\partial g}{\partial h}\bigg|_{h=0}^{h\gg1} = (X-2)\left(\frac{e\phi_0}{m}\right)^{1/2}\frac{mV}{T}\bar{f}_0. \tag{4.6}$$

The length $2X$ appears because the hole, and therefore the dissipation region, extends to infinity. Normalising using $\langle P\rangle_h = 2XP$ gives the power dissipated to be

$$P = \left(1-\frac{2}{X}\right)\frac{mn_0kV\Delta_0}{\pi^{1/2}v_e^2}\left(\frac{e\phi_0}{T}\right)^{5/2}e^{-V^2/v_e^2}\int_0^\infty \mathrm{d}v_\perp v_\perp v_{\perp z}^2 e^{-mv_\perp^2/2T}, \tag{4.7}$$

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where $\Delta = 2^{3/2}(\nu v_{\perp z}^2/kv_e^3)(T/e\phi_0)^{3/2} \equiv 2^{3/2}\Delta_0 v_{\perp z}^2/v_e^2$ and $\nu \approx \nu_e v_e^3/2V^3$. Integrating gives

$$P = \left(1 - \frac{2}{X}\right)\frac{(Z+2)\nu_e n_0 T v_e^2}{2\pi^{1/2}(Z+1)V^2}\left(\frac{e\phi_0}{T}\right)e^{-V^2/v_e^2}. \tag{4.8}$$

To see this result implies the dissipation or damping rate γ_{hole} is very small within a finite hole region of extent $2X \gg 1$, the island power dissipated is balanced by the slow decay of the hole potential to obtain

$$P = \gamma_{hole}\left\langle E_{||}^2\right\rangle_h/4\pi = \gamma_{hole} n_0 T k^2\lambda_D^2 e^2\left\langle(\partial\phi/\partial x)^2\right\rangle_h/T^2. \tag{4.9}$$

Using $\langle\phi^2\rangle_h = 4\phi_0^2/3$, $\langle\phi^3\rangle_h = 16\phi_0^3/15$, (3.10) and (3.11) gives

$$\begin{aligned} k^2\lambda_D^2\left\langle\left(\frac{\partial\phi}{\partial x}\right)^2\right\rangle_h &= -k^2\lambda_D^2\left\langle\phi\frac{\partial^2\phi}{\partial x^2}\right\rangle = \left[\frac{v_e^2}{V^2}\left(1+\frac{3v_e^2}{2V^2}\right) - \frac{ZT}{T_i}\right]\langle\phi^2\rangle_h \\ &+ \frac{e}{2T}\left(\frac{Z^2T^2}{T_i^2} + \frac{v_e^2}{2V^2}\right)\langle\phi^3\rangle_h = \frac{4}{3}\left[\frac{v_e^2}{V^2}\left(1+\frac{3v_e^2}{2V^2}\right) - \frac{ZT}{T_i}\right]\phi_0^2 \\ &+ \frac{8e}{15T}\left(\frac{Z^2T^2}{T_i^2} + \frac{v_e^2}{2V^2}\right)\phi_0^3 = \frac{16}{15}k^2\lambda_D^2\phi_0^2 \end{aligned} \tag{4.10}$$

and thereby yields

$$P = \frac{16}{15}\gamma_{hole} n_0 T k^2\lambda_D^2\left(\frac{e\phi_0}{T}\right)^2. \tag{4.11}$$

As a result, the island or hole decay or damping rate is

$$\begin{aligned} \gamma_{hole} &= \frac{15(Z+2)\nu_e v_e^2 e^{-V^2/v_e^2}}{32\pi^{1/2}(Z+1)k^2\lambda_D^2 V^2(e\phi_0/T)} \\ &= \frac{15\sqrt{3}(Z+2)\nu_e v_e^2 e^{-V^2/v_e^2}}{16\pi^{1/2}(Z+1)k\lambda_D V^2(Z^2T^2/T_i^2 + v_e^2/2V^2)^{1/2}(e\phi_0/T)^{3/2}}, \end{aligned} \tag{4.12}$$

with the last form more suggestive than the first. The limit considered here assumes $\Delta_0 = (\nu_e/kv_e)(v_e^3/2V^3)(T/e\phi_0)^{3/2} \ll 1$, giving $\gamma_{hole} \sim (V\Delta_0/\lambda_D)e^{-V^2/v_e^2}$ if $V^2/v_e^2 \approx 3$ and $ZT/T_i > 1/2$. As a result, this decay rate can be very small – much smaller than the sinusoidal Landau (1946) damping rate for a Maxwellian

$$\gamma_L = \frac{\pi^{1/2}kV^2}{2k^2\lambda_D^2 v_e}e^{-3/2-1/2k^2\lambda_D^2}. \tag{4.13}$$

Consequently, a hole is allowed to be very long lived and be many Debye lengths long.

The rate γ_{hole} differs from the BGK damping rate used by Valentini (2008) because he employed the sinusoidal results of Zakharov & Karpman (1963) rather than a solitary hole as considered here and for which ϕ_0^2 terms are required.

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5. Discussion

The preceding calculation demonstrates the key role collisions can play whenever a piecewise continuous matching of distribution functions is required at the separatrix of a phase space island. The separatrix between the island bound and unbound electrons is then likely to be enclosed by a narrow collisional boundary layer, with no vortex or phase space mixing allowed in the bound response. That is, the collisional solubility constraint severely restricts the form of the bound solution, as it does in other situations like nonlinear plasma waves in the limit of very weak collisions (Zahkarov & Karpman 1963; Catto 2025*a*), galactic bar behaviour in a disc galaxy (Hamilton *et al.* 2023), intense lower hybrid current drive in a weakly collisional plasma (Catto 2025*b*) and weakly collisional transport in the quasisymmetric stellarator with a single helicity error field (Catto 2025*c*). Even though the lowest order perturbed electron distribution function allowing a hole potential is independent of collision frequency, its form is constrained by the collision operator by a solubility condition on the highly localised solitary phase space island structure moving in the background plasma. The discontinuous behaviour of the distribution function is then self-consistently determined by matching across the separatrix (without the need to resolve this narrow collisional boundary layer). The dissipation in the collisional boundary layer associated with the electrons colliding with the background plasma is very weak so the island and hole potential can be very long lived. Unlike for a collisionally unconstrained solution, no singular behaviour arises in the collisionally constrained solution which also permits electron depletion. Perhaps, the existence of holes is confirmation there are collisional boundary layers as well as the proof of the collisional nature of Landau damping.

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Declaration of interests

The author reports no conflict of interest.

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