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# QUANTILE-BASED ASYMMETRIC DYNAMICS OF REAL GDP GROWTH

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This paper studies asymmetric dynamics of real GDP growth by estimating linear and nonlinear quantile persistence over different parts of the conditional distribution for six major developed economies. Several novel quantile-based hypotheses are motivated in this paper and tested for the steepness asymmetry of real GDP growth that hypothesizes that contractions are steeper than expansions. The empirical results show that quantile persistence is generally high at far lower tails, thus requiring much longer half-lives to reverting negative deviations to the mean of real GDP growth and hence leading to gradual economic recoveries. By contrast, less persistence in far upper tails tends to generate sharp and short economic downturns that adjust positive deviations towards the mean of real GDP growth so as to cause abrupt economic recessions. In particular, this asymmetry in quantile persistence strongly supports the steepness asymmetry conjecture, robust to the presence of structural breaks and potential nonlinearities in real GDP growth.

**Keywords:** Distributional Persistence, Steepness Asymmetry, Half-lives, Quantile Unit Root Test, Threshold Quantile Autoregression

## 1. INTRODUCTION

When a boom ends, the downturn is generally sharp and short. When growth resumes, the recovery is more gradual because adjustment is costly and hence protracted due to endogenous delay in investments [Gale (1996) and Nieuwerburgh and Veldkamp (2006)]. In an influential paper, Sichel (1993) has labeled this cyclical asymmetry as "steepness," characterized by sharply falling contractions and gradual expansions. This type of asymmetry is fundamental in both theoretical and empirical characterizations of an economy [Ramsey and Rothman (1996) and Psaradakis and Sola (2003)].

Contrary to the widespread belief, little evidence has been found in the literature for the steepness asymmetry of real output data. Applying Neftci's (1984) second-order Markov procedure, Falk (1986) has failed to find any support for asymmetry in the US real output series. While, by deriving necessary and sufficient moment conditions, Clements and Krolzig (2003) have shown significant steepness of expansions, their result is contrary to the conventional hypothesis of steepness of contractions in real output series. Using the moment-based approach

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of Sichel (1993), many studies have not yet found significant steepness for real output series.[1]

On the other hand, a number of studies have found the different behavior of the economy during expansions and contractions based on the Hamilton (1989) type of Markov-switching models.[2] In addition, Beaudry and Koop (1993) find that negative innovations in the US real output are much less persistent than positive ones, thus suggesting asymmetry in persistence as one source of asymmetric dynamics.[3] By contrast, Hess and Iwata (1997) report weak asymmetric persistence in the US real GDP data and mixed results for international data.

Different from previous studies which exclusively focus on asymmetry in conditional mean of real output series, this paper examines asymmetric dynamics of real GDP growth by estimating linear and nonlinear quantile persistence over different parts of the conditional distribution. The quantile-based approach has several important statistical properties that circumvent the issues in the conventional mean analysis. First, quantile persistence allows for different and potentially asymmetric speed of adjustment across different locations of the data distribution [Koenker and Xiao (2006)]. In contrast to the constant speed adjustment assumed in previous studies,[4] this quantile feature reproduces the fact that real output following shocks drawn from the tails of the shock distribution might exhibit a different pattern of persistence than real output close to the mean.[5] Second, the quantile unit root test of Koenker and Xiao (2004) has better test power in non-Gaussian environments than the existing unit root tests for conditional mean. Third, quantile autoregression allows for unit-root-like tendencies or even temporarily explosive behavior [*Corollary 1* of Koenker and Xiao (2006)]. Moreover, the quantile approach is robust to outliers with bounded influential functions and free of distribution assumptions [Koenker (2005)]. The moment-based test of Sichel (1993) is very sensitive to outliers and tends to produce arbitrarily large values with least meaningful interpretations. These advantages are also reserved for nonlinear quantile autoregressions.

As the first paper to investigate asymmetric dynamics of real GDP growth through regression quantiles,[6] I offer three major contributions. First, this paper proposes an alternative inference for steepness based on the tests for asymmetric quantile persistence. Consider the case where real GDP falls in its deep lower tails, that is, 5% or lower quantiles, due to extreme adverse shocks that cause large negative deviations from its long-run mean. In this case, the persistence of the deep lower tails precisely represents the adjusting speed of resuming economic growth from recessions towards the long-run mean (the process of economic recovery or expansion). Similarly, the persistence of the high upper tails reflects the adjusting speed of real GDP from economic booms (due to large positive shocks) towards the long-run mean. Testing asymmetric persistence for extreme lower and upper tails can thus be used to assess the potential steepness asymmetry of real GDP growth.

Second, I propose the novel quantile-based hypotheses testable for asymmetric quantile persistence and explicitly linked to steepness asymmetry. Specifically,

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volatility of real GDP growth is assumed a measurable function of the past of real GDP growth so that quantile persistence is affected by error variances of real GDP growth. The main asymmetric hypothesis predicts that, if it increased real GDP volatility, a negative deviation from the mean would produce much higher quantile persistence at the deep lower tails than the corresponding high upper tails. The simulation result based on this hypothesis shows much longer time to reverting the negative deviation to the mean than a positive shock of the same magnitude, and hence economic recovery would be gradual. However, if a (possibly large) positive deviation raised real GDP growth volatility, the hypothesis would predict high quantile persistence in its high upper tails. In this regard, the simulation result shows that the persistence over different quantiles of real GDP growth would be more symmetric. This result partly explains the mixed findings in Hess and Iwata (1997) and Elwood (1998). In addition, round peaks could also be observed due to the high upper tail data persistence, justifying to some extent the empirical finding in McQueen and Thorley (1993) that business cycles have sharp troughs and round peaks.

Third and importantly, the empirical tests in this paper strongly support asymmetric quantile persistence for real GDP growths of six major developed economies. In particular, the main hypothesis is supported by real GDP growths of the USA, Canada, Germany, France, and European Union (with 28 countries) which have much higher persistence at quantiles lower than 10% than at quantiles higher than 90%. By contrast, the quantile persistence of the UK data appears to be more symmetric than that of other economies, as the UK quantile persistence is high in both extreme lower and upper tails. Moreover, these results are robust to sequentially tested structural breaks and potential nonlinearities modeled by threshold quantile autoregression. Consequently, international evidence of steepness asymmetry of real GDP growth has been found significant in this study.

The rest of this paper is organized as follows. Section 2 motivates quantile-based asymmetric dynamics of real GDP growth. Section 3 proposes several novel asymmetric hypotheses which are testable. A simulation illustrates insightful implications for the steepness asymmetry of real GDP growth. Section 4 reports empirical evidence obtained from six developed economies. Section 5 concludes the paper.

## 2. MOTIVATION

Consider real GDP growth rate denoted by $y_t$ at time $t$. For the purpose of this paper, I focus on the persistence of real GDP growth modeled in an autoregressive model as

$$y_t = \alpha + \rho y_{t-1} + \sum_{j=1}^{q} \beta_j \triangle y_{t-j} + \sigma \varepsilon_t, \tag{1}$$

where $\varepsilon_t$ is an i.i.d. random variable with zero mean and unit variance, and $\sigma^2$ is a homogeneous variance. Large values of $\rho$ represent high persistence of

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conditional mean of $y_t$. In the limiting case where $\rho = 1$, $y_t$ behaves as a unit root and explosive for $\rho > 1$.

The measure of persistence in (1) focuses on dynamic characteristics of real GDP growth at the center of its distribution, and thus neglects the possibility that these features might vary in other parts of the distribution of $y_t$. To see this issue, one can obtain a quantile autoregressive (QAR) process implied by (1) as

$$Q_{y_t}(\tau|\mathcal{F}_{t-1}) = \alpha(\tau) + \rho(\tau)y_{t-1} + \sum_{j=1}^{q} \beta_j(\tau)\triangle y_{t-j}, \tag{2}$$

where, for $\tau \in (0,1)$, $\alpha(\tau) = \alpha + \sigma Q_{\varepsilon_t}(\tau)$, $\rho(\tau) = \rho$ and $\beta_j(\tau) = \beta_j$ for $j = 1, \ldots, q$.[7] $Q_{y_t}(\tau|\mathcal{F}_{t-1})$ represents the $\tau$th-quantile of $y_t$ conditional on information $\mathcal{F}_{t-1}$ available at time $t-1$, and $Q_{\varepsilon_t}(\tau)$ is the theoretical $\tau$th-quantile of a distribution assumed for $\varepsilon_t$. It is easy to see from (2) that conditional mean autoregressive models imply location quantile processes where effects of conditioning variables on a dependent variable are independent of $\tau$. Therefore, $\rho(\tau) = \rho$ in (2) is constant over the distribution of $y_t$, and hence it cannot represent quantile persistence of $y_t$ over $\tau$.

To address this issue, one can relax the restriction of homogeneous variance in (1) by assuming that $\varepsilon_t$ has time-varying conditional variances, denoted by $\sigma_t^2$. For instance, assume that conditional volatility of real GDP growth is functions of its past, that is, $\sigma_t = exp\left(a_0 + a_1 y_{t-1} + \sum_{j=1}^{q} b_j \triangle y_{t-j}\right)$.[8] Then, one can apply the first-order Taylor expansion evaluated at the long-run mean of real GDP growth, $\mu = E(y_t)$, to linearize the conditional standard deviation as

$$\sigma_t = \mu_0 \left[1 + a_1(y_{t-1} - \mu) + \sum_{j=1}^{q} b_j \triangle y_{t-j}\right], \tag{3}$$

where $\mu_0 = exp(a_0 + a_1\mu)$. Further, (3) can be substituted into the heteroskedastic version of (1) to yield the $\tau$th-quantile of $y_t$ as expressed in (2), with $\alpha(\tau) = \alpha + (1 - a_1\mu)\mu_0 Q_{\varepsilon_t}(\tau)$, $\rho(\tau) = \rho + a_1 Q_{\varepsilon_t}(\tau)\mu_0$, and $\beta_j(\tau) = \beta_j + b_j Q_{\varepsilon_t}(\tau)\mu_0$ for $j = 1, \ldots, q$.

The result shows that quantile persistence, $\rho(\tau)$, of $y_t$ depends on $\tau$, which generalizes conditional mean persistence, $\rho$, over $\tau$. If one assumes a symmetric distribution, that is, the standard normal distribution, for $\varepsilon_t$, then $\rho(\tau = 0.5) = \rho$ due to $Q_{\varepsilon_t}(\tau = 0.5) = 0$. To this end, asymmetry in persistence can be tested for $\rho(\tau)$ by the null hypothesis, $H_0 : \rho(\tau) = \rho(\tau')$, against the alternative, $H_1 : \rho(\tau) \neq \rho(\tau')$, for any $\tau, \tau' \in (0,1)$ and $\tau \neq \tau'$. In this regard, persistence behaviors of $y_t$ can be investigated by taking various values of $\tau \in (0,1)$ over different parts of the distribution of $y_t$, while conventional autoregressive models allow only for examining persistence of conditional central tendency.

Of paramount interest, this paper defines a mean-reversion process of real GDP growth dynamics as an adjustment speed of $y_t$ towards its mean in cases where $y_t$ deviates from below ($y_{t-1} - \mu < 0$) or above ($y_{t-1} - \mu > 0$) the mean $\mu$.

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**TABLE 1.** Volatility effects of real GDP growth deviations

| Estimates | USA | CA | UK | GM | FR | EU |
|---|---|---|---|---|---|---|
| $a_1$ | −0.048 | −0.053 | 0.008 | −0.065 | −0.055 | −0.104 |
| | (−5.524) | (−2.145) | (0.253) | (−5.737) | (−0.728) | (−2.086) |
| $a_1^-$ | −0.077 | −0.089 | −0.071 | −0.132 | −0.201 | −0.218 |
| | (−3.072) | (−3.718) | (−2.204) | (−4.178) | (−11.69) | (−1.791) |
| $a_1^+$ | −0.019 | −0.017 | 0.088 | 0.002 | 0.089 | 0.011 |
| | (−1.895) | (−1.874) | (7.812) | (0.076) | (3.108) | (0.153) |

*Notes*: This table reports the parameter estimates for the quarterly real GDP growth rates from the following EGARCH models. The mean equation is specified as $y_t = \alpha + \rho y_{t-1} + \sigma_t \varepsilon_t$, and two variance processes are considered as (i) $ln(\sigma_t) = \omega_0 + \omega_1 ln(\sigma_{t-1}) + \omega_2 |\varepsilon_{t-1}| + a_1 \varepsilon_{t-1}$; and (ii) $ln(\sigma_t) = \omega_0 + \omega_1 ln(\sigma_{t-1}) + \omega_2 |\varepsilon_{t-1}| + a_1^- \varepsilon_{t-1} \mathbb{I}(\varepsilon_{t-1} < 0) + a_1^+ \varepsilon_{t-1} \mathbb{I}(\varepsilon_{t-1} \geq 0)$. $\sigma_t^2$ is conditional variance and $\mathbb{I}(\cdot)$ is an indicator function. $t$-statistics are reported in parentheses.

