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Modification and Numerical Method for the Jiles-Atherton Hysteresis Model

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Abstract. The Jiles-Atherton (J-A) model is a commonly used physics-based model in describing the hysteresis characteristics of ferromagnetic materials. However, citations and interpretation of this model in literature have been non-uniform. Solution methods for solving numerically this model has not been studied adequately. In this paper, through analyzing the mathematical properties of equations and the physical mechanism of energy conservation, we point out some unreasonable descriptions of this model and develop a relatively more accurate, modified J-A model together with its numerical solution method. Our method employs a fixed point method to compute anhysteretic magnetization. We obtain the susceptibility value of the anhysteretic magnetization analytically and apply the 4th order Runge-Kutta method to the solution of total magnetization. Computational errors are estimated and then precisions of the solving method in describing various materials are verified. At last, through analyzing the effects of the accelerating method, iterative error and step size on the computational errors, we optimize the numerical method to achieve the effects of high precision and short computing time. From analysis, we determine the range of best values of some key parameters for fast and accurate computation.

AMS subject classifications: 65L06, 34C55, 39B12, 82D40

Key words: Modified J-A model, numerical solution method, error estimation, optimization.

1 Introduction

As a type of functional material, the giant magnetostrictive material (GMM) can generate a field-induced deformation under external magnetic field. With the help of some necessary devices, the material can output required displacement and force [1]. The material

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has many advantages just as high output force, quick responding speed and high Curie temperature, etc. So its application has become a research hotspot [2, 3]. As a kind of ferromagnetic material, giant magnetostrictive material has the hysteresis characteristics. That is to say, the magnetization of the material changes after the magnetic field changing in the time domain. And a residual magnetization exists even the external magnetic field is changed to 0.

Micro-magnetism theory has been the most recognized method to explain the formation of magnetic hysteresis. According to the theory, magnetic domains in different shapes and sizes exist in the ferromagnetic material. The connections and interactions between these domains lead to irreversible movements of the domains. The irreversible movements cause energy losses in the process of forming magnetization, and then the magnetic hysteresis exists. In describing the hysteresis, there were two kinds of models, respectively named as physics-based and phenomenon-based models. The physics-based models considered the micro-mechanism of hysteresis process, including Stoner-Wolhfath model [4], Jiles-Atherton model [5, 6] and Smith free-energy model [7]. The phenomenon-based models, such as Preisach model [8] and neural network model [9], just considered the external behavior of the magnetization caused by the external magnetic field while not the internal mechanism.

As a commonly used hysteresis model, the Jiles-Atherton (J-A) model has been modified for many versions. These versions have different mathematical expressions in describing the magnetization. The inconsistencies are mainly in three aspects, respectively of the format of anhysteretic magnetization [10,11], energy conservation equation [12–14] and the necessary of introduced parameter δ_M [15, 16]. These uncertain citations have made these modified J-A models quite difficult to recognize and are really inconvenient for further modeling. Though almost all expressions have been proved in different references [17–20], we notice that some inaccuracies in their manuscripts could be recognized directly from physical or mathematical points of view.

Numerical methods for solving the J-A model also require careful examination. With nonlinear forms, the J-A model cannot be solved analytically. Therefore, the numerical solution is quite important for precise model or control in applying the J-A hysteresis model. However, the solution process and calculation error have rarely been studied [21,22]. In fact, solving the J-A model involves solving nonlinear algebraic equations, nonlinear differential equations and the derivative of nonlinear function, some numerical methods just like interpolation, iteration or fitting methods should be employed. If the calculating method were set unreasonably, the error of the calculated would not be acceptably low.

So the accurate modification and effective solution method, combined with the error analysis, for the J-A hysteresis model are really needed. In this paper, three uninformed points in various J-A model expressions are analyzed. Then a modified J-A model, which could accord to the physical mechanism and mathematical properties of the equations at the same time, is presented and verified. Referring to the form of modified J-A model, the fixed point method and 4^{th} order Runge-Kutta method are employed in the numer-

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ical solution method. Error estimation of presented computational method is also done and the numerical method is proved to be effectively on different materials mathematically. At last, some optimizations are done on the numerical solution method. From the optimization, different solution methods can be achieved to meet different requirements on computing precision or speed.

2 Modification of the J-A model

Five equations are introduced in J-A model to describe the relationship between the external magnetic field H, effective magnetic field H_e, anhysteretic magnetization M_{an}, irreversible magnetization M_{irr}, reversible magnetization M_{rev} and total magnetization M. Commonly recognized, the model takes modified Langevin equation to fit the anhysteretic magnetization and deduces the expression of the hysteresis from the equation of Energy Conservation. Though the based mechanism is the same, the expressions of J-A model are not uniformed in many references.

2.1 Format of anhysteretic magnetization

In describing the anhysteretic magnetization M_{an}, there are two expressions

$$M_{an} = M_s\left(\coth\frac{H+\alpha M_{an}}{a} - \frac{a}{H+\alpha M_{an}}\right), \tag{2.1a}$$

$$M_{an} = M_s\left(\coth\frac{H+\alpha M}{a} - \frac{a}{H+\alpha M}\right), \tag{2.1b}$$

where α is the internal coupling parameter, M_s is the saturation magnetization, a is the anhysteretic shape coefficient. Both equations (2.1a) [11] and (2.1b) [12–15] take the Langevin equation $L(*) = \coth(*) - 1/(*)$ while the expressions of $*$ are different. As $M_s = 0$ when $H = 0$, the $*$ of $\coth(*) - 1/(*)$ should also be 0 when $H = 0$. Simultaneously, the only zero-point of $L(*)$ meets $\lim_{*\to 0} L(*) = 0$. Apparently, Eq. (2.1a) can't meet the requirement as the total magnetization M is not 0 under $H = 0$ for the existence of residual magnetization. On the contrary, $\frac{H+\alpha M_{an}}{a}$ in Eq. (2.1a) can meet the mathematical requirement. So Eq. (2.1a) is the right expression of M_{an}.

