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Pollution thresholds under uncertainty

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ABSTRACT. This study investigates the optimal use of 'dirty' technologies for economic development and when to stop polluting (either temporarily or forever). This question is addressed within a real option framework where three different kinds of options arise: for stopping pollution temporarily or forever and for re-starting. The possibility to shelve a polluting technology temporarily allows for a more conservationist policy. Costs associated with the option of a re-start lead to hystereses effects that complicate empirical cross-country comparisons of environmental policies.

1. Introduction

This paper addresses the question: how far can and should a country (or a region) test the resilience of its natural resources (the environment at large or, in particular, air, soil, and water) in its pursuit of development? Famous in this respect is Larry Summer's memo during his stint at the World Bank asking why poor countries do not exchange their clean (under-polluted) environment for dollars fostering development. And in the economic research an entire literature – first empirical (Grossman and Krueger, 1995) and then theoretical (e.g. Stokey (1998)) – has focused on what has been called environmental Kuznets curves, i.e. the phenomenon that emissions increase with economic activity at low income levels but decline once incomes surpass a critical level. This suggests (aside from the dependence on incomes and related issues like institutions, corruption, etc.) a different explanation: poor countries may indeed follow Larry Summers' advice and use pollution as a cost saving device to grow and to reduce emissions later, not only because of greener preferences at higher incomes, but also because of the degradation of the environment associated with this kind of development. This leads to the question of how long can or should such a development strategy go on given the increasing external environmental costs. A topical example is the price tag that China seems to pay for its vast recent economic progress (making it to the cover of *The Economist*, 2004). A recent historical and negative example was unsustainable development, in particular in the Soviet Union, where e.g. Feshbach (1995) conjectures that exceeding the resilience of the environment (air, water) and of humans was a major force in purging communism.

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I prefer the term *threshold* to integrate, on the one hand, the related questions of resilience and sustainability (among the most topical issues of environmental economics at least since the 'definition' of sustainability in the famous Bruntland report, WCED (1987)) and, on the other hand, quite diverse approaches. One approach is that of intertemporal efficiency, which according to Heal (2001) must be linked to sustainability. Within such deterministic intertemporal optimization models as advocated in Heal (2001), a number of papers have attributed the existence of thresholds (between ignoring or taking care of the resilience of, e.g., a shallow lake) to non-concavities (see a recent issue of the journal *Environmental and Resource Economics*, 'The economics of non-convex ecosystems', published also as a book, Dasgupta and Mäler (2004)), although this phenomenon persists even broader, see Feichtinger and Wirl (2000), the survey in Deissenberg *et al.* (2004) and Wirl (2004a, b) with reference to renewable resource exploitation and growth. A related yet slightly different line of research focuses on irreversibilities, in particular a special issue of *Resource and Energy Economics* (2000) and a recent contribution by Fisher and Narain (2003). Another approach, and this is the one taken in this paper, is that of real options (see Dixit and Pindyck, 1994). This allows the incorporation of uncertainty, which is a crucial feature of many ecological systems and which is ignored in the above mentioned deterministic models. This real options approach has been applied to environmental problems, in particular to global warming, e.g. in Conrad (1997, 2000), Saphores and Carr (2000), and Pindyck (2000, 2002). In fact, those papers are the starting point for the following investigation. While they focus on irreversible stopping, this study allows also for temporary shelving of polluting technologies (at a cost) and analyzes the differences between the different kinds of stopping. That is, the question is not only when to stop polluting, but also when to restart emitting and when to exit. The model is introduced in section 2 and analyzed in section 3. An example presented in section 4 substitutes for the lack of explicit analytical results and section 5 investigates special cases that allow for closed-form solutions.