Note that a negative deviation below $\mu$ is most likely seen in lower tails ($\tau < 0.5$) of $y_t$, and a positive deviation above $\mu$ in upper tails ($\tau > 0.5$) of $y_t$. In this paper, the process of reverting deviations to $\mu$ is measured by half-lives (HLs) of a shock hitting real GDP growth, computed as $HL(\tau) = \ln(0.5) / \ln(\rho(\tau))$.[9] In the case of $\rho(\tau) \geq 1$ half-lives are set to $HL(\tau) = \infty$, and $HL(\tau) = 0$ for $-1 < \rho(\tau) \leq 0$. The higher values HLs are, the longer time mean-reversion processes take.

In general, a negative shock hitting real GDP growth, such as $y_t - \mu < 0$, is expected to cause higher uncertainty in future consumption so as to increase volatility of $y_t$, while a positive shock ($y_t - \mu > 0$) to $y_t$ reduces instability of real GDP growth [Ebell (2001) and Nieuwerburgh and Veldkamp (2006)]. Therefore, one expects a negative coefficient, $a_1 < 0$ in (3). In order to gain some preliminary evidence on the sign of $a_1$, I employ real GDP growth data of USA, Canada (CA), UK, Germany (GM), France (FR), and European Union (EU with 28 countries) described in Section 4 to estimate the EGARCH models of Nelson (1991).[10] Table 1 reports the EGARCH estimates of $a_1$, $a_1^-$ and $a_1^+$ with $t$-statistics in parentheses. The table shows that $a_1$ is significantly negative for real GDP growths of the USA, CA, GM, and EU at 5% significance level, while it is insignificant for real GDP growths of the UK and FR. However, the estimates from the UK and France further show that a (possibly large) positive shock to these economies could countercyclically worsen the stability of real GDP growth, as their estimates of $a_1^+$ are significantly positive.[11]

## 3. TESTABLE HYPOTHESES

Based on the result, $\rho(\tau) = \rho + a_1 Q_{\varepsilon_t}(\tau)\mu_0$, where $Q_{\varepsilon_t}(\tau)$ monotonically increases with $\tau$, I thus consider the following testable asymmetric hypotheses for quantile persistence, $\rho(\tau)$, over $\tau \in (0, 1)$, as

$$\rho(\tau) < \rho \quad \text{if } a_1 < 0,\ Q_{\varepsilon_t}(\tau) > 0, \tag{4}$$

$$\rho(\tau) > \rho \quad \text{if } a_1 < 0,\ Q_{\varepsilon_t}(\tau) < 0, \tag{5}$$

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$$\rho(\tau) > \rho \quad \text{if } a_1 > 0,\ Q_{\varepsilon_t}(\tau) > 0, \tag{6}$$

$$\rho(\tau) < \rho \quad \text{if } a_1 > 0,\ Q_{\varepsilon_t}(\tau) < 0, \tag{7}$$

and, for $\tau, \tau' \in (0, 1)$ and $\tau < \tau'$

$$\rho(\tau) > \rho(\tau') \quad \text{if } a_1 < 0, \tag{8}$$

$$\rho(\tau) < \rho(\tau') \quad \text{if } a_1 > 0. \tag{9}$$

Note that the scenario of $a_1 > 0$ and $Q_{\varepsilon_t}(\tau) < 0$ in (7) and (9) is somewhat counterintuitive, because $Q_{\varepsilon_t}(\tau) < 0$ indicates lower tails, where a negative shock is expected to increase volatility so that $a_1 < 0$, contrary to $a_1 > 0$. Nonetheless, I include this scenario for generality to be tested in the empirical section.

In Table 2, the sample unconditional quantiles of real GDP growths are positive for $\tau > 0.1$ but negative for $\tau \leq 0.1$. Therefore, the hypothesis (4) holds for $a_1 < 0$ with $\rho\ (\tau > 0.1) < \rho$, implying that quantile persistence for $\tau > 0.1$ is less than conditional mean persistence. However, for $\tau \leq 0.1$, the hypothesis (5) holds for $a_1 < 0$ with $\rho\ (\tau \leq 0.1) > \rho$, indicating persistence of far lower tails higher than conditional mean persistence.

The hypothesis (8) suggests that $HL(\tau) > HL(\tau')$ for $\tau, \tau' \in (0, 1)$, $\tau < \tau'$ and $a_1 < 0$. Using the sample quantiles from Table 2, one can obtain $HL\,(\tau \to 0) \gg HL\left(\tau' \to 1\right)$. This novel finding is crucial in that it provides insightful implications as follows. (1) If real GDP growth deviated from its mean to its left tail, for instance, $\tau < 0.5$, it would take longer time to adjust back to the mean than a deviation to its right tail, such as $\tau > 0.5$. (2) If real GDP growth deviated from its mean towards its high upper tails, such as $\tau \to 1$, an economic boom could be "peak-reversion" within a short period of time. (3) On the other hand, if real GDP growth deviated from its mean to its deep lower tails, that is, $\tau \to 0$, where recessions and/or crises are often triggered, it would cost a much longer time to recover the growth from such extreme adverse shocks. Importantly, the combination of these implications theoretically predicts the steepness asymmetry of real GDP growth.

To further illustrate these hypotheses, I simulate half-lives, plotted in Figure 1, to provide several important observations as follows.[12] First, Figure 1(a) explains the steepness asymmetry of business cycles which is characterized by gradual upward slopes during expansions (slow recovery) and steep downward slopes during recessions (quick decline).[13] Specifically, Figure 1(a) shows that, when an extreme adverse shock hitting real GDP growth, that is, $\tau \to 0$, the high values of half-lives indicate that the recovery from the negative shock is more gradual and costs a relatively longer period of time. By contrast, for a large positive shock as $\tau \to 1$, half-lives are much shorter, implying that, when a boom ends, the downturn is generally sharp and short by quickly returning to the mean of real GDP growth.

However, if the hypotheses (6) and (9) occurred, that is, the estimates of UK and France in Table 1, Figure 1(b) would exhibit much longer half-lives for high

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**TABLE 2.** Data sample statistics summary

| Statistics | USA | CA | UK | GM | FR | EU |
|---|---|---|---|---|---|---|
| Starting period | 1947Q4 | 1961Q4 | 1975Q4 | 1991Q4 | 1980Q4 | 1995Q4 |
| #Obs. | 277 | 220 | 165 | 101 | 145 | 85 |
| Mean | 3.196 | 2.807 | 2.265 | 1.345 | 1.779 | 1.704 |
| Std.dev. | 2.625 | 5.277 | 2.125 | 2.056 | 1.488 | 1.809 |
| $\hat{\rho}_1$ | 0.852 | 0.886 | 0.858 | 0.834 | 0.902 | 0.912 |
| $\hat{\rho}_2$ | 0.590 | 0.668 | 0.690 | 0.561 | 0.740 | 0.710 |
| $\hat{\rho}_3$ | 0.293 | 0.405 | 0.485 | 0.247 | 0.533 | 0.456 |
| $\hat{\rho}_4$ | 0.032 | 0.163 | 0.261 | −0.073 | 0.323 | 0.210 |
| $\hat{\rho}_{25}$ | 0.023 | 0.016 | −0.105 | 0.114 | 0.133 | 0.021 |
| $Q(25)$ | <0.001 | <0.001 | <0.001 | <0.001 | <0.001 | <0.001 |
| $Q^2(25)$ | <0.001 | <0.001 | <0.001 | <0.001 | <0.001 | <0.001 |
| Jarque–Bera test | 0.018 | 0.065 | <0.001 | <0.001 | <0.001 | <0.001 |
| $\hat{Q}_{y_t}(\tau = 0.05)$ | −1.520 | −6.238 | −1.732 | −1.709 | −0.355 | −1.832 |
| $\hat{Q}_{y_t}(\tau = 0.1)$ | −0.200 | −3.385 | −0.212 | −0.408 | 0.304 | −0.294 |
| $\hat{Q}_{y_t}(\tau = 0.2)$ | 1.300 | −1.444 | 1.307 | 0.226 | 0.927 | 1.139 |
| $\hat{Q}_{y_t}(\tau = 0.3)$ | 2.080 | 0.266 | 1.850 | 0.715 | 1.153 | 1.563 |
| $\hat{Q}_{y_t}(\tau = 0.4)$ | 2.700 | 1.912 | 2.208 | 1.256 | 1.416 | 1.815 |
| $\hat{Q}_{y_t}(\tau = 0.5)$ | 3.100 | 2.839 | 2.451 | 1.594 | 1.732 | 1.978 |
| $\hat{Q}_{y_t}(\tau = 0.6)$ | 3.800 | 4.126 | 2.815 | 1.774 | 2.175 | 2.254 |
| $\hat{Q}_{y_t}(\tau = 0.7)$ | 4.400 | 5.679 | 3.182 | 1.991 | 2.414 | 2.497 |
| $\hat{Q}_{y_t}(\tau = 0.8)$ | 5.180 | 7.575 | 3.631 | 2.600 | 2.857 | 2.874 |
| $\hat{Q}_{y_t}(\tau = 0.9)$ | 6.340 | 9.126 | 4.383 | 3.628 | 3.662 | 3.349 |
| $\hat{Q}_{y_t}(\tau = 0.95)$ | 7.520 | 10.45 | 5.110 | 4.373 | 4.341 | 3.714 |

*Notes*: All quarterly data series end at 2016Q4 except that Canada data ends at 2016Q3. $\hat{\rho}_k$ is the sample autocorrelation of order $k$. $\hat{Q}_{y_t}(\tau)$ is the sample quantile at $\tau$. $p$-values are reported for Ljung–Box tests on (squared) GDP growth denoted by ($Q^2(25)$) $Q(25)$ with 25 lags. $p$-values are also reported for Jarque–Bera test for the null hypothesis of normality. EU contains 28 countries.

upper tails of real GDP growth. In this case, half-lives in Figure 1(b) become more symmetric. This result may partly explain the mixed findings in Hess and Iwata (1997) and Elwood (1998) whose results cannot reject symmetric persistence of real output growth. Moreover, the longer half-lives at high upper tails instead produce round peaks of real GDP growth, consistent with McQueen and Thorley (1993) who present evidence that business cycles can be characterized by sharp troughs and round peaks.

## 4. EMPIRICAL EVIDENCE

This section tests the quantile-based asymmetric hypotheses for empirical evidence. In particular, quantile persistence of real GDP growth over different parts of its distribution, hereafter also referred to as distributional persistence, is estimated from (2) by applying the quantile autoregression of Koenker and

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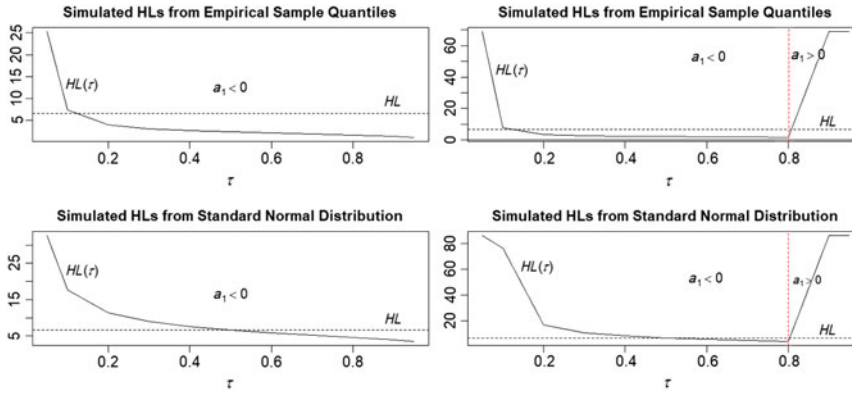



**FIGURE 1.** Simulations of half-lives. Based on Table 1, the simulation takes $a_1 = -0.048$ from the US estimate, and $a_1 = -0.071$ for $\tau \leq 0.8$ and $a_1 = 0.088$ for $\tau > 0.8$ from the UK estimates. Standard normal distribution is used as an example of symmetric distributions. $\rho = 0.9$ is calibrated from the EGARCH mean equation estimations (not reported in Table 1). $\mu_0$ is normalized to one. In the case of $\rho(\tau) \geq 1$, $HL(\tau) = ln(0.5)/ln(0.99)$ is used for graphical illustrations.