2.2 Differential equation of energy conservation

In describing the energy conservation, the most popular differential expression [14–16] is

$$M_{irr} = M_{an} - \delta k\frac{\mathrm{d}M_{irr}}{\mathrm{d}H_e}. \tag{2.2}$$

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In this equation, the irreversible magnetostatic energy is seemed as equal to the total anhysteretic magnetostatic energy minus the hysteresis loss. From Eq. (2.2), another differential expression [10,12,13] is deduced and referenced frequently

$$\frac{\mathrm{d}M_{\mathrm{irr}}}{\mathrm{d}H}=\frac{M_{\mathrm{an}}-M_{\mathrm{irr}}}{\delta k-\alpha(M_{\mathrm{an}}-M_{\mathrm{irr}})}. \tag{2.3}$$

Simultaneously, almost all papers [10–13] have accepted the definition of the magnetic effective field H_{e}

$$H_{\mathrm{e}}=H+\alpha M. \tag{2.4}$$

However, Eq. (2.3) is derived from Eq. (2.2) under the assumption of $H_{\mathrm{e}}=H+\alpha M_{\mathrm{irr}}$ while not Eq. (2.4). That is to say, Eq. (2.3) is not equivalent to Eq. (2.2) if Eq. (2.4) is introduced, though both of them are referred to the same energy equation. It can be proved simply as follow. Take some manipulations for Eq. (2.2) and one gets

$$\frac{\mathrm{d}M_{\mathrm{irr}}}{\mathrm{d}H}=\frac{M_{\mathrm{an}}-M_{\mathrm{irr}}}{\delta k}\frac{\mathrm{d}H_{\mathrm{e}}}{\mathrm{d}H}. \tag{2.5}$$

Eq. (2.3) could be easily acquired by substituting $H_{\mathrm{e}}=H+\alpha M_{\mathrm{irr}}$ into Eq. (2.5) while not be correct under Eq. (2.4), which is quite referenced by most authors. But fortunately, computed results from Eqs. (2.2) and (2.3) are quite similar, even the same, under most conditions. And maximum error occurs only when the magnetization reversibility c is quite close to 1. Fig. 1 shows the difference between the two calculated magnetizations under $c=0.99$. Though the two equations have almost the same computing effects under most conditions, they are different indeed.

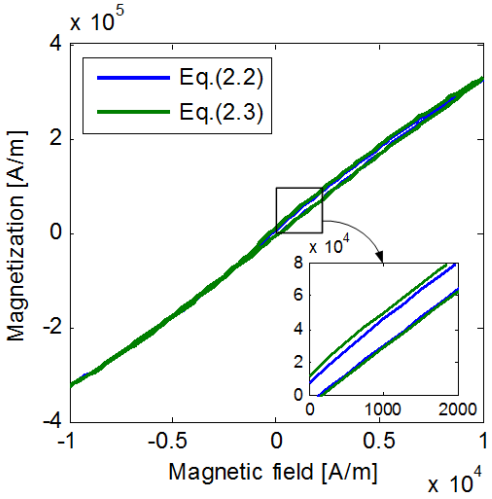


Figure 1: Computing difference between Eqs. (2.2) and (2.3).

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The authors of [11] seem to have made the mistake that Eqs. (2.2) and (2.3) were the same. However, they presented one more important point of view based on the magnetizing mechanism of ferromagnets. They pointed out that the total magnetostatic energy, caused by total magnetization M while not $M_{\rm irr}$, equals the anhysteretic magnetostatic energy minus the energy loss caused by the magnetic hysteresis. And the correct energy conservation equation is given by [11]

$$M = M_{\rm an} - \delta k \frac{{\rm d}M_{\rm irr}}{{\rm d}H_{\rm e}}. \tag{2.6}$$

Eq. (2.6) is correct in describing the magnetizing mechanism, while it is inaccurate for existence of the negative incremental susceptibility. To guarantee positive susceptibility under all magnetic fields, we shall introduce an important parameter.

2.3 Remove negative susceptibility

Another inconsistent point in literature is the introduction of the parameter $\delta_{\rm M}$. In quite a few references, $\delta_{\rm M}$ is not referred [11–14], while some papers take this parameter into consideration [15, 16]. The use of $\delta_{\rm M}$ can guarantee to avoid the unphysical stages of the J-A model for minor loops, in which incremental susceptibility becomes negative. Its value is

$$z = \begin{cases} 0, & {\rm d}H/{\rm d}t < 0 \text{ and } M_{\rm an} - M > 0, \\ 0, & {\rm d}H/{\rm d}t > 0 \text{ and } M_{\rm an} - M < 0, \\ 1, & \text{in other cases.} \end{cases} \tag{2.7}$$

The form without parameter $\delta_{\rm M}$ may lead to negative increment of magnetization. Take different amplitudes of the magnetic field H and draw the magnetizations in Fig. 2. From Fig. 2, the J-A model without $\delta_{\rm M}$ will lead to negative increments and the phenomenon will get more apparent with the magnetic field amplitude decreasing. While the model with $\delta_{\rm M}$ can guarantee positive incremental susceptibilities all the time.

With one set of parameters, computed magnetizations from the J-A model should not change its increasing trends when magnetic field amplitudes change. To satisfy this requirement, $\delta_{\rm M}$ is really needed in J-A model. Join this parameter into Eq. (2.6) (multiply by $M_{\rm an} - M$), and one gets the modified differential equation of the energy conservation

$$\frac{{\rm d}M_{\rm irr}}{{\rm d}H_{\rm e}} = \delta_{\rm M} \frac{M_{\rm an} - M}{\delta k}. \tag{2.8}$$

It should be noted that the effect of $\delta_{\rm M}$ in removing negative susceptibility is also suitable to Eqs. (2.2) and (2.3). Energy equation without considering $\delta_{\rm M}$ must lead to the negative incremental susceptibility.

2.4 Form of the modified J-A model

There are no conflicts in citing the relationship between $M_{\rm an}$, $M_{\rm irr}$, $M_{\rm rev}$ and M. Combining these equations with Eqs. (2.1a) and (2.8), the accurate expression of modified J-A

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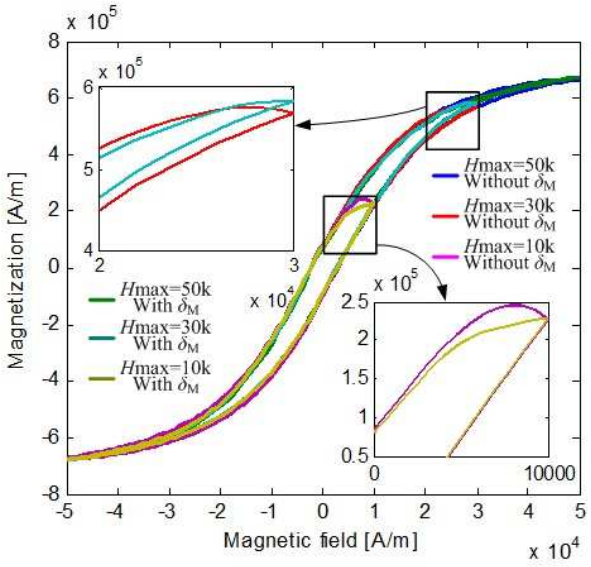