2. The model

Current use of the environment (air, water, soil) of the magnitude $x(t) = \bar{x}$ constant (to apply real options theory) in period t delivers the current flow of (also constant) benefits u. However, this usage increases a stock of pollution P, which is described by an Itô-process

$$dP(t) = x(t)dt + \sigma dz(t), \; P(0) = P_0. \tag{1}$$

This process extends the familiar accumulation of a flow, $dP/dt = x$, for uncertainty, σdz, where σ is the standard error and the uncertainty or noise, dz, is the increment of a normalized Wiener process (or Brownian motion). However this stochastic process $P(t)$ is not only a simple book-keeping variable, but measures an underlying force, say the degree of environmental degradation, which is increased by emissions, but which is not completely determined by physical accounting. The classical example is global warming, where temperature is a random process affected by, but not exclusively, the greenhouse effect. Similarly, the water quality in a lake

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depends on the amount of effluents discharged, but not exclusively, for example warm water fosters algae growth, but not cold water. In short, P captures (indirectly) the aspect of the uncertainty of external cost, which is described by a quadratic function of this underlying process

$$C(P) = \frac{c}{2}P^2. \tag{2}$$

These costs (the coefficient $c > 0$ determines the convexity and also the level) are independent of current actions and have to be covered even after pollution has stopped depending on the autonomous development of P, which may continue deteriorating, even absent further pollution: e.g., global temperatures can increase or fall, absent any carbon emissions; species may get extinct without hunting or poisoning, and stopping effluents will not remove all uncertainty about the water quality of a lake.

A benevolent government will maximize the expected net present value of benefits minus costs by choosing among the following actions facing the trade-off between benefits and costs:

1. Pollute, $x(t) = \bar{x}$. This allows the benefit u to accrue.
2. Suspend pollution, $x = 0$, but do not scrap the polluting technology, and rather maintain it at the period costs γ for potential future uses. This argument is often used against phasing out nuclear power completely, namely to keep this source as an option for future uses, because otherwise (with complete exit and shutdown) the corresponding know how will be lost.
3. Re-start pollution, which requires the fixed cost K.
4. Shut down the pollution generating technology forever at no cost except that with it all future uses and benefits are sacrificed.

This should be sufficient to sketch the types of transitions between the different phases, although it can be easily extended to accommodate for further policies, such as that active abatement reduces the emissions to $\eta\bar{x}$, $0 < \eta < 1$, which reduces the net surplus to $\theta u, 0 < \theta < 1$ due to the abatement costs. It is worth mentioning that related 'entry and exit' decisions in the different case of price dynamics (geometric Brownian) are analyzed in Brennan and Schwartz (1985) for a copper mine and in Dixit (1989).

3. Analysis

The analysis considers the different actions described above in isolation before joining them.

3.1. Polluting

Given the strategy of actively polluting, $x(t) = \bar{x}$ and enjoying the corresponding benefit u, the Bellman equation for the value function $V(P)$ is

$$rV(P) = u - \frac{c}{2}P^2 + \bar{x}V'(P) + \frac{\sigma^2}{2}V''(P), \tag{3}$$

where $r > 0$ denotes the subjective time preference of the decision maker, see e.g. Dixit and Pindyck (1994: 107, equation (10)). By the principle of

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superposition, the general solution of (3) is given by a particular solution of (3) plus the solution of the autonomous part

$$V(P) = \left[\frac{u}{r} - \frac{cP^2}{2r} - \frac{c\sigma^2}{2r^2} - \frac{c\bar{x}P}{r^2} - \frac{c\bar{x}^2}{r^3}\right] + c_1 e^{\alpha_1 P} + c_2 e^{\alpha_2 P}, \tag{4}$$

where

$$\alpha_1 = \frac{-\bar{x} + \sqrt{\bar{x}^2 + 2\sigma^2 r}}{\sigma^2} > 0 \quad \text{and} \quad \alpha_2 = \frac{-\bar{x} - \sqrt{\bar{x}^2 + 2\sigma^2 r}}{\sigma^2} < 0, \tag{5}$$

are the roots of the characteristic equation of the homogeneous part of the differential equation (3). The linear combination of the exponential terms is the general solution of the autonomous part of the differential equation and the two coefficients, c_1, c_2, account for the integration constants (or two boundary conditions) associated with the second-order differential equation (3). The term between the squared brackets in (4) is a particular solution of (3) (differentiation proves this) and this has, as is common in these kinds of analyses, an interesting economic interpretation. It is the aggregate net present value of benefits minus costs supposing that the flow of pollution $\bar{x}$ is maintained forever