Xiao (2006) who allow for systematic influences of conditioning variables on the location, scale, and shape of the conditional distribution of a response variable. Specifically, I consider $\tau = \{0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95\}$ for the estimation of distributional persistence, where $\tau = \{0.05, 0.95\}$ represent extreme lower and upper tails of real GDP growth.

### 4.1. Data

I estimate distributional persistence for the quarterly real GDP (seasonally adjusted) data taken from Federal Reserve Economic Data for six developed economies, including the USA, Canada, the UK, Germany, France, and European Union (with 28 countries). Quarterly real GDP growth rates are computed in a year-over-year (YoY) basis.[14] Due to data availability, the sample periods vary across these economies: 1947Q4–2016Q4 for the USA with 277 observations, 1961Q4–2016Q3 for Canada with 220 observations, 1975Q4–2016Q4 for the UK with 165 observations, 1991Q4–2016Q4 for Germany with 101 observations, 1980Q4–2016Q4 for France with 145 observations, and 1995Q4–2016Q4 for European Union with 85 observations.

Table 2 summarizes data statistics, and Figure 2 plots time series of real GDP growths. Among the economies, the USA has the highest real GDP growth mean, while France has the lowest sample standard deviation. All data series appear to have significant autocorrelations. For instance, their first-order autocorrelations have the range from 0.852 to 0.912, and the Ljung–Box tests on real GDP growths ($Q(25)$) reject the null of joint zero autocorrelations up to 25 lags. Importantly, the

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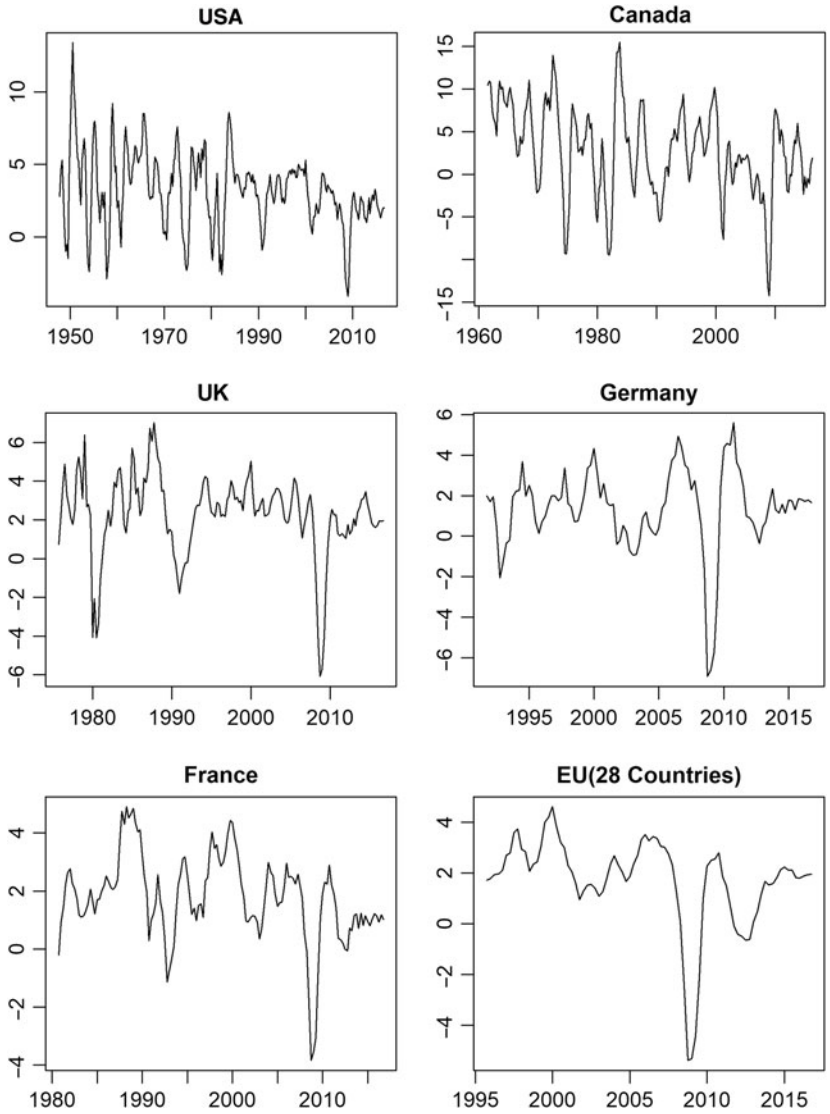



**FIGURE 2.** Time series plots of real GDP growths. The quarterly real GDP (seasonally adjusted) data of five developed economies, taken from Federal Reserve Economic Data, include the USA, Canada, the UK, Germany, France, and European Union (with 28 countries). Due to data availability, the sample periods vary across the economies.

Ljung–Box tests on squared real GDP growths have also rejected the null of joint zero autocorrelations up to 25 lags, implying that variances of real GDP growths are not constant (non-homogeneous). Moreover, the Jarque–Bera tests reject data normality for all data series.

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### 4.2. Distributional Persistence and Half-life Estimates

Figure 3 plots the estimates of distributional persistence, $\rho(\tau)$, of real GDP growths with 95% confidence intervals (shaded areas). In the empirical estimation of (2), $q = \{6, 6, 13, 5, 5, 2\}$ are selected by AIC for USA, CA, UK, GM, FR, and EU, respectively. The overall message taken from the figure is that quantile persistence is significant and decreases with $\tau$, supporting the hypothesis (8), that is, $\rho(\tau) > \rho(\tau')$ for $\tau < \tau'$ and $a_1 < 0$. For instance, distributional persistence of French real GDP growth is $\hat{\rho}(\tau < 0.5) = 0.964$ and $\hat{\rho}(\tau > 0.5) = 0.865$ over its lower and upper tails, respectively. This 9.9% difference in quantile persistence between lower and upper tails costs France about 14 quarters more to reverting a negative deviation than a positive deviation of the same magnitude to its mean.

The exception is that distributional persistence of the UK real GDP growth appears to be a "U" shape and more symmetric than real GDP growths of other economies. In particular, it has higher persistence in extreme lower and upper tails but lower persistence in the middle part of the distribution (i.e., $\tau \in (0.4, 0.6)$). Nonetheless, this empirical result supports the hypothesis (8) in extreme lower tails ($a_1 < 0$) and the hypothesis (9) in extreme upper tails ($a_1 > 0$); that is, for an existent value, $\tau^*$, $\rho(\tau) > \rho(\tau')$ for $a_1 < 0$ and $\tau < \tau' \leq \tau^*$, and $\rho(\tau) < \rho(\tau')$ for $a_1 > 0$ and $\tau^* \leq \tau < \tau'$. The sign change in $a_1$ from negative to positive most likely occurs in upper tails, i.e., $\tau^* > 0.5$ in the UK real GDP growth estimation.

Importantly, the estimates of all data series contain the value of 1 in their 95% confidence intervals of lower tails. Additionally, the UK confidence intervals of upper tails also contain the value of 1. This estimation result indicates the possibility of a unit root at a quantile level of $\tau$, such as $\rho(\tau) = 1$ cannot be rejected. A mean-reversion tendency that adjusts deviations back to the mean can happen only if $\rho(\tau) < 1$ [Koenker and Xiao (2006)]. In this regard, I conduct the quantile unit root test of Koenker and Xiao (2004) on the null hypothesis, $H_0 : \rho(\tau) = 1$, while unit roots of conditional means are tested for $H_0 : \rho = 1$ by the standard ADF test without trend. Table 3 reports the unit root test results. In the table, $t_n(\tau)$ and $\hat{\delta}^2(\tau)$ are computed based on Section 3 of Koenker and Xiao (2004). Critical values, $CV(\tau)$, are obtained by fitting a polynomial of order 2 regression (see their original paper on p. 778) on the table in Appendix A.1 of Koenker and Xiao (2004) for intermediate values of $\hat{\delta}^2(\tau)$. The null hypothesis of a quantile unit root at $\tau$ cannot be rejected if $t_n(\tau) > CV(\tau)$ at 5% significance level, marked in bold values in the table.

Table 3 shows that the test statistics for all data series cannot reject quantile unit roots at the extreme lower tail, $\tau = 0.05$, while they are statistically stationary at conditional mean. Moreover, real GDP growths of France and the UK have quantile unit roots at lower quantiles of $\tau < 0.5$ and the upper tail of $\tau = 0.9$, respectively. In fact, this empirical result is consistent with Corollary 1 of Koenker and Xiao (2006), saying that, a random variable, $y_t$, is globally stationary but can still display local (and asymmetric) unit-root-like persistency in the presence of certain type of shocks (i.e., negative shocks at $\tau = 0.05$ in Table 3).[15]

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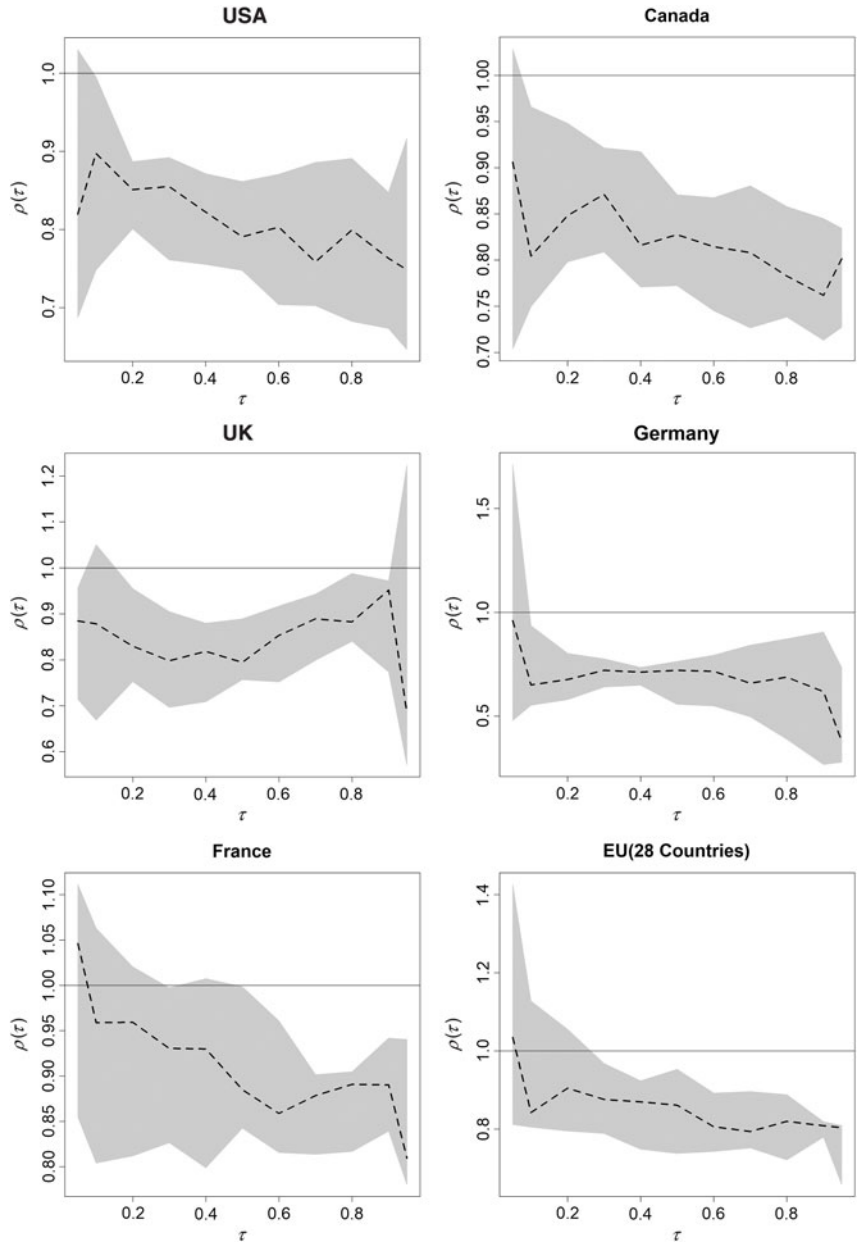



**FIGURE 3.** Plots of distributional persistence estimates over $\tau$ of real GDP growths with 95% confidence intervals (shaded areas).