Figure 2: Effect of introducing parameter δ_{M}.

model is written as

$$\begin{cases} H_{\mathrm{e}} = H + \alpha M, \\ M_{\mathrm{s}}\left(\coth\dfrac{H+\alpha M_{\mathrm{an}}}{a} - \dfrac{a}{H+\alpha M_{\mathrm{an}}}\right), \\ M = M_{\mathrm{irr}} + M_{\mathrm{rev}}, \\ M_{\mathrm{rev}} = c(M_{\mathrm{an}} - M_{\mathrm{irr}}), \\ \dfrac{\mathrm{d}M_{\mathrm{irr}}}{\mathrm{d}H_{\mathrm{e}}} = \delta_{\mathrm{M}}\dfrac{M_{\mathrm{an}} - M}{\delta k}, \end{cases} \tag{2.9}$$

where c is the magnetization reversibility, k is the irreversible loss coefficient, δ is the sign parameter (equals to 1 when $\mathrm{d}H/\mathrm{d}t > 0$ while -1 when $\mathrm{d}H/\mathrm{d}t < 0$).

Removing the intermediate variables to solve the magnetization easily, Eq. (2.9) can be rewritten into the following two equations

$$M_{\mathrm{an}} = M_{\mathrm{s}}\left(\coth\frac{H+\alpha M_{\mathrm{an}}}{a} - \frac{a}{H+\alpha M_{\mathrm{an}}}\right), \tag{2.10}$$

$$\frac{\mathrm{d}M}{\mathrm{d}H} = \frac{\delta k c\frac{\mathrm{d}M_{\mathrm{an}}}{\mathrm{d}H} + \delta_{\mathrm{M}}(1-c)(M_{\mathrm{an}} - M)}{\delta k - \alpha\delta_{\mathrm{M}}(1-c)(M_{\mathrm{an}} - M)}. \tag{2.11}$$

In order to visualize the calculated results, the giant magnetostrictive material is taken as an example of ferromagnetic materials. The main parameters are shown in Table 1.

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Table 1: Parameters of the giant magnetostrictive material.

Parameters (symbol) [unit]	Value
Internal coupling parameter (α)	-0.001
Saturation magnetization (M_s) [A/m]	8.0×10^5
Reversible coefficient (c)	0.18
Anhysteretic shape coefficient (a) [A/m]	7800
Irreversible loss coefficient (k) [A/m]	3208

3 Solution of the modified J-A model

3.1 Solve anhysteretic magnetization Man and its susceptibility

It could be easily concluded that values of M_{an} can be determined directly by solving Eq. (2.10). For the nonlinear form, M_{an} can't be solved analytically and some numerical methods are needed. Interval partitioning method, fixed point iteration, tangent method (Newton iterative method) and secant method are commonly used numerical methods [22]. Considering the form of Eq. (2.10), the interval partitioning method is inconvenience as the method requires initial interval of M_{an} value. When numerous M_{an} values under different values of the magnetic field H are required, the initial intervals at some points can't be determined so appropriately. As the derivative information is not given, the tangent method is not suitable either. For the secant method, as two initial M_{an} values are needed at each point, the method is not so convenient at the same time, and the solving precision is difficult to compute and control. In contrast, the fixed-point iteration method may be the most appropriate method. The fixed-point iteration just needs one not so accurate initial value (but the value will affect the computing time), which is relatively effortless compared to other methods. Furthermore, the calculating errors can be deduced by using some theorems or principles, and is easily controlled by adjusting the number of iterations. So the fixed point iteration is chosen in this paper.

It should be noted that one time of fixed point iteration should be done at each value of the magnetic field H. Under each time of the iteration, H is seen as a "constant" value. Simultaneously, Initial values of M_{an} under all points are set as 0. For convenience, the iterative function could be chosen directly as the right side of Eq. (2.4). The iterative approach is

$$M_{an}^{i+1}=M_s\left(\coth\frac{H+\alpha M_{an}^i}{a}-\frac{a}{H+\alpha M_{an}^i}\right)\Big|_{M_{an}^0=0,\ i=0,1,2,\cdots},\tag{3.1}$$

where the superscript i is the iterative time, M_{an}^0 is the initial value of M_{an}, and M_{an}^{i+1} is the M_{an} value after $i+1$ iterative times. The iteration could not be executed at the point of $H=0$. As the value of M_{an} is 0, so $(0,0)$ should be added to the iterative results of M_{an}.

From Eq. (2.11), susceptibility of the anhysteretic magnetization $\mathrm{d}M_{an}/\mathrm{d}H$ is required in computing M. As the M_{an} value has been obtained, the value of $\mathrm{d}M_{an}/\mathrm{d}H$ can be calculated by some numerical differential methods directly. However, this numerical

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solution will introduce more calculating errors except for the transmission error (As the error is also in computation, the calculation error will change under computing, and the new calculation error is defined as the transmission error). To avoid this condition, $\mathrm{d}M_{\mathrm{an}}/\mathrm{d}H$ is better to obtain its analytic formula from M_{an} and then be computed by substitution. This computing method will only introduce the transmission error.