$$E\left[\int_0^\infty (u - C)e^{-rt}\,dt\right] = \left(\frac{u}{r} - \frac{cP^2}{2r}\right) - \frac{c\sigma^2}{2r^2} - \frac{c\bar{x}P}{r^2} - \frac{c\bar{x}^2}{r^3}, \tag{6}$$

since Ito's calculus yields for the expected change in the costs (associated with the pollution stock)

$$\frac{1}{dt} E\,dC = cP\bar{x} + \frac{c\sigma^2}{2}, \tag{7}$$

that is, (6) reflects the fundamental term. Therefore the exponential terms in (4) account for the benefit from the possibility to stop polluting. This option is definitely of positive value so that the coefficients c_1 and c_2, which determine the level of the value of this option, must be non-negative. However, this option becomes worthless for low values of P (i.e. for $P \to -\infty$) because future stopping is then very unlikely. This requires $c_2 = 0$ so that the second exponential term vanishes in (4) and the value function is

$$V = \left[\frac{\bar{u}}{r} - \frac{cP^2}{2r} - \frac{c\sigma^2}{2r^2} - \frac{c\bar{x}P}{r^2} - \frac{c\bar{x}^2}{r^3}\right] + c_1 e^{\alpha_1 P},\ c_1 > 0. \tag{8}$$

It is important to note that entering this domain is not free (except for starting with pollution, i.e. at $t = 0$) and any re-entry requires paying the entry costs K.

3.2. Stopping

Pollution is suspended if the stock of pollution has reached too high levels, i.e. if it exceeds the critical level denoted $\bar{P}$ (and remains below the threshold of a final and irreversible shutdown). If $x = 0$ evolution of the stock of pollution is reduced to

$$dP = \sigma dz. \tag{9}$$

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No pollution implies no benefits, and in fact keeping the option alive causes additional per period costs γ leading to the Bellman equation

$$rV = -\gamma - \frac{c}{2}P^2 + \frac{\sigma^2}{2}V''(P). \tag{10}$$

The same procedure as sketched in the previous sub-section yields the general solution

$$V(P) = \left[-\frac{\gamma}{r} - \frac{cP^2}{2r} - \frac{c\sigma^2}{2r^2}\right] + k_1 e^{\beta_1 P} + k_2 e^{\beta_2 P}, \tag{11}$$

$$\beta_1 = \frac{\sqrt{2r}}{\sigma} > 0, \quad \beta_2 = -\frac{\sqrt{2r}}{\sigma} < 0. \tag{12}$$

The term between the squared parentheses in (11) is a particular solution of (10) yielding the fundamental term (the expected costs supposing that both, pollution and exiting, are impossible), while the exponential terms account for the two different actually opposite options: of future pollution and of a definite and irreversible exit.

3.3. Exit

The decision to stop polluting entirely and forever leaves only the costs due to future pollution and thus the 'value' is

$$V(P) = -\frac{cP^2}{2r} - \frac{c\sigma^2}{2r^2}. \tag{13}$$

3.4. Joining

The different segments of the value functions are joined by applying the familiar value matching and smooth pasting conditions (see Dixit and Pindyck, 1994: 109, equation (15)) at the crucial threshold levels (in increasing order):

1. Re-polluting, i.e. jumping from $x = 0$ to $x = \bar{x}$ at the cost K. The corresponding threshold is denoted $\underline{P}$.
2. Stopping pollution, i.e. jumping from $x = \bar{x}$ to $x = 0$, yet bearing the costs γ per unit of time to keep the technology ready rather than scrapping it entirely. The corresponding threshold is $\bar{P}$ and exceeds the starting level $\underline{P}$ due to the entry fee K (and these thresholds coincide for $K = 0$). This induces an overlap $\underline{P} < \bar{P}$, i.e. emissions for $P \in [\underline{P} < \bar{P}]$ only if current emissions are positive ($x = \bar{x}$), but not if the polluting technology is on hold ($x = 0$, but no exit); Dixit (1989) discusses such 'hystereses' effects in detail.
3. Exit, shutdown with no possibility of a comeback, i.e. an irreversible transition to a clean technology sacrificing the (flow of) benefit u forever. The corresponding threshold is denoted by $\hat{P}$. This is optimal if the poor (environmental) prospects do not justify the ongoing expenditures γ and the re-entry fee K.