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**TABLE 3.** Quantile unit root test and distributional half-life estimates

| Countries | | *AR* | *QAR*(τ) 0.05 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| USA | $t_n(\tau)$ | −3.373 | **−2.111** | **−1.694** | −4.005 | −4.445 | −5.282 | −6.192 | −5.386 | −6.531 | −4.751 | −4.370 | −4.010 |
| | $\hat{\delta}^2(\tau)$ | | 0.203 | 0.261 | 0.243 | 0.268 | 0.267 | 0.270 | 0.307 | 0.296 | 0.262 | 0.207 | 0.248 |
| | $CV(\tau)$ | −2.872 | −2.282 | −2.358 | −2.335 | −2.366 | −2.365 | −2.368 | −2.411 | −2.399 | −2.359 | −2.287 | −2.341 |
| | $HL(\tau\|t_n(\tau))$ | 3.362 | ∞ | ∞ | 4.299 | 4.427 | 3.547 | 2.954 | 3.164 | 2.510 | 3.095 | 2.566 | 2.394 |
| CA | $t_n(\tau)$ | −4.071 | **−1.491** | −4.155 | −3.444 | −3.179 | −4.914 | −4.767 | −5.368 | −5.166 | −5.337 | −5.259 | −4.003 |
| | $\hat{\delta}^2(\tau)$ | | 0.101 | 0.199 | 0.235 | 0.282 | 0.313 | 0.298 | 0.279 | 0.257 | 0.240 | 0.184 | 0.217 |
| | $CV(\tau)$ | −2.875 | −2.124 | −2.276 | −2.325 | −2.382 | −2.417 | −2.401 | −2.379 | −2.353 | −2.331 | −2.255 | −2.301 |
| | $HL(\tau\|t_n(\tau))$ | 3.913 | ∞ | 3.185 | 4.210 | 5.026 | 3.410 | 3.664 | 3.378 | 3.256 | 2.830 | 2.551 | 3.131 |
| UK | $t_n(\tau)$ | −2.894 | **−2.054** | **−2.093** | −3.612 | −5.129 | −4.448 | −5.085 | −3.635 | −2.705 | −2.734 | **−1.020** | −5.402 |
| | $\hat{\delta}^2(\tau)$ | | 0.095 | 0.150 | 0.189 | 0.214 | 0.163 | 0.218 | 0.198 | 0.302 | 0.154 | 0.201 | 0.183 |
| | $CV(\tau)$ | −2.880 | −2.112 | −2.204 | −2.261 | −2.297 | −2.223 | −2.303 | −2.275 | −2.405 | −2.210 | −2.278 | −2.254 |
| | $HL(\tau\|t_n(\tau))$ | 3.846 | ∞ | ∞ | 3.709 | 3.070 | 3.457 | 3.018 | 4.368 | 5.896 | 5.553 | ∞ | 1.828 |
| GM | $t_n(\tau)$ | −5.229 | **−0.321** | −3.803 | −3.772 | −4.973 | −5.855 | −5.690 | −4.908 | −5.037 | −3.694 | −4.473 | −6.281 |
| | $\hat{\delta}^2(\tau)$ | | 0.148 | 0.236 | 0.311 | 0.270 | 0.254 | 0.283 | 0.312 | 0.304 | 0.286 | 0.242 | 0.162 |
| | $CV(\tau)$ | −2.891 | −2.200 | −2.326 | −2.415 | −2.369 | −2.349 | −2.384 | −2.416 | −2.408 | −2.388 | −2.333 | −2.222 |
| | $HL(\tau\|t_n(\tau))$ | 1.998 | ∞ | 1.610 | 1.770 | 2.110 | 2.037 | 2.111 | 2.064 | 1.661 | 1.846 | 1.438 | 0.712 |
| FR | $t_n(\tau)$ | −2.893 | **0.782** | **−0.867** | **−0.925** | **−1.609** | **−1.672** | −2.788 | −3.731 | −3.182 | −2.902 | −2.559 | −3.576 |
| | $\hat{\delta}^2(\tau)$ | | 0.102 | 0.199 | 0.291 | 0.316 | 0.348 | 0.355 | 0.336 | 0.304 | 0.315 | 0.152 | 0.118 |
| | $CV(\tau)$ | −2.882 | −2.124 | −2.275 | −2.393 | −2.421 | −2.454 | −2.462 | −2.442 | −2.408 | −2.420 | −2.207 | −2.151 |
| | $HL(\tau\|t_n(\tau))$ | 6.599 | ∞ | ∞ | ∞ | ∞ | ∞ | 5.678 | 4.551 | 5.351 | 5.994 | 5.961 | 3.278 |
| EU | $t_n(\tau)$ | −4.669 | **0.394** | −3.077 | −2.539 | −3.957 | −4.300 | −4.635 | −6.827 | −7.297 | −4.361 | −4.021 | −4.810 |
| | $\hat{\delta}^2(\tau)$ | | 0.076 | 0.148 | 0.241 | 0.233 | 0.253 | 0.243 | 0.213 | 0.198 | 0.179 | 0.107 | 0.032 |
| | $CV(\tau)$ | −2.897 | −2.078 | −2.200 | −2.332 | −2.322 | −2.348 | −2.335 | −2.295 | −2.274 | −2.247 | −2.133 | −1.994 |
| | $HL(\tau\|t_n(\tau))$ | 4.216 | ∞ | 4.026 | 6.890 | 5.213 | 4.956 | 4.626 | 3.207 | 2.996 | 3.482 | 3.257 | 3.171 |

*Notes*: The unit root is tested for the AR model estimates by the standard ADF test without trend. The quantile unit root test of Koenker and Xiao (2004) is used for $H_0 : \rho(\tau) = 1$. $t_n(\tau)$ and $\hat{\delta}^2(\tau)$ are computed from Section 3 of Koenker and Xiao Koenker and Xiao (2004). The critical values, $CV(\tau)$, are obtained by fitting a polynomial of order 2 regression on the table in Appendix A.1 of Koenker and Xiao (2004) for intermediate values of $\hat{\delta}^2(\tau)$. The quantile unit root null hypothesis cannot be rejected if $t_n(\tau) > CV(\tau)$ at 5% significance level, marked as bold values. The half-lives are computed as $HL(\tau) = ln(0.5)/ln(\rho(\tau))$. $HL(\tau|t_n(\tau)) = HL(\tau)$ if $t_n(\tau) < CV(\tau)$ (the quantile unit root is rejected) and $HL(\tau|t_n(\tau)) = \infty$ if $t_n(\tau) > CV(\tau)$ (the quantile unit root cannot be rejected).

Table 3 also reports the half-lives computed as $HL(\tau) = ln\,(0.5)\,/ln\,(\rho\,(\tau))$. Following Nikolaou (2008), Tsong and Lee (2011), and Cicek and Akar (2013), I set $HL(\tau) = \infty$ for $\rho\,(\tau) \geq 1$. In addition, conditional on the quantile unit root test results, I consider $HL\,(\tau|t_n(\tau)) = \infty$ if $t_n(\tau) > CV(\tau)$, where the quantile unit root is not rejected, and $HL\,(\tau|t_n(\tau)) = HL(\tau)$ if $t_n(\tau) < CV(\tau)$, where the quantile unit root is rejected.

Figure 4 plots distributional half-lives calculated in Table 3.[16] The figure shows that, except the UK, the half-lives of all other real GDP growths strongly support the hypothesis (8) of asymmetric quantile persistence as illustrated in Figure 1(a), and hence the steepness asymmetry in real GDP growth. In particular, the high half-life values at the extreme lower tails lead to slow recovery from adverse/negative shocks to real GDP growth. On the other hand, the short half-lives at the high upper tails sharply and quickly revert peaks to downturns at the end of economic booms. By contrast, the high half-life value at the upper tail, $\tau = 0.9$, of the UK real GDP growth would generate round peaks. Nonetheless, the figure shows that the round peaks can still decline quickly due to the low half-life value at $\tau = 0.95$. In addition, these half-life estimates over different parts of real GDP growth distributions also support the hypotheses (4)–(6).

### 4.3. Distributional Persistence and Half-life Estimates under Possible Structural Breaks

Omitting structural breaks in unit root tests can lead to a spurious persistence of a process [Perron (1989) and Manzan and Zerom (2015)]. For the purpose of robustness, this subsection attempts to quantify the impact of possible structural breaks on the empirical results in Table 3. Applying sequential testing procedures of Bai (1997), I test the number ($b$) of structural breaks at 5% significance level for $H_0 : b = l$ vs $H_1 : b = l + 1$ with a maximum of $b = 5$ breaks and 15% sample trimming for each of the data series.[17]

The test is conducted in two steps. First, I test structural breaks in means of the data series. The test results reported in Table 4 show that no structural breaks have statistically been found in German real GDP growth, while the test has dated breaks (denoted by $b_l$) for other data series. Structural breaks of $y_t$ split the sample period to $b_l + 1$ segments. I construct a dummy variable vector of dimension $(b_l + 1)$ for $y_t$, denoted by $D_t^l$. For $i = 1, \ldots, b_l + 1$, the $i$th element of $D_t^l$ takes the value of 1 if the $i$th segment contains time $t$ and otherwise 0. Running an OLS regression of $y_t$ on $D_t^l$, I thus obtain fitted residuals as $\hat{e}_t = y_t - \hat{\beta}' D_t^l$ for each of the data series, where $\beta$ is a $(b_l + 1)$ vector of parameters. Second, defining $\eta_t = \sqrt{\frac{\pi}{2}}\,|\hat{e}_t|$, I follow McConnell and Perez-Quiros (2000) to test structural breaks in $\eta_t$ by the sequential test procedures.[18] The detected breaks (denoted by $b_v$) reported in Table 4 are thus considered as the breaks in the variance of $y_t$ by controlling its mean breaks.[19] Note that $b_l$ and $b_v$ are allowed to be different. Similarly, using the dated breaks in error variances, I construct a dummy

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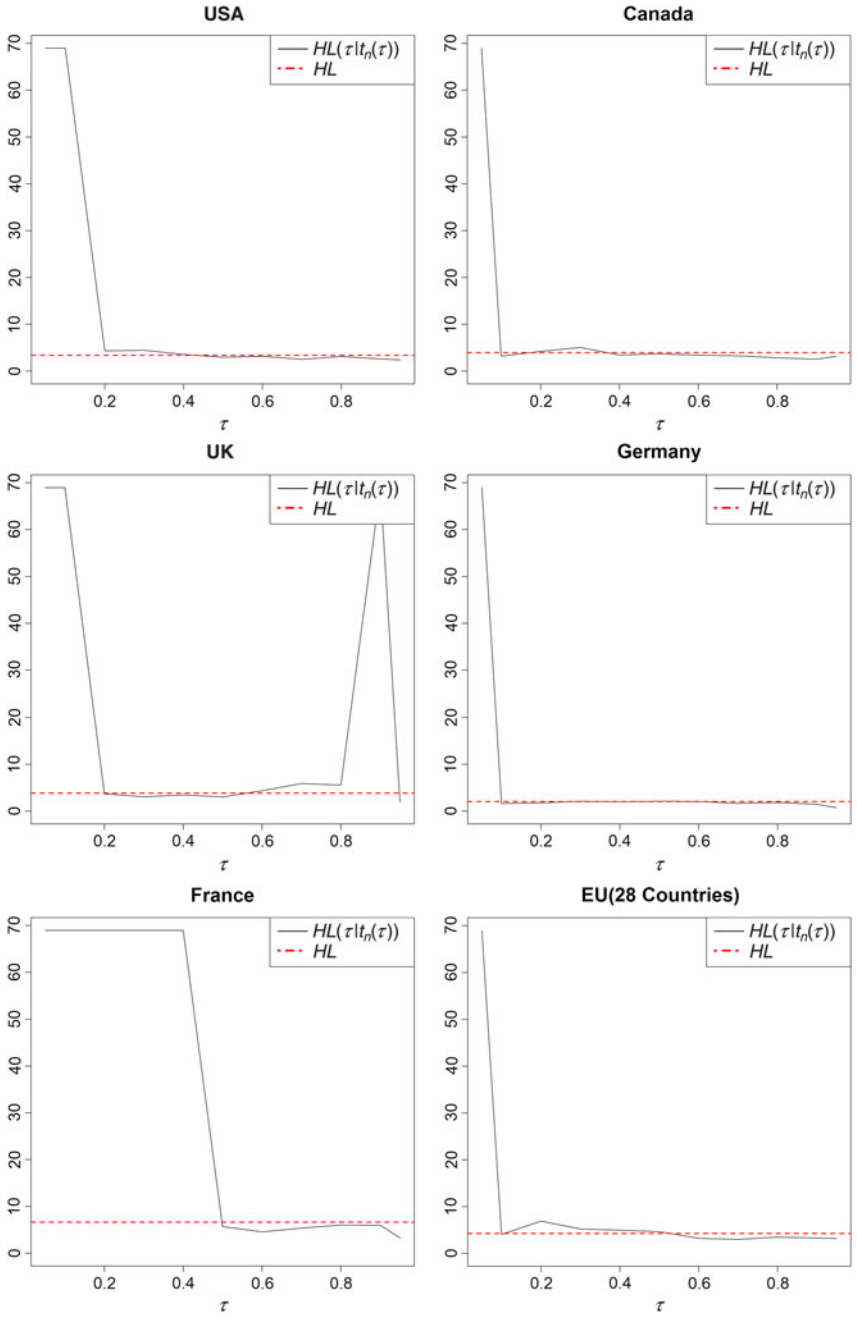



**FIGURE 4.** Plots of distributional half-life estimates over $\tau$ of real GDP growths. *HL* represents a half-life value estimated from conditional mean autoregressive models, and $HL(\tau|t_n(\tau))$ represents a distributional half-life value at $\tau$ conditional on quantile unit-root test results.