Derive two sides of Eq. (2.10) with respect to H and one gets

$$\frac{\mathrm{d}M_{\mathrm{an}}}{\mathrm{d}H}=M_{\mathrm{s}}\left[\frac{-\frac{1}{a}(1+\alpha\frac{\mathrm{d}M_{\mathrm{an}}}{\mathrm{d}H})}{\sinh^2(\frac{H+\alpha M_{\mathrm{an}}}{a})}-\frac{-a(1+\alpha\frac{\mathrm{d}M_{\mathrm{an}}}{\mathrm{d}H})}{(H+\alpha M_{\mathrm{an}})^2}\right]. \tag{3.2}$$

After some manipulations, one gets the susceptibility of the anhysteretic magnetization

$$\frac{\mathrm{d}M_{\mathrm{an}}}{\mathrm{d}H}=\frac{a^2\sinh^2(\frac{H+\alpha M_{\mathrm{an}}}{a})-(H+\alpha M_{\mathrm{an}})^2}{\frac{a}{M_{\mathrm{s}}}(H+\alpha M_{\mathrm{an}})^2\sinh^2(\frac{H+\alpha M_{\mathrm{an}}}{a})-\alpha[a^2\sinh^2(\frac{H+\alpha M_{\mathrm{an}}}{a})-(H+\alpha M_{\mathrm{an}})^2]}. \tag{3.3}$$

Eq. (3.3) is not suitable under the condition of $H=0$, $M_{\mathrm{an}}=0$. And the value of $\mathrm{d}M_{\mathrm{an}}/\mathrm{d}H$ at $H=0$ is complemented by its limit value. From computed results in the Appendix, the limit is equal to

$$\left.\frac{\mathrm{d}M_{\mathrm{an}}}{\mathrm{d}H}\right|_{H=0}\triangleq\lim_{H\to 0}\frac{\mathrm{d}M_{\mathrm{an}}}{\mathrm{d}H}=\frac{1}{\frac{3a}{M_{\mathrm{s}}}-\alpha}. \tag{3.4}$$

3.2 Solve the total magnetization

To solve the nonlinear ordinary differential equation of Eq. (2.11), the 4th order Runge-Kutta method could be employed. Imposing $\varphi(H,M)=\frac{\delta kc\frac{\mathrm{d}M_{\mathrm{an}}}{\mathrm{d}H}+\delta_{\mathrm{M}}(1-c)(M_{\mathrm{an}}-M)}{\delta k-\alpha\delta_{\mathrm{M}}(1-c)(M_{\mathrm{an}}-M)}$, the computing approach is

$$\begin{cases} M_{n+1}=M_n+\frac{h}{6}(k_1+2k_2+2k_3+k_4),\\ k_1=\varphi(H_n,M_n),\\ k_2=\varphi(H_n+\frac{h}{2},M_n+\frac{1}{2}hk_1),\\ k_3=\varphi(H_n+\frac{h}{2},M_n+\frac{1}{2}hk_2),\\ k_4=\varphi(H_n+h,M_n+hk_3), \end{cases} \tag{3.5}$$

where the subscript n is a label of the calculating point, h is the computing step-size of H, M_{n+1} and H_{n+1} are respectively the values of M and H at the $n+1$ times of computing.

To reduce computing errors, the step-size h should be designed carefully. Computing M requires the information of M_{an} and $\mathrm{d}M_{\mathrm{an}}/\mathrm{d}H$, and the ideal computing method is calculating M value by known points directly while not introducing some interpolation methods. That is because the interpolation method used to compute the unknown values of M_{an} and $\mathrm{d}M_{\mathrm{an}}/\mathrm{d}H$ must lead to higher computing errors. From Eq. (3.5), calculating distance of magnetic field in computing the total magnetization M is $h/2$. To avoid introducing unknown values of M_{an} and $\mathrm{d}M_{\mathrm{an}}/\mathrm{d}H$, $h/2$ should be integer numbers as high

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as, at least be equal to, the iterative distance of H in computing M_{an}. That is to say, computing step size h is better to be set as even numbers as high as the iterative distance in Eq. (3.1).

3.3 Computing process

The initial magnetization is calculated firstly with the first value $M_0 = 0$ $(H = 0)$. Then points on fall curve are computed subsequently and the initial point is exactly the last computing result from initial magnetization. Similarly, the initial magnetization of rise curve is precisely the last magnetization in fall curve. The rise and fall curves will change until the steady-state computation is reached. Under the steady state condition, the initial and final values of the fall curve are respectively the final and initial values of the rise curve. And the last two curves are selected as the total magnetization. An example of computed results is shown in Fig. 3.

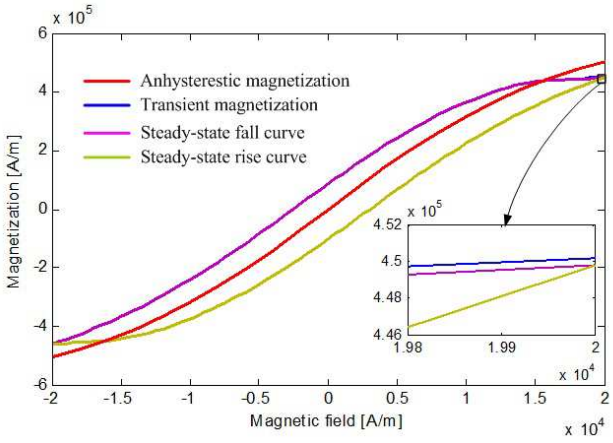


Figure 3: Calculated magnetizations.

4 Estimation of the computational error

Computational errors of the presented numerical method should be estimated to verify the solution mathematically. In computing anhysteretic magnetization M_{an}, calculation error is always lower than the iterative error. Then the computational error will be transferred to be the transmission error in calculating $\mathrm{d}M_{\text{an}}/\mathrm{d}H$. These errors will also be transmitted in calculating the total magnetization M.

4.1 Transmission error in solving $\mathrm{d}M_{\text{an}}/\mathrm{d}H$

Calculation error of computing M_{an} gets its maximum value of the iterative error which is labeled as ΔM_{an}. Supposing $\Delta\frac{\mathrm{d}M_{\text{an}}}{\mathrm{d}H}$ is the error in computing $\frac{\mathrm{d}M_{\text{an}}}{\mathrm{d}H}$, the expression of

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can be calculated using error transferring formula from Eq. (3.3)

$$\Delta\frac{\mathrm{d}M_{\rm an}}{\mathrm{d}H}\approx\frac{\partial\frac{\mathrm{d}M_{\rm an}}{\mathrm{d}H}}{\partial M_{\rm an}}\Delta M_{\rm an}=\frac{2x^4\sinh x\cosh x-2x\sinh^4 x}{[\frac{a}{M_{\rm s}}x^2\sinh^4 x-\alpha(\sinh^2 x-x^2)]^2}\frac{\alpha}{a}\Delta M_{\rm an}, \tag{4.1}$$

where $x=(H+\alpha M_{\rm an})/a$. Also, the transmission error $\Delta\frac{\mathrm{d}M_{\rm an}}{\mathrm{d}H}$ at $H=0$ is also 0. After calculation, $\Delta\frac{\mathrm{d}M_{\rm an}}{\mathrm{d}H}$ is quite less than the iterative error $\Delta M_{\rm an}$, about 10^{-4} times of $\Delta M_{\rm an}$.