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**TABLE 4.** Test for structural breaks

| Estimates | USA | CA | UK | GM | FR | EU |
|---|---|---|---|---|---|---|
| Break dates in means | 2000Q4 | 1974Q1<br>1983Q1<br>2000Q4 | 2008Q1 | ∼ | 2008Q1 | 2008Q1<br>2013Q3 |
| Break dates in error variances | 1984Q4<br>2008Q1 | 1974Q4<br>1991Q4<br>2008Q3 | 1992Q3<br>2008Q1 | 2006Q1<br>2011Q3 | 1987Q3<br>1993Q4<br>2011Q4 | 2008Q3<br>2011Q3 |

*Notes*: The sequential structural break test of Bai (1997) is used to test the number of structural breaks at 5% significance level for $H_0 : b = l \; vs. \; H_1 : b = l + 1$ with a maximum of $b = 5$ breaks and 15% sample trimming. The general (robust) form of the $F$-statistic in Bai and Perron (2003*a*) for the test is used in place of the difference in sums-of-squared residuals described in Bai (1997) and Bai and Perron (1998). Critical value and response surface computations are provided by Bai and Perron (2003*b*). The symbol, ∼, represents no breaks are detected.

variable, $D_t^v$. Then, I fit $\hat{e}_t = \sigma_t \varepsilon_t$ to a normal distribution with standard deviation $\sigma_t = \sigma' D_t^v$ and $\sigma = \left(\sigma_1, \ldots, \sigma_{b_v+1}\right)'$. For $i = 1, \ldots, b_v + 1$, the $i$th element of $\sigma$ is the standard deviation of $e_t$ in the $i$th segment of the sample period. Finally, I apply the fitted residuals, $\hat{\varepsilon}_t = \hat{e}_t / \hat{\sigma}_t$, to (1) and (2) with $q = \{10, 5, 5, 5, 3, 1\}$.

Figure 5 plots the estimates of distributional persistence, $\rho(\tau)$, with the treatment of data structural breaks. Similar to those from Figure 3, distributional persistence of the USA, Canadian, German and French real GDP growths is also highly significant and supports the asymmetric hypothesis (8) in that quantile persistence decreases with $\tau$ in Figure 5. In addition, the significant UK estimates still support both hypotheses (8) and (9) due to higher quantile persistence in its lower and upper tails than that in the middle part of its distribution.

Nonetheless, different from Figure 3, the EU estimates present an opposite trend that quantile persistence increases with $\tau$ in Figure 5. While the UK estimates have a "U"-shaped quantile persistence, suggesting sign changes in $a_1$ from (8) to (9), the EU estimates support (9) with monotonically increased quantile persistence with $\tau$, which indicates $a_1 > 0$ over $\tau$ without switching signs.

From Figure 5, the possibility of a quantile unit root cannot be ruled out by the treatment of data structural breaks, as the 95% confidence intervals contain the value of 1 in the upper tails of EU real GDP growth and the lower tails of other data series. In this regard, Table 5 also reports the test results of quantile unit roots. The table shows that the lower tails ($\tau \leq 0.1$) of real GDP growths of Canada, the UK, Germany and France, and the EU upper tails ($\tau \geq 0.6$) cannot reject unit-root-like quantile persistency, while the US real GDP growth appears to be stationary across different parts of the distribution. Conditional on the quantile unit root test results, half-lives are calculated in Table 5 by the same criteria as those in Table 3.

Figure 6 plots distributional half-lives calculated in Table 5. In line with the overall conclusions drawn from Figure 4 without the treatment of structural

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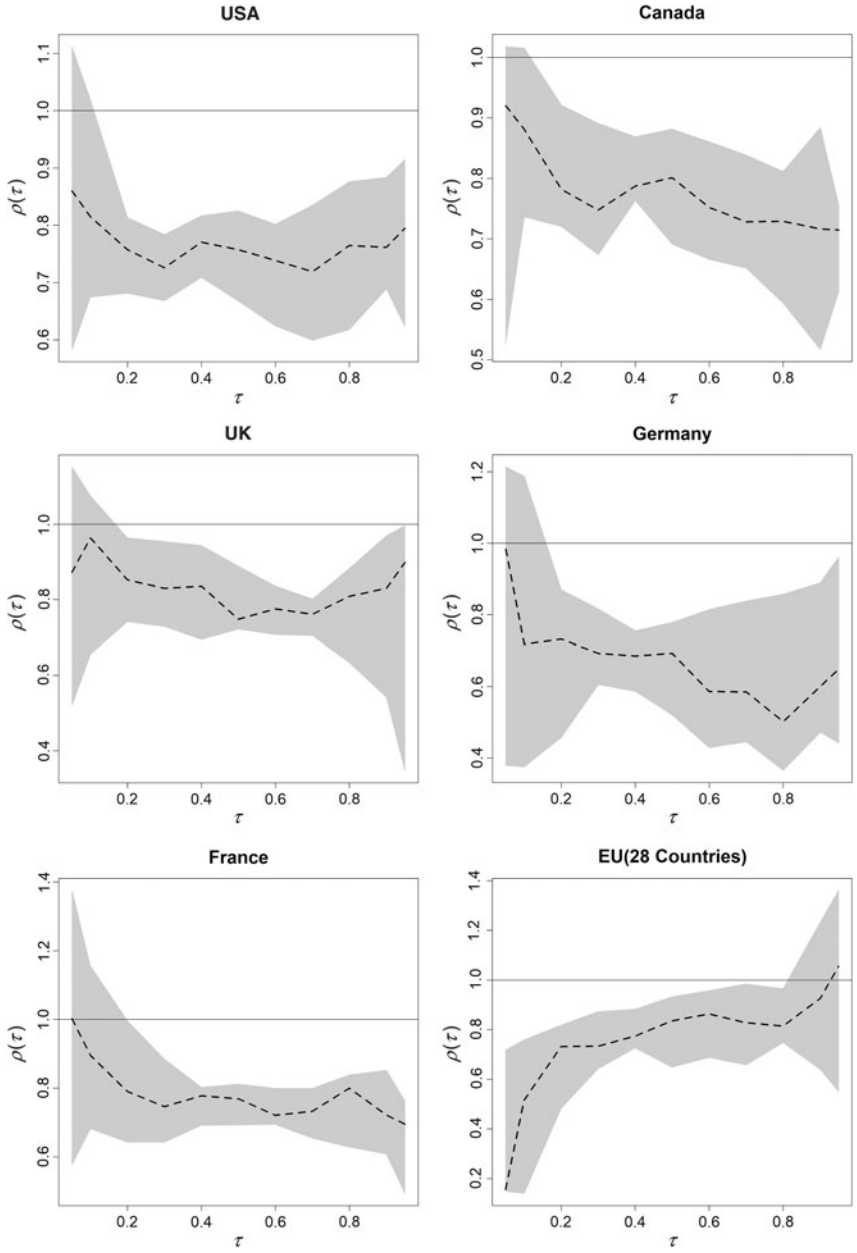



**FIGURE 5.** Plots of distributional persistence estimates over $\tau$ under structural breaks. Structural breaks are sequentially tested for means and variances of the data series and treated separately.

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**TABLE 5.** Quantile unit root test and distributional half-life estimates under structural breaks

| | | | $QAR(\tau)$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Countries | | $AR$ | 0.05 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 0.95 |
| USA | $t_n(\tau)$ | −5.616 | −2.296 | −3.759 | −4.991 | −5.688 | −4.658 | −5.085 | −5.351 | −5.521 | −4.576 | −4.726 | −3.594 |
| | $\hat{\delta}^2(\tau)$ | | 0.125 | 0.211 | 0.256 | 0.298 | 0.349 | 0.335 | 0.352 | 0.328 | 0.249 | 0.148 | 0.145 |
| | $CV(\tau)$ | −2.872 | −2.163 | −2.293 | −2.352 | −2.401 | −2.456 | −2.441 | −2.458 | −2.434 | −2.342 | −2.201 | −2.196 |
| | $HL(\tau\|t_n(\tau))$ | 2.500 | 4.597 | 3.384 | 2.496 | 2.164 | 2.659 | 2.496 | 2.287 | 2.102 | 2.584 | 2.542 | 3.021 |
| CA | $t_n(\tau)$ | −5.558 | **−1.081** | **−1.965** | −3.782 | −4.584 | −4.292 | −4.126 | −5.050 | −5.263 | −4.862 | −3.053 | −3.346 |
| | $\hat{\delta}^2(\tau)$ | | 0.145 | 0.199 | 0.287 | 0.319 | 0.335 | 0.371 | 0.347 | 0.321 | 0.293 | 0.161 | 0.082 |
| | $CV(\tau)$ | −2.875 | −2.195 | −2.277 | −2.389 | −2.424 | −2.441 | −2.477 | −2.454 | −2.426 | −2.395 | −2.220 | −2.090 |
| | $HL(\tau\|t_n(\tau))$ | 2.412 | ∞ | ∞ | 2.819 | 2.383 | 2.896 | 3.124 | 2.428 | 2.185 | 2.194 | 2.078 | 2.063 |
| UK | $t_n(\tau)$ | −3.612 | **−1.438** | **−0.343** | **−2.354** | −2.925 | −3.310 | −5.322 | −4.690 | −4.950 | −2.897 | −1.996 | −1.346 |
| | $\hat{\delta}^2(\tau)$ | | 0.249 | 0.274 | 0.410 | 0.390 | 0.369 | 0.342 | 0.317 | 0.342 | 0.312 | 0.216 | 0.152 |
| | $CV(\tau)$ | −2.879 | −2.343 | −2.373 | −2.512 | −2.495 | −2.475 | −2.448 | −2.422 | −2.449 | −2.416 | −2.300 | −2.206 |
| | $HL(\tau\|t_n(\tau))$ | 3.240 | ∞ | ∞ | ∞ | 3.718 | 3.851 | 2.387 | 2.721 | 2.540 | 3.261 | ∞ | 6.494 |
| GM | $t_n(\tau)$ | −3.651 | **−0.101** | **−1.988** | −2.765 | −3.592 | −4.038 | −3.680 | −4.487 | −4.134 | −5.188 | −4.139 | −3.733 |
| | $\hat{\delta}^2(\tau)$ | | 0.070 | 0.212 | 0.384 | 0.376 | 0.411 | 0.376 | 0.415 | 0.361 | 0.367 | 0.159 | 0.030 |
| | $CV(\tau)$ | −2.891 | −2.067 | −2.295 | −2.489 | −2.481 | −2.513 | −2.482 | −2.517 | −2.468 | −2.473 | −2.217 | −1.990 |
| | $HL(\tau\|t_n(\tau))$ | 1.813 | ∞ | ∞ | 2.217 | 1.889 | 1.832 | 1.889 | 1.296 | 1.294 | 1.005 | 1.359 | 1.600 |
| FR | $t_n(\tau)$ | −5.238 | **0.021** | **−1.516** | −3.585 | −5.586 | −5.344 | −5.503 | −6.473 | −5.503 | −3.802 | −3.552 | −3.360 |
| | $\hat{\delta}^2(\tau)$ | | 0.108 | 0.228 | 0.356 | 0.412 | 0.397 | 0.426 | 0.423 | 0.399 | 0.392 | 0.300 | 0.293 |
| | $CV(\tau)$ | −2.882 | −2.136 | −2.316 | −2.463 | −2.515 | −2.501 | −2.526 | −2.524 | −2.503 | −2.497 | −2.403 | −2.395 |
| | $HL(\tau\|t_n(\tau))$ | 12.626 | ∞ | ∞ | 2.942 | 2.371 | 2.761 | 2.639 | 2.119 | 2.231 | 3.107 | 2.124 | 1.910 |
| EU | $t_n(\tau)$ | −3.976 | −7.560 | −5.259 | −3.139 | −3.747 | −3.386 | −2.576 | **−2.163** | **−2.202** | **−2.180** | **−0.686** | 0.428 |
| | $\hat{\delta}^2(\tau)$ | | 0.311 | 0.255 | 0.362 | 0.355 | 0.348 | 0.352 | 0.342 | 0.334 | 0.338 | 0.324 | 0.189 |
| | $CV(\tau)$ | −2.897 | −2.415 | −2.350 | −2.468 | −2.461 | −2.454 | −2.458 | −2.449 | −2.440 | −2.444 | −2.430 | −2.262 |
| | $HL(\tau\|t_n(\tau))$ | 1.747 | 0.375 | 1.048 | 2.221 | 2.238 | 2.711 | 3.869 | ∞ | ∞ | ∞ | ∞ | 68.968 |

*Notes*: The unit root is tested for the AR model estimates by the standard ADF test without trend. The quantile unit root test of Koenker and Xiao (2004) is used for $H_0 : \rho(\tau) = 1$. $t_n(\tau)$ and $\hat{\delta}^2(\tau)$ are computed from Section 3 of Koenker and Xiao (2004). The critical values, $CV(\tau)$, are obtained by fitting a polynomial of order 2 regression on the table in Appendix A.1 of Koenker and Xiao of (2004) for intermediate values of $\hat{\delta}^2(\tau)$. The quantile unit root null hypothesis cannot be rejected if $t_n(\tau) > CV(\tau)$ at 5% significance level, marked as bold values. The half-lives are computed as $HL(\tau) = ln(0.5)/ln(\rho(\tau))$. $HL(\tau|t_n(\tau)) = HL(\tau)$ if $t_n(\tau) < CV(\tau)$ (the quantile unit root is rejected) and $HL(\tau|t_n(\tau)) = \infty$ if $t_n(\tau) > CV(\tau)$ (the quantile unit root cannot be rejected).

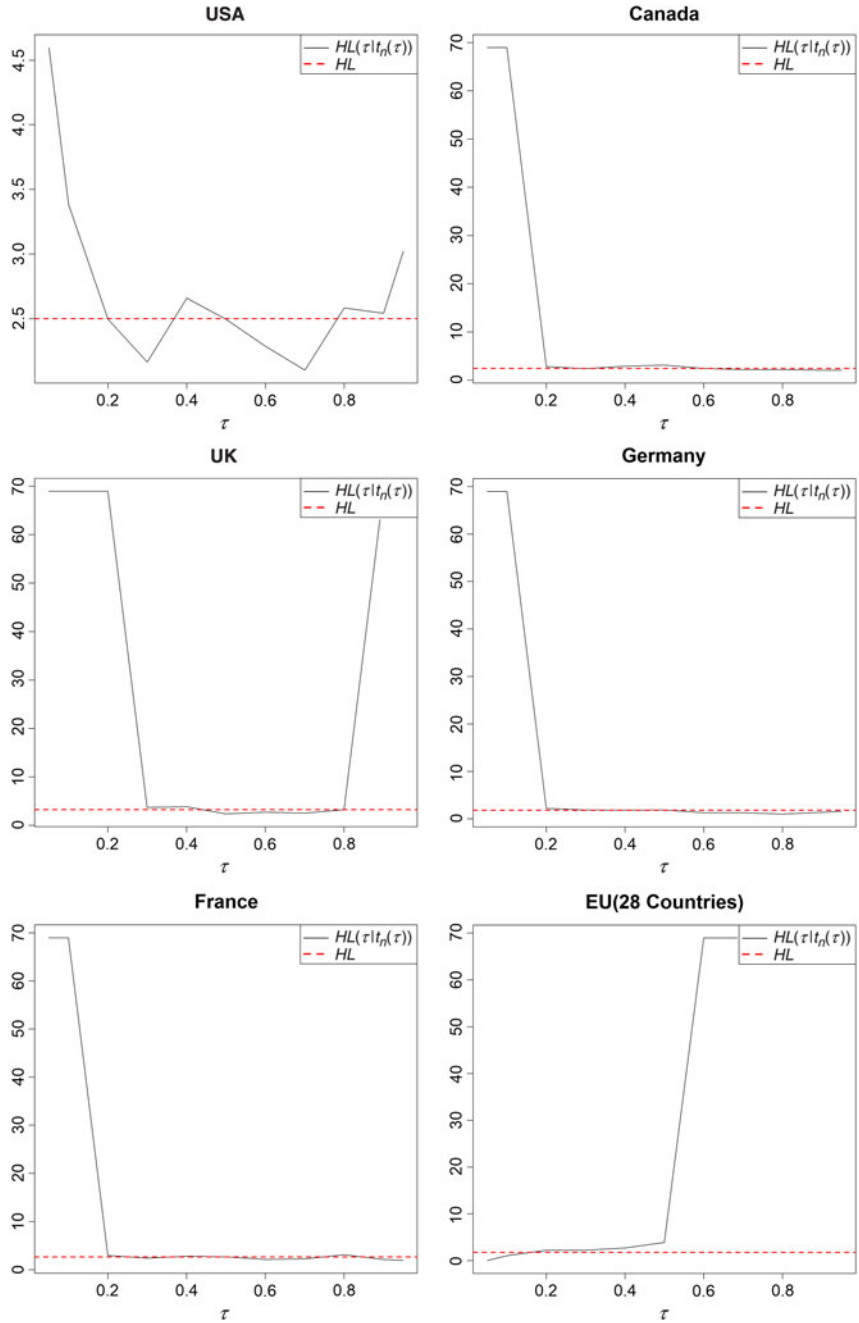



**FIGURE 6.** Plots of distributional half-life estimates over $\tau$ under structural breaks. Structural breaks are sequentially tested for means and variances of the data series and treated separately. *HL* represents a half-life value estimated from conditional mean autoregressive models, and $HL(\tau|t_n(\tau))$ represents a distributional half-life value at $\tau$ conditional on quantile unit-root test results.

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breaks, Figure 6 also supports the asymmetric hypotheses and hence the steepness asymmetry in real GDP growths that contractions are steeper than expansions. Specifically, the EU half-lives support the counter-intuitive hypotheses (7) and (9) with $a_1 > 0$ and $Q_{\varepsilon_t}(\tau) < 0$ due to $HL(\tau \leq 0.2|t_n(\tau)) < HL$ and $HL(\tau|t_n(\tau)) < HL(\tau'|t_n(\tau'))$ for $\tau < \tau'$, respectively. Nonetheless, the monotonic increase in the EU half-lives from lower to upper tails does not support the steepness asymmetry conjecture in real GDP growth, since this result implies that expansions are steeper than contractions in real GDP growth dynamics. However, accounting for potential nonlinearities in the EU data has reverted this counter result in the next subsection.

### 4.4. Nonlinear Distributional Persistence and Half-life Estimates

Encompassing asymmetries in macroeconomic dynamics requires more flexible and possibly nonlinear econometric models. For instance, the finding in McConnell and Perez-Quiros (2000) that a structural break presents in the US output growth volatility in 1984 suggests that time-varying volatility of real GDP growth may take a form, such as $\sigma_t = \sigma_1 \mathbb{I}(y_{t-1} < c) + \sigma_2 \mathbb{I}(y_{t-1} \geq c)$, where $c$ is a self-exciting threshold parameter to describe economic states labeled by $\sigma_1 > \sigma_2$. Applying this state-dependent volatility to (1), I further obtain the implied quantile process of (2) as a threshold quantile autoregressive (TQAR) model

$$Q_{y_t}(\tau|\mathcal{F}_{t-1}) = \begin{cases} \alpha_1(\tau) + \rho_1(\tau) y_{t-1} + \sum_{j=1}^{q} \beta_{1j}(\tau) \Delta y_{t-j} & \text{if } y_{t-1} < c(\tau) \\ \alpha_2(\tau) + \rho_2(\tau) y_{t-1} + \sum_{j=1}^{q} \beta_{2j}(\tau) \Delta y_{t-j} & \text{if } y_{t-1} \geq c(\tau), \end{cases} \quad (\mathbf{10})$$

with $\alpha_k(\tau) = \alpha + \left(1 - a_{1,k}\mu\right) \mu_{0k} Q_{\varepsilon_t}(\tau)$, $\rho_k(\tau) = \rho + a_{1k} Q_{\varepsilon_t}(\tau) \mu_{0k}$, and $\beta_{kj}(\tau) = \beta_j + b_{kj} Q_{\varepsilon_t}(\tau) \mu_{0k}$ for $j = 1, \ldots, q$ and $k = 1, 2$. $c(\tau)$ is an unknown (endogenous) threshold parameter to be estimated. Equation (10) implies that the first endogenous regime ($y_{t-1} < c(\tau)$) is associated with negative shocks and deviations, while the second endogenous regime ($y_{t-1} \geq c(\tau)$) is associated with positive shocks and deviations. In the estimation of (10), I set $q = 2$ for all data series in order to keep a parsimonious parameter space for statistical inference. See Cai and Stander (2008) and Galvao et al. (2011) for detailed estimation methods and asymptotic properties.

Figure 7 plots the estimates of distributional threshold persistence, $\rho_1(\tau)$ and $\rho_2(\tau)$, from real GDP growths with 95% confidence intervals (shaded areas).[20] In this figure, the $x$-axis for $\rho_2\left(\tau|y_{t-1} \geq c(\tau)\right)$ is valued as $\tau + 0.9$, so that the value of 0.95 is equivalent to $\tau = 0.05$ for $\rho_2\left(\tau|y_{t-1} \geq c(\tau)\right)$, and so on. Basically, the figure shows that quantile persistence in the negative shock regimes, $\rho_1\left(\tau|y_{t-1} < c(\tau)\right)$, decreases with $\tau$ in support of the hypothesis (8) of asymmetric quantile persistence. By contrast, quantile persistence of the UK and German real GDP growths in the positive shock regimes appears to increase with $\tau$, supporting the

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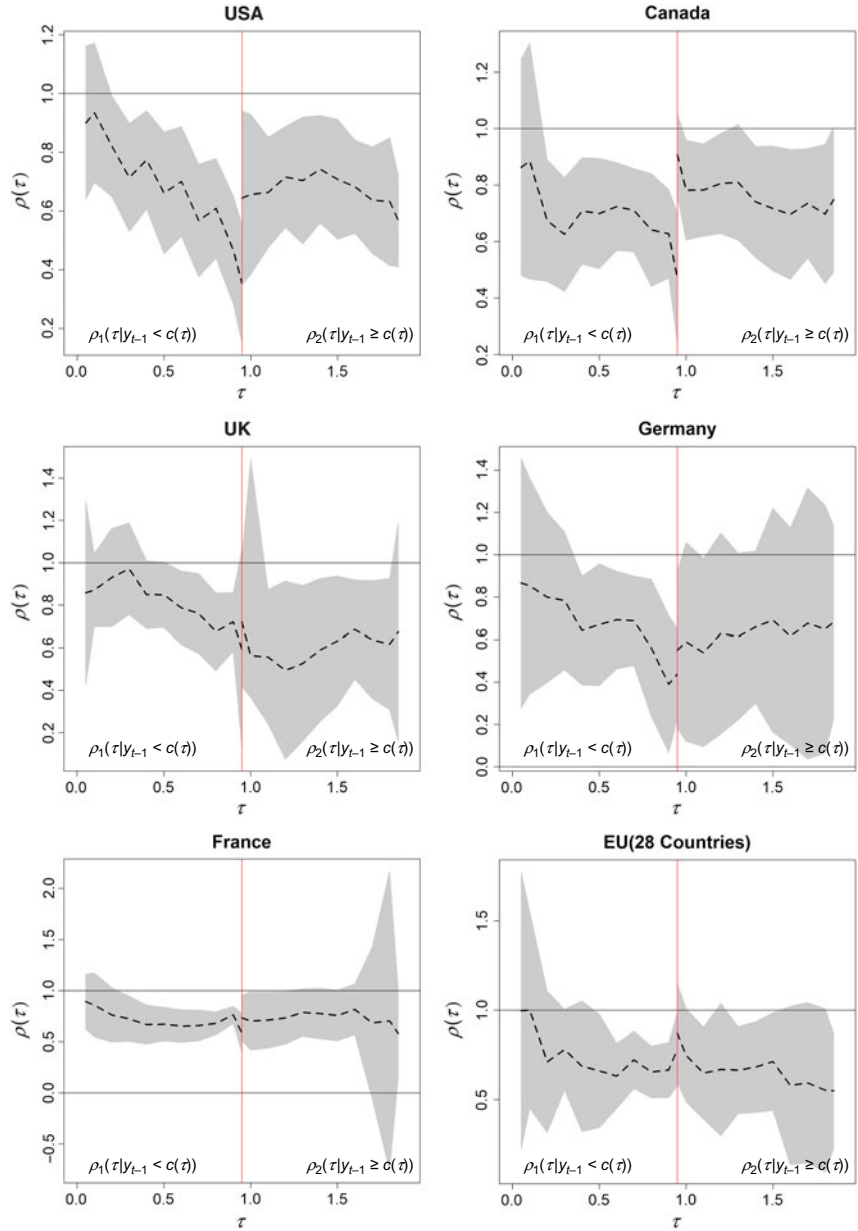



**FIGURE 7.** Plots of the threshold distributional persistence estimates, $\rho_1(\tau\,|y_{t-1} < c(\tau))$ and $\rho_2\big(\tau\,|y_{t-1} \geq c(\tau)\big)$, over $\tau$ with 95% confidence intervals (shaded areas). $c(\tau)$ is the threshold parameter. In this figure, the $x$-axis for $\rho_2\big(\tau\,|y_{t-1} \geq c(\tau)\big)$ is valued as $\tau + 0.9$.