4.2 Errors in computing M

In computing the total magnetization M, the $4^{\rm th}$ order Runge-Kutta method will introduce some local truncation errors. Three kinds of errors, respectively as the iterative error in calculating anhysterestic magnetization $\Delta M_{\rm an}$, transmission error in computing susceptibility of anhysterestic magnetization $\Delta\frac{\mathrm{d}M_{\rm an}}{\mathrm{d}H}$ and the local truncation error, will be transmitted when computing M.

4.2.1 Local truncation error

The local truncation error is caused using the $4^{\rm th}$ order Runge-Kutta method in calculating M. As the calculated magnetization curves are sufficiently smooth functions, from Taylor's expansion one gets

$$M(H+h)=M(H)+\sum_{n=1}^{4}\frac{h^n}{n!}M^{(n)}(H)+\frac{h^5}{5!}M^{(5)}(\xi), \tag{4.2}$$

where ξ is a point in the interval $(H,H+h)$. As the employed method is a single-step computing method, the calculating error from x_n to x_{n+1}, without considering previous errors, is $h^5M^{(5)}(\xi)/5!$. Supposing E as the maximum value of $M^{(5)}(H)$ in interval $(H,H+h)$, the error maximum is $h^5E/5!$. And maximum value of the local truncation error $\Delta_1 M$ can be written by

$$\Delta_1 M=\frac{h^5}{5!}E. \tag{4.3}$$

Use "diff" function in MATLAB [23] in solving the 5^{th} order derivative of M with respect to H, then the local truncation error can be roughly determined.

4.2.2 Transmission error

Imposing y the function of n independent variables $x_1,x_2,\cdots,x_n$, when the increments of all independent variables are little, one gets the function increments

$$\Delta y\approx\frac{\partial y}{\partial x_1}\Delta x_1+\frac{\partial y}{\partial x_2}\Delta x_2+\cdots+\frac{\partial y}{\partial x_n}\Delta x_n. \tag{4.4}$$

This increment function is also the error transfer formula, when the computing errors of independent variables are quite little. Δx_i represents the computational error of x_i and

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Δy represents the computational error of the functional variable y. From Eq. (3.5), k_1 is seen as the function of independent variables of M, $M_{\rm an}$ and $\mathrm{d}M_{\rm an}/\mathrm{d}H$ and then Δk_1 is calculated by

$$\begin{aligned}\Delta k_1 &\approx \frac{\partial\varphi(H,M)}{\partial(\mathrm{d}M_{\rm an}/\mathrm{d}H)}\cdot\Delta\frac{\mathrm{d}M_{\rm an}}{\mathrm{d}H}+\frac{\partial\varphi(H,M)}{\partial M_{\rm an}}\cdot\Delta M_{\rm an}+\frac{\partial\varphi(H,M)}{\partial M}\cdot\Delta M\\ &=\frac{\delta kc}{\delta k-\alpha\delta_{\rm M}(1-c)(M_{\rm an}-M)}\Delta\frac{\mathrm{d}M_{\rm an}}{\mathrm{d}H}+\frac{(1-c)\delta k(1+\alpha c\frac{\mathrm{d}M_{\rm an}}{\mathrm{d}H})}{[\delta k-\alpha\delta_{\rm M}(1-c)(M_{\rm an}-M)]^2}(\Delta M_{\rm an}-\Delta M).\end{aligned} \tag{4.5}$$

Then k_2 could be seen as the function of M, $M_{\rm an}$, $\mathrm{d}M_{\rm an}/\mathrm{d}H$ and k_1, and the expression of Δk_2 is

$$\begin{aligned}\Delta k_2 &\approx \frac{\partial\varphi(H+\frac{h}{2},M+\frac{hk_1}{2})}{\partial(\mathrm{d}M_{\rm an}/\mathrm{d}H)}\cdot\Delta\frac{\mathrm{d}M_{\rm an}}{\mathrm{d}H}+\frac{\partial\varphi(H+\frac{h}{2},M+\frac{hk_1}{2})}{\partial M_{\rm an}}\cdot\Delta M_{\rm an}\\ &\quad+\frac{\partial\varphi(H+\frac{h}{2},M+\frac{hk_1}{2})}{\partial M}\cdot\Delta M+\frac{\partial\varphi(H+\frac{h}{2},M+\frac{hk_1}{2})}{\partial k_1}\cdot\Delta k_1\\ &=\frac{\delta kc}{\delta k-\alpha\delta_{\rm M}(1-c)(M_{\rm an}-M-\frac{hk_1}{2})}\Delta\frac{\mathrm{d}M_{\rm an}}{\mathrm{d}H}\\ &\quad+\frac{(1-c)\delta k(1+\alpha c\frac{\mathrm{d}M_{\rm an}}{\mathrm{d}H})}{[\delta k-\alpha\delta_{\rm M}(1-c)(M_{\rm an}-M-\frac{hk_1}{2})]^2}\Big(\Delta M_{\rm an}-\Delta M-\frac{h}{2}\Delta k_1\Big).\end{aligned} \tag{4.6}$$

In the same way, Δk_3 and Δk_4 could be obtained as

$$\begin{aligned}\Delta k_3 &\approx \frac{\delta kc}{\delta k-\alpha\delta_{\rm M}(1-c)(M_{\rm an}-M-\frac{hk_2}{2})}\Delta\frac{\mathrm{d}M_{\rm an}}{\mathrm{d}H}\\ &\quad+\frac{(1-c)\delta k(1+\alpha c\frac{\mathrm{d}M_{\rm an}}{\mathrm{d}H})}{[\delta k-\alpha\delta_{\rm M}(1-c)(M_{\rm an}-M-\frac{hk_2}{2})]^2}\Big(\Delta M_{\rm an}-\Delta M-\frac{h}{2}\Delta k_2\Big),\end{aligned} \tag{4.7}$$

$$\begin{aligned}\Delta k_4 &\approx \frac{\delta kc}{\delta k-\alpha\delta_{\rm M}(1-c)(M_{\rm an}-M-\frac{hk_3}{2})}\Delta\frac{\mathrm{d}M_{\rm an}}{\mathrm{d}H}\end{aligned} \tag{4.8}$$

$$\begin{aligned}&\quad+\frac{(1-c)\delta k(1+\alpha c\frac{\mathrm{d}M_{\rm an}}{\mathrm{d}H})}{[\delta k-\alpha\delta_{\rm M}(1-c)(M_{\rm an}-M-\frac{hk_3}{2})]^2}(\Delta M_{\rm an}-\Delta M-h\Delta k_3).\end{aligned} \tag{4.9}$$

At last, M_{n+1} is seen as the function of M_n, Δk_1, Δk_2, Δk_3, Δk_4, and the transmission error $\Delta_2 M_{n+1}$ calculated from ΔM_n is

$$\Delta_2 M_{n+1}=\Delta M_n+\frac{h}{6}(\Delta k_1+2\Delta k_2+2\Delta k_3+\Delta k_4)\big|_{M=M_n}, \tag{4.10}$$

where ΔM_n represents the transmission error of each computed point.