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hypothesis (9). Quantile persistence of French real GDP growth is statistically insignificant at $\tau = 0.9$. In Figure 7, the 95% confidence intervals contain the value of 1 in many cases.[21]

Figure 8 plots distributional half-lives calculated from Figure 7 as $HL_1\left(\tau|y_{t-1} < c(\tau)\right) = ln\,(0.5)\,/ln\left(\rho_1\left(\tau|y_{t-1} < c(\tau)\right)\right)$ and $HL_2\left(\tau|y_{t-1} \geq c(\tau)\right) = ln\,(0.5)\,/ln\left(\rho_2\left(\tau|y_{t-1} \geq c(\tau)\right)\right)$. In cases where $\rho\,(\tau)$ is statistically insignificant, I set $HL(\tau) = 0$. Similarly, the $x$-axis for $HL_2\left(\tau|y_{t-1} \geq c(\tau)\right)$ is valued as $\tau + 0.9$. All data series in the negative shock regimes have much higher half-life estimates from their lower tails than their corresponding upper tails, implying slow recovery from an adverse shock to real GDP growth. Round peaks are possibly produced from German and French real GDP growths due to the increased half-life estimates at their high upper tails in the positive shock regimes. However, French real GDP growth would still have a quick decline for $\tau > 0.7$ where the half-life estimates dramatically drop in its positive shock regime.

For the purpose of robustness, Figure 9 demonstrates the estimates of distributional threshold persistence, $\rho_1(\tau)$ and $\rho_2(\tau)$, from real GDP growths with the treatment of data structural breaks. Figure 10 plots distributional half-lives calculated from Figure 9. As seen, the overall conclusions obtained from Figures 7 and 8 remain unaltered in support of the asymmetric hypotheses (8)–(9) for the steepness asymmetry and potential round peaks of real GDP growths, while some detailed estimates may be different. For instance, quantile persistence in the negative shock regimes decreases with $\tau$, except that the UK real GDP growth exhibits similar "U"-shaped quantile persistence in its negative shock regime. However, by modeling potential nonlinearities, the EU half-lives in the negative shock regime are decreasing with $\tau$, no longer supporting the counter-intuitive hypotheses (7) and (9) with $a_1 > 0$ and $Q_{\varepsilon_t}(\tau) < 0$ as shown in Figure 6. In the EU positive shock regime quantile persistence is statistically insignificant for $\rho_2\ (\tau > 0.4|t_n(\tau))$.

## 5. CONCLUSION

This paper studies asymmetric dynamics of real GDP growth by estimating quantile persistence over different parts of real GDP growth distributions for six major developed economies. The empirical results provide strong evidence for the steepness asymmetry and round peaks of real GDP growth. In particular, this paper finds that quantile persistence of real GDP growth is generally higher in its lower tails than the corresponding upper tails. This finding implies a slow recovery from a negative shock to real GDP growth and a sharp and quick decline at the end of economic booms. In some cases, round peaks can be obtained due to increased half-lives at high upper tails. These empirical results are robust to the presence of structural breaks and potential nonlinearities in real GDP growth.

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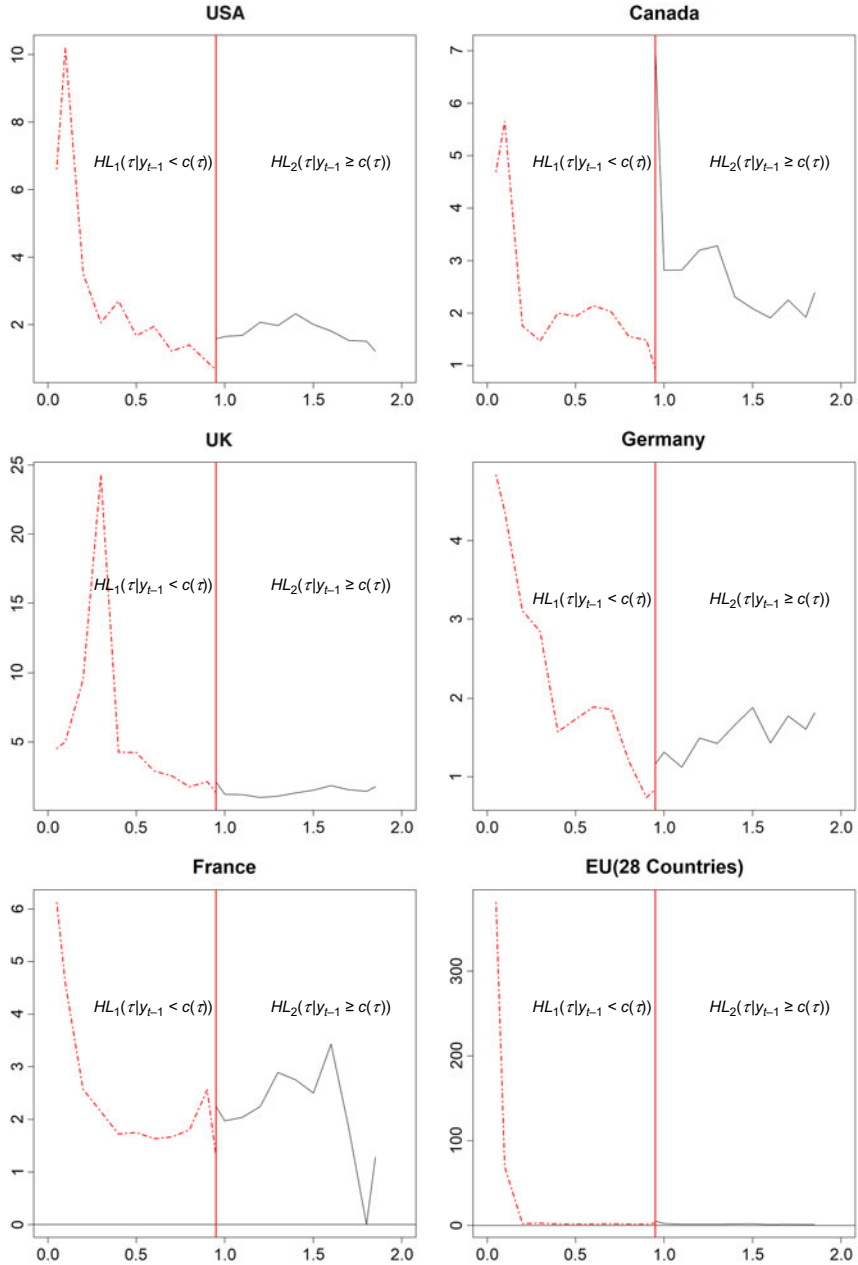



**FIGURE 8.** Plots of the threshold distributional half-life estimates, $HL_1(\tau\,|y_{t-1} < c(\tau))$ and $HL_2\big(\tau\,|y_{t-1} \geq c(\tau)\big)$, over $\tau$. $c(\tau)$ is the threshold parameter. In this figure, the $x$-axis for $HL_2\big(\tau\,|y_{t-1} \geq c(\tau)\big)$ is valued as $\tau + 0.9$.

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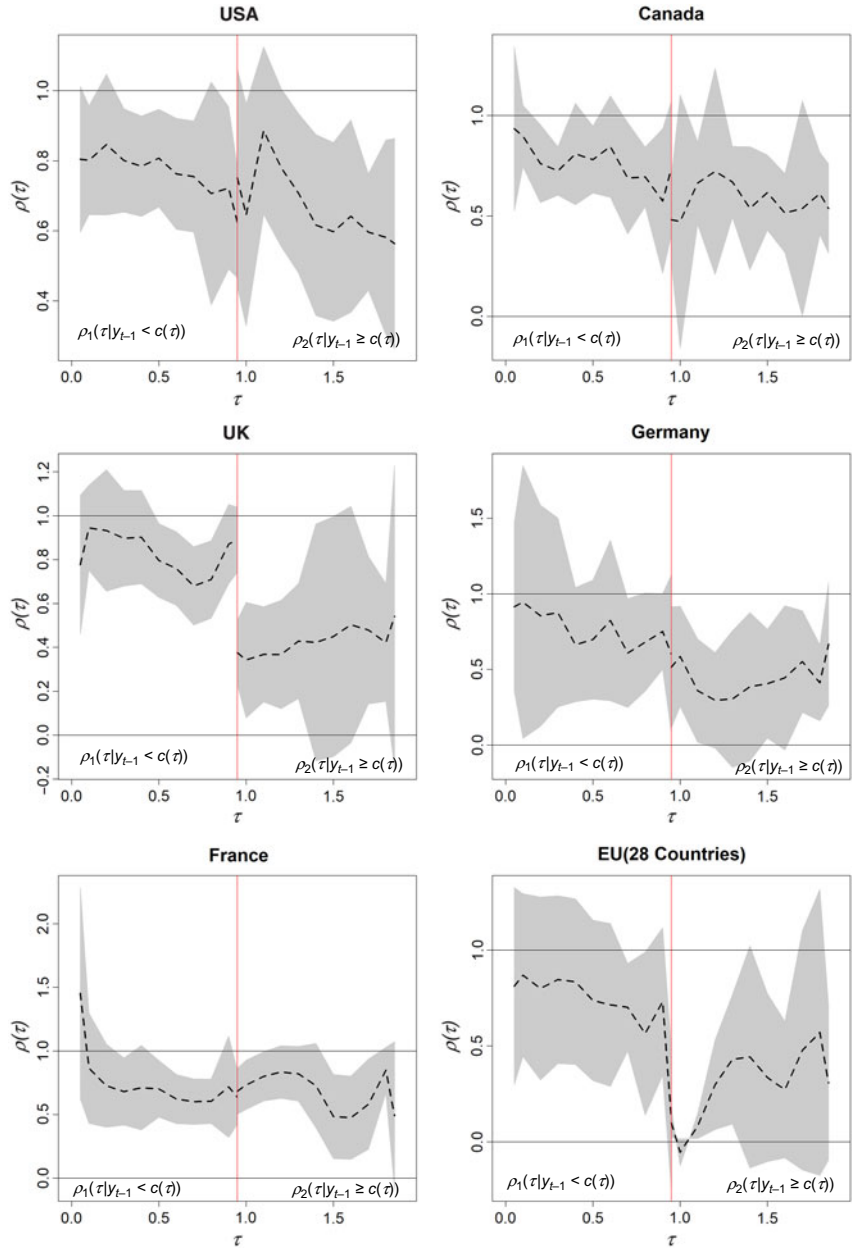



**FIGURE 9.** Plots of the threshold distributional persistence estimates over $\tau$ under structural breaks, $\rho_1(\tau|y_{t-1} < c(\tau))$ and $\rho_2\big(\tau|y_{t-1} \geq c(\tau)\big)$, with 95% confidence intervals (shaded areas). $c(\tau)$ is the threshold parameter. In this figure, the $x$-axis for $\rho_2\big(\tau|y_{t-1} \geq c(\tau)\big)$ is valued as $\tau + 0.9$. Structural breaks are sequentially tested for means and variances of the data series and treated separately.

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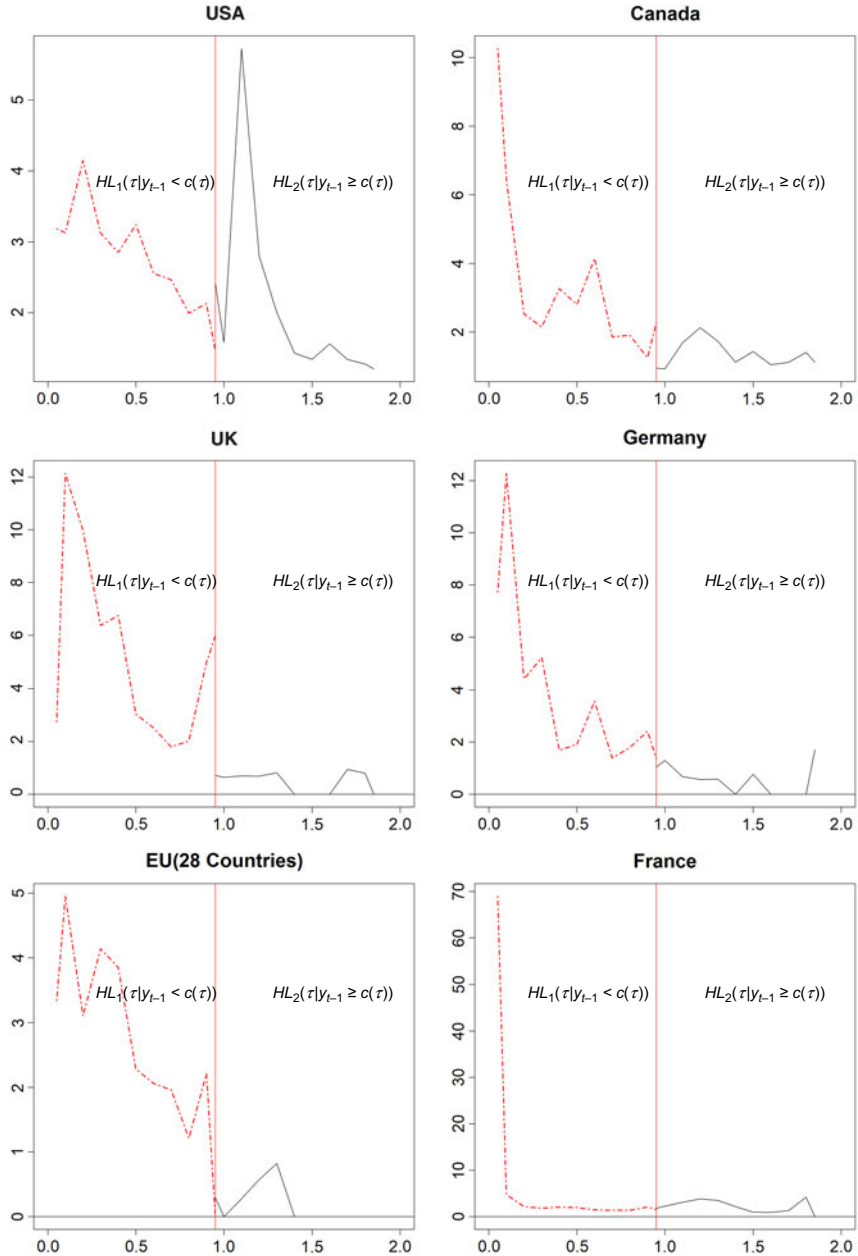



**FIGURE 10.** Plots of the threshold distributional half-life estimates over $\tau$ under structural breaks, $HL_1(\tau\,|y_{t-1} < c(\tau))$ and $HL_2\big(\tau\,|y_{t-1} \geq c(\tau)\big)$. $c(\tau)$ is the threshold parameter. In this figure, the $x$-axis for $HL_2\big(\tau\,|y_{t-1} \geq c(\tau)\big)$ is valued as $\tau + 0.9$. Structural breaks are sequentially tested for means and variances of the data series and treated separately.

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## SUPPLEMENTARY MATERIAL

To view supplementary material for this article, please visit https://doi.org/10.1017/S1365100519000063.

### NOTES

1. See, for example, Verbrugge and Verbrugge (1997), Razzak (2001), Psaradakis and Sola (2003), among others.

2. See, for example, Filardo (1994), Diebold and Rudebusch (1996), Chauvet (1998), Nelson et al. (2001), Ullah (2005), Knüppel (2009), Hamilton (2016), among many others.

3. As discussed in Elwood (1998), the question of asymmetry in persistence is distinct from the issue of the steepness asymmetry. The latter assesses persistence or duration of real output during expansions separately from those during recessions. In contrast, the former observes the lasting influence of shocks to real output, regardless of where they occur in the business cycle.

4. Threshold and Markov-switching autoregressions, used in, that is, Elwood (1998), Clements and Krolzig (2003), among others, allow for changes in the adjusting speed of mean-reversion over regimes. However, within each identified regime, the adjustment is still constant. This is inconsistent with the characterization of the steepness asymmetry, as the conditional mean of a bad regime does not necessarily represent economic recessions or depressions generally associated with deep lower tails.

5. See, for example, Nikolaou (2008), Ferreira (2011), Tsong and Lee (2011), Wolters (2012), Cicek and Akar (2013), Chevapatrakul and Paez-Farrell (2014), Manzan and Zerom (2015), Wolters and Tillmann (2015), Liu (2018), among others.

6. The studies referenced in footnote 5 also employ quantile regression to study asymmetric dynamics, but for different macroeconomic variables, such as inflation, interest rate, exchange rate, etc.

7. The transition from (1) to (2) is an immediate consequence of equivariance to monotone transformations [see the theorem of equivariance to monotone transformations in Koenker (2005), p. 39 and Liu (2016), p. 357].

8. For the purpose of illustration, I leave other possible driving sources out of this paper.

9. See, for example, Choi et al. (2004), Murray and Papell (2005), Rossi (2005), Nikolaou (2008), Tsong and Lee (2011), Cicek and Akar (2013), Kunkler and MacDonald (2015), among others.

10. I consider the mean equation as $y_t = \alpha + \rho y_{t-1} + \sigma_t \varepsilon_t$, and two conditional volatility processes as $M1: ln(\sigma_t) = \omega_0 + \omega_1 ln(\sigma_{t-1}) + \omega_2 |\varepsilon_{t-1}| + a_1 \varepsilon_{t-1}$, and $M2: ln(\sigma_t) = \omega_0 + \omega_1 ln(\sigma_{t-1}) + \omega_2 |\varepsilon_{t-1}| + a_1^- \varepsilon_{t-1} \mathbb{I}(\varepsilon_{t-1} < 0) + a_1^+ \varepsilon_{t-1} \mathbb{I}(\varepsilon_{t-1} \geq 0)$, where $\mathbb{I}(\cdot)$ is an indicator function. Note that $\varepsilon_t = (y_t - \mu_t)/\sigma_t$, where $\mu_t = E_{t-1}(y_t) = \alpha + \rho y_{t-1}$, is the standardized deviation of real GDP growth from its conditional mean.

11. Recent studies, for example, Brunnermeier et al. (2012), Adrian and Brunnermeier (2016), and Liu (2017), have shown that systemic risk is countercyclically accumulated in the background of economic booms.

12. The sample quantiles of the USA and UK real GDP growths in Table 2 are used for Figure 1(a) and (b), respectively. Based on the results in Table 1, I choose $a_1 = -0.048$ from the US estimate, and $a_1 = -0.071$ for $\tau \leq 0.8$ and $a_1 = 0.088$ for $\tau > 0.8$ from the UK estimates. Standard normal distribution is used as an example of symmetric distributions. $\rho = 0.9$ is calibrated from the estimates of the mean equations of the EGARCH models (not reported in Table 1). $\mu_0$ is normalized to one. Note that at some extreme tails, such as $\tau = 0.05, 0.95$, $\rho(\tau)$ has values greater than one. For the purpose of graphical illustration, I set $HL(\tau) = ln(0.5)/ln(0.99) = 69$ quarters (about 17 years) in cases of $\rho(\tau) \geq 1$, which is a reasonable large number to represent $\infty$ in the context of this paper.

13. The steepness asymmetry of business cycles have graphically been shown in Figure 1(b) of Sichel (1993) and Knüppel (2009), Figures 1 and 2 of Gale (1996), and Figure 1 of Kim and Piger (2002). Gale (1996) explains that the steepness asymmetry of business cycles can be caused by

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endogenous delay in investments during a recession which prolongs the recovery from a recession and heightens the effect of the boom. Also see, for example, McQueen and Thorley (1993), Beaudry and Koop (1993), Hess and Iwata (1997), Clements and Krolzig (2003), Psaradakis and Sola (2003), Jovanovic (2006), Nieuwerburgh and Veldkamp (2006), Ordonez (2013), among others.

14. Quarterly real GDP growth rates are computed on a YoY basis (12 month rate of change) as $y_t = log\,(GDP_t/GDP_{t-4}) \times 100\%$, which has been applied in a variety of macroeconomic research topics, see, for example, Cristadoro et al. (2013), Alessi and Detken (2014), Herbst-Murphy (2015), Higgins and Zha (2015), Higgins et al. (2016), among others. Compared to percent changes from previous quarters (QoQ), this YoY-based growth rate is smoother. Nonetheless, the conclusions obtained in this paper are unaltered by using the QoQ-based growth rate, while persistence estimated from the QoQ rate is lower than that from the YoY rate. This paper reports the results estimated from the YoY rate for easier illustrations due to its larger difference in estimated persistence between lower and upper tails.

15. Their Corollary 1 also indicates that even with $\rho(\tau) > 1$ over some range of quantiles, as long as $E\,(\rho(\tau))^2 < 1$, $y_t$ can still be covariance stationary in the long run. Thus, a QAR process may allow for some (transient) forms of explosive behavior while maintaining stationary in the long run. Also see the empirical illustration in Liu (2016), section 7.1.

16. Similar to the simulation in Figure 1, I use $HL(\tau) = ln\,(0.5)\,/ln\,(0.99) \approx 69$ to represent $\infty$ for the purpose of graphical illustration.

17. The general (robust) form of the $F$-statistic in Bai and Perron (2003*a*) is used in place of the difference in sums-of-squared residuals described in Bai (1997) and Bai and Perron (1998). Critical value and response surface computations are provided by Bai and Perron (2003*b*). In this section, I employ EViews to conduct the sequential structural break test.

18. McConnell and Perez-Quiros (2000) state that if $e_t$ follows a normal distribution, $\sqrt{\frac{\pi}{2}}\left|\hat{e}_t\right|$ is an unbiased estimator of the standard deviation of $e_t$. They test structural breaks in $\sqrt{\frac{\pi}{2}}\left|\hat{e}_t\right|$ for the US output volatility.

19. The sequential structural break test of Bai (1997) strongly supports the presence of a structural break in 1984Q4 for the US real GDP growth volatility, similar to the result in McConnell and Perez-Quiros (2000) who find a break dated in 1984Q1. However, the test does not support a structural break in the US real GDP growth volatility during the recent economic recession period of 2008–2009, different from the dated breaks of other data series. This is mainly because the increased variance of the US real GDP growth during this period has returned to its prior-crisis level since 2010, so that it appears to be more temporary, compared to the break dated in 1984 which have stronger permanent effects on the US real GDP growth fluctuations. However, considering the high volatility level of the US real GDP growth during this recession period, I date a break in 2008Q1 to reflect extreme economic fluctuations during this period, although its effect seems temporary.

20. Over the different values of $\tau$ considered in this paper, the average threshold estimate for the USA is 3.577 with the range from 3.099 to 4.299, 2.947 for Canada with the range from 2.033 to 3.580, 2.466 for the UK with range from 2.154 to 3.073, 1.731 for Germany with the range from 1.141 to 1.893, 2.350 for France with the range from 1.991 to 3.693, and 1.831 for EU with the range from 1.582 to 2.275. Except France, these threshold parameter estimates vary closely around the sample means of real GDP growths reported in Table 2. To preserve space, the detailed estimation values are not reported here, but available upon request to the author.

21. A quantile unit root test for TQAR models is not available in the literature, and the development of such a test is beyond the scope of this paper.

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