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4.2.3 Total error and visualization

It can be concluded from Eq. (3.5) that the error truncation is done after the transmission, and so the two kinds of errors should be superimposed linearly. As the two errors are respectively the increment and decrement with respect to exact results, the total error ΔM is written as

$$\Delta M = \Delta_2 M - \Delta_1 M. \tag{4.11}$$

Concluding above results, the three kinds of errors in computing the magnetization, shown in Eqs. (4.3), (4.10) and (4.11) respectively, are drawn in Fig. 4. Compared with the amplitude of magnetization, the computational errors are quite low. That is to say, the numerical computing method, using fixed-point to solve M_{an}, computing $\mathrm{d}M_{\text{an}}/\mathrm{d}H$ analytically and reaching M values through the 4^{th} order Runge-Kutta method, has very low errors. So the numerical solving method is verified to be quite accurate.

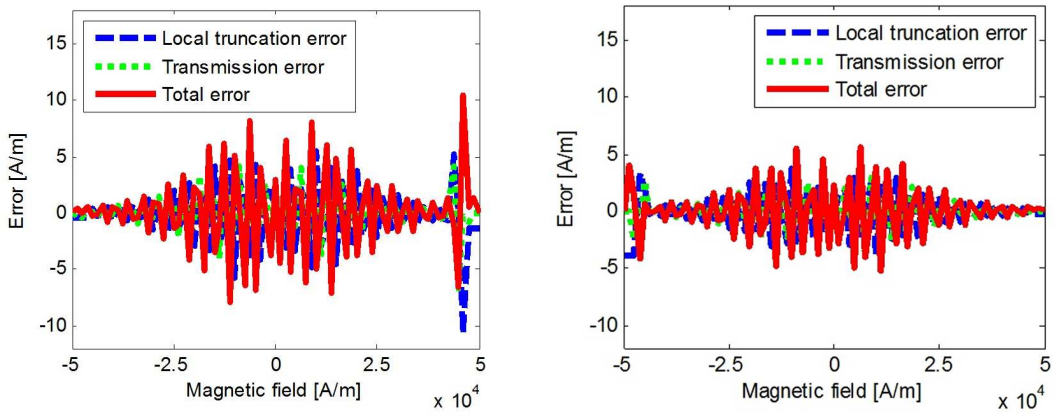


Figure 4: Three kinds of magnetization errors: (a) calculation errors of the fall curve; (b) calculation errors of the rise curve.

4.3 Errors under different parameters

Fig. 4 just gives the computational errors under one set of parameter, which represents the example of giant magnetostrictive material. More materials should be considered to verify the numerical computing method. We have done many computations and found that the error's trends under different parameters are similar while the error amplitudes change. So take the maximum value of the absolute value of the total errors (including rise and fall curves) as the indicator of describing computing precision.

There are four parameters of a, α, c and k. As the parameter a influences only the precision of computing M_{an} which can be controlled by adjusting the iterative precision, the effect of a on computing M is not necessary to be discussed. Change the other three

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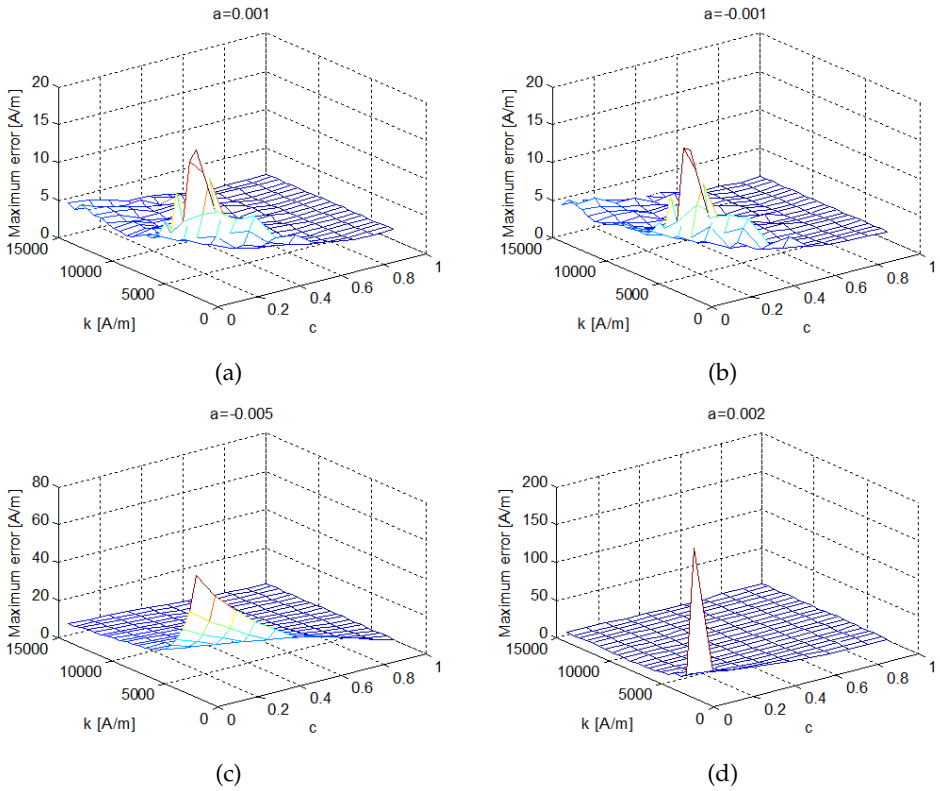


Figure 5: Calculation errors under different parameters: (a) α=-0.005; (b) α=-0.001; (c) α=0.001; (d) α=-0.002.

parameters to simulate different materials and draw the maximum errors in Fig. 5. From computational results with various parameters, total errors of the numerical computing method are really low. So the presented numerical method is proved to be effective in calculating magnetization of different materials.

This action also gives the parameter design technique for modified J-A model. From computation, higher errors occur with lower k and higher α. When the two parameters of k and α are set unreasonably, calculated magnetization curves are not smooth any more, which means that the modified J-A model is not applicable under these conditions. From our error estimations, to introduce modified J-A model in describing magnetization of magnetic materials, the value of k is better higher than 1/3 of the value of a, and the value of parameter α is better less than 0.0025.

5 Optimization of the solution method

The solving method proposed in Section 3 has quite high precision while really long computing time. For magnetic materials or devices, J-A hysteresis model is always just

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one sub-model of the overall model. Such long operating time will extend the whole calculating time severely. In addition, sometimes the calculating accuracy is not required so high as other sub-models of the whole model may produce quite higher errors than the J-A model. So, computing methods are better to reach different precisions and computing time to meet various requirements. As solving the $\mathrm{d}M_{\mathrm{an}}/\mathrm{d}H$ and M is quick while the iteration spends really long time, the speed optimization is concentrated mainly on the process of solving M_{an}.

5.1 Iterative acceleration

To reduce the iterating time on computing M_{an}, some acceleration methods or initial value optimization can be introduced. Two commonly used convergence acceleration methods are Aitken and Steffensen accelerations [21]. Both methods adjust the original solutions to approach the exact solution more quickly and their iterating laws are respectively shown in Eqs. (5.1) and (5.2).

$$\widehat{M}_{\mathrm{an}}^{i+1}=M_{\mathrm{an}}^{i}-\frac{(M_{\mathrm{an}}^{i+1}-M_{\mathrm{an}}^{i})^2}{M_{\mathrm{an}}^{i+2}-2M_{\mathrm{an}}^{i+1}+M_{\mathrm{an}}^{i}}, \tag{5.1}$$

$$\begin{cases} Y_{\mathrm{an}}^{i+1}=M_{\mathrm{s}}\left(\coth\dfrac{H+\alpha M_{\mathrm{an}}^{i}}{a}-\dfrac{a}{H+\alpha M_{\mathrm{an}}^{i}}\right), \\ Z_{\mathrm{an}}^{i+1}=M_{\mathrm{s}}\left(\coth\dfrac{H+\alpha Y_{\mathrm{an}}^{i}}{a}-\dfrac{a}{H+\alpha Y_{\mathrm{an}}^{i}}\right), \\ M_{\mathrm{an}}^{i+1}=M_{\mathrm{an}}^{i}-\dfrac{(Y_{\mathrm{an}}^{i}-M_{\mathrm{an}}^{i})^2}{Z_{\mathrm{an}}^{i}-2Y_{\mathrm{an}}^{i}+M_{\mathrm{an}}^{i}}. \end{cases} \tag{5.2}$$

Another approach to accelerating the iteration is optimizing the initial value. Without changing the iterative precision, the iterating initial value can affect the iterative times to a certain extent. The closer to the exact solution of the initial value, the less computing time will be taken. So a reasonable initial value is effective for reducing the computing time.

In Section 3.1, all initial values of M_{an} in iterations are set as 0. To approach exact solutions, initial values of the linear form K_1H and tanh form $K_2\tanh(H/K_3)$ are introduced to make contrasts. Parameters in these expressions can be determined roughly through substituting several points. Three kinds of initial value curves are shown in Fig. 6. Exerting different acceleration methods and initial values on the iterative methods, iterative numbers and time are shown in Table 2. Through computation, taking initial values in tanh form and Aitken acceleration can acquire fastest computing speed.

5.2 Computation under different iterative errors

The iterative error ΔM_{an} has direct effect on the computing precision and time. Iterative errors in Sections 3 and 4 are set as 10^{-6} A/m, and the computation reaches high precision

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Table 2: Computing time under different conditions.

	Initial formats	0	linear	tanh
without acceleration	Step number	1443	1395	1289
	Computing time[s]	380.32	371.92	350.86
Aitken acceleration	Step number	949	915	851
	Computing time[s]	322.78	292.02	272.08
Steffensen acceleration	Step number	601	601	599
	Computing time[s]	364.03	356.24	325.13

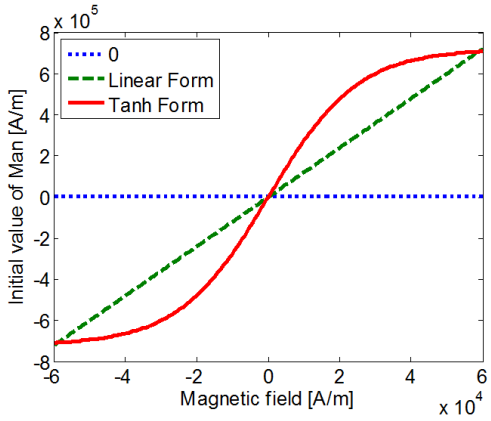


Figure 6: Different forms of the initial values.

while slow computing speed. Increase ΔM_{an} from 10^{-6} A/m to 100 A/m, maximum errors in computing M are shown in Fig. 7. It is apparent that computational maximum error increases with ΔM_{an} increases. The growth rate of the maximum error is quite high when ΔM_{an} is higher than 10 A/m, so ΔM_{an} is best to be set less than 10 A/m to guarantee low computational error.

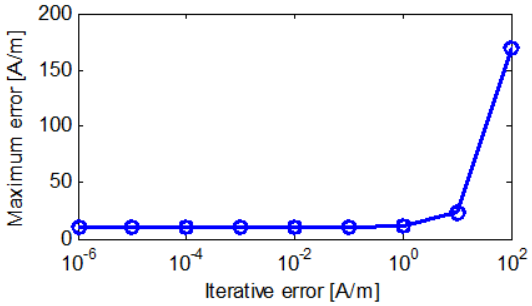


Figure 7: Maximum errors under different iterative errors.

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Iterative error can also change the computational time through increasing or decreasing iterative numbers. Imposing the value of ΔM_{an} higher than 1 A/m and introducing the initial value in tanh form, from our computations, the iterative number changes to 0. That is to say, the initial value can meet the precision requirement and then the iterative operation is no need anymore. This condition gives the chance of predicting M_{an} by a fitting function in tanh form, which will spend little time. So, if less computing time were more required and the precision were not needed so strictly, M_{an} value can be directly fitted as the function of $K_2\tanh(H/K_3)$ with spending almost no time.

5.3 Computation under different step-sizes

The value of step-size h also has influences on the calculating precision. Reasonable value of h is proved to be even numbers of the iterative error ΔM_{an}. When $h=2\Delta M_{an}$, highest numbers of points are acquired directly through 4^{th} order Runge-Kutta method. Another method is also widely used that selecting higher h value to obtain less magnetization points and then computing other points through Spline Interpolation method. However, the computational results from interpolation method have really deviations with the ones without interpolation.

Take $h/\Delta M_{an}$ respectively as 2, 4, 6, 8, 10 and draw the maximum deviations, compared to the magnetization under $h/\Delta M_{an}=2$, in Fig. 8. It can be indicated that deviations get higher with the ratio (also the value of h) increasing. When the ratio is higher than 8, the deviation gets higher than 1 kA/m. As the computational error under $h=2\Delta M_{an}$ has been proved to be quite low (the maximum value is less than 12 A/m) in Section 4, the calculated error will be quite high with high value of $h/\Delta M_{an}$. So, for accurate computation, the ratio of $h/\Delta M_{an}$ is better to be lower than 6.

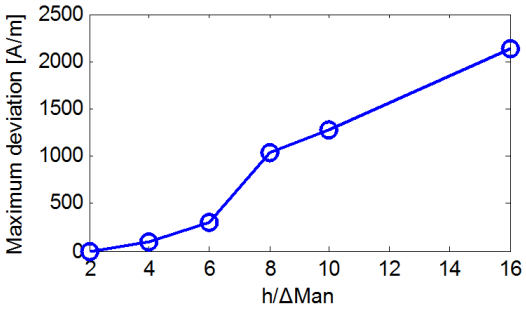


Figure 8: Computing deviations under different values of $h/\Delta M_{an}$.

6 Conclusions

Referring to the uncertain citations of the J-A hysteresis model, traditional model is modified in three aspects. Firstly, determine the form of the anhysteretic magnetization from

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the property of Langevin function. Secondly, rewrite the Energy Conservation expression from the physical point of view. At last, parameter δ_{M} is proved to be necessary to remove negative incremental susceptibilities.

A numerical solution method for the modified model is presented and its accuracy is verified with the help of error estimating. The anhysteretic magnetization is calculated by the fixed-point iterative method and its derivative with respect to magnetic field is analytically deduced. The total magnetization is obtained by the 4^{th} Runge-Kutta method. The solving method is verified in describing magnetic hysteresis of different materials as the computational errors are proved to be quite low.

The solving method is optimized through analyzing the effects of acceleration methods, iterative error and step size on the computation. To save computing time, we introduce the Aitken acceleration method combined with initial value in tanh form in the iterating process of solving anhysteretic magnetization. In addition, function in tanh form could be seen as the fitting function of the anhysteretic magnetization directly if short computing time were quite needed and the precision are not so strictly required.

Design techniques of the iterative error and step size are also reached. To guarantee low computational errors, the value of ΔM_{an} is best to be set less than 10 A/m and the ratio of $h/\Delta M_{\mathrm{an}}$ is better to be even number lower than 6.

Acknowledgments

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Appendix

The computing process of $\Delta\frac{\mathrm{d}M_{\mathrm{an}}}{\mathrm{d}H}|_{H=0}$ is shown as follow.

Supposing $x=(H+\alpha M_{\mathrm{an}})/a$, one gets

$$\frac{\mathrm{d}M_{\mathrm{an}}}{\mathrm{d}H}=\frac{a^2\sinh^2 x-(ax)^2}{\frac{a}{M_{\mathrm{s}}}(ax)^2\sinh^2(x)-\alpha[a^2\sinh^2(x)-(ax)^2]}=\frac{\sinh^2 x-x^2}{\frac{a}{M_{\mathrm{s}}}x^2\sinh^2(x)-\alpha[\sinh^2(x)-x^2]}.$$

Derivatives of the hyperbolic functions are:

$$\begin{cases}(\sinh x)'=\cosh x,\ (\sinh^2 x)'=2\sinh x\cosh x,\ (\sinh x\cosh x)'=\cosh^2 x+\sinh^2 x,\\ (\cosh x)'=\sinh x,\ (\cosh^2 x)'=2\sinh x\cosh x,\ (\sinh^2 x+\cosh^2 x)'=4\sinh x\cosh x.\end{cases}$$

And then

$$\begin{aligned}(\sinh^2 x-x^2)^{(4)}&=(2\sinh x\cosh x-2x)^{(3)}\\&=[2(\sinh^2 x+\cosh^2 x)-2]''\\&=(8\sinh x\cosh x)'\\&=8(\sinh^2 x+\cosh^2 x),\end{aligned}$$

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$$\begin{aligned}(x^2\sinh^2 x)^{(4)} &= (2x\sinh^2 x+2x^2\sinh x\cosh x)^{(3)}\\ &= [2\sinh^2 x+8x\sinh x\cosh x+2x^2(\sinh^2 x+\cosh^2 x)]''\\ &= [12\sinh x\cosh x+12x(\sinh^2 x+\cosh^2 x)+8x^2\sinh x\cosh x]'\\ &= 24(\sinh^2 x+\cosh^2 x)+64x\sinh x\cosh x+8x^2(\sinh^2 x+\cosh^2 x).\end{aligned}$$

Therefore,

$$\begin{aligned}&\lim_{x\to 0}(\sinh^2 x-x^2)^{(4)} = \lim_{x\to 0} 8(\sinh^2 x+\cosh^2 x) = 8,\\ &\lim_{x\to 0}(x^2\sinh^2 x)^{(4)} = \lim_{x\to 0}[24(\sinh^2 x+\cosh^2 x)+0+0] = 24.\end{aligned}$$

At last, one gets

$$\frac{\mathrm{d}M_{\mathrm{an}}}{\mathrm{d}H}|_{H=0} = \lim_{x\to 0}\frac{(\sinh^2 x-x^2)^{(4)}}{[\frac{a}{M_{\mathrm{s}}}x^2\sinh^2 x-\alpha(\sinh^2 x-x^2)]^{(4)}} = \frac{8}{24\frac{a}{M_{\mathrm{s}}}-8\alpha} = \frac{1}{\frac{3a}{M_{\mathrm{s}}}-\alpha}.$$

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