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Applying now the above-mentioned conditions of value matching and smooth pasting at these three levels to the solutions of the value functions (8), (11), and (13) yields six equations for the six unknowns, the coefficients, c_1, k_1, k_2, and the three thresholds, $\underline{P}$, $\bar{P}$, $\hat{P}$. This system is given below

$$
\begin{aligned}
V(\underline{P}_-) &= \left[\frac{u}{r} - \frac{c\underline{P}^2}{2r} - \frac{c\sigma^2}{2r^2} - \frac{c\bar{x}\underline{P}}{r^2} - \frac{c\bar{x}^2}{r^3}\right] + c_1 e^{\alpha_1 \underline{P}} - K \\
&= \left[-\frac{\gamma}{r} - \frac{c\underline{P}^2}{2r} - \frac{c\sigma^2}{2r^2}\right] + k_1 e^{\beta_1 \underline{P}} + k_2 e^{\beta_2 \underline{P}} = V(\underline{P}_+), \\
V'(\underline{P}_-) &= \left[-\frac{c\underline{P}}{r} - \frac{c\bar{x}}{r^2}\right] + c_1\alpha_1 e^{\alpha_1 \underline{P}} \\
&= -\frac{c\underline{P}}{r} + k_1\beta_1 e^{\beta_1 \underline{P}} + k_2\beta_2 e^{\beta_2 \underline{P}} = V'(\underline{P}_+), \\
V(\bar{P}_-) &= \left[\frac{u}{r} - \frac{c\bar{P}^2}{2r} - \frac{c\sigma^2}{2r^2} - \frac{c\bar{x}\bar{P}}{r^2} - \frac{c\bar{x}^2}{r^3}\right] + c_1 e^{\alpha_1 \bar{P}} \\
&= \left[-\frac{\gamma}{r} - \frac{c\bar{P}^2}{2r} - \frac{c\sigma^2}{2r^2}\right] + k_1 e^{\beta_1 \bar{P}} + k_2 e^{\beta_2 \bar{P}} = V(\bar{P}_+), \\
V'(\bar{P}_-) &= \left[-\frac{c\bar{P}}{r} - \frac{c\bar{x}}{r^2}\right] + c_1\alpha_1 e^{\alpha_1 \bar{P}} \\
&= -\frac{c\bar{P}}{r} + k_1\beta_1 e^{\beta_1 \bar{P}} + k_2\beta_2 e^{\beta_2 \bar{P}} = V'(\bar{P}_+), \\
V(\hat{P}_-) &= \left[-\frac{\gamma}{r} - \frac{c\hat{P}^2}{2r} - \frac{c\sigma^2}{2r^2}\right] + k_1 e^{\beta_1 \hat{P}} + k_2 e^{\beta_2 \hat{P}} \\
&= \left[-\frac{c\hat{P}^2}{2r} - \frac{c\sigma^2}{2r^2}\right] = V(\hat{P}_+), \\
V'(\hat{P}_-) &= \left[-\frac{c\hat{P}}{r} - \frac{c\bar{x}}{r^2}\right] + k_1\beta_1 e^{\beta_1 \hat{P}} + k_2\beta_2 e^{\beta_2 \hat{P}} \\
&= \frac{-c\hat{P}}{r} = V'(\hat{P}_+). \qquad (14)
\end{aligned}
$$

This system looks awful, but can be simplified substantially, because it is linear in the coefficients and nonlinear only in the critical levels. The latter (linear plus exponential terms containing the thresholds) impedes an explicit closed-form solution but would still allow, in principle, the derivation of qualitative results analytically by applying the implicit function theorem to the reduced-form equations for the levels resulting from the value-matching conditions and substituting the coefficients as functions of the three threshold levels. However, due to the complexity of these cumbersome and lengthy expressions, these properties are shown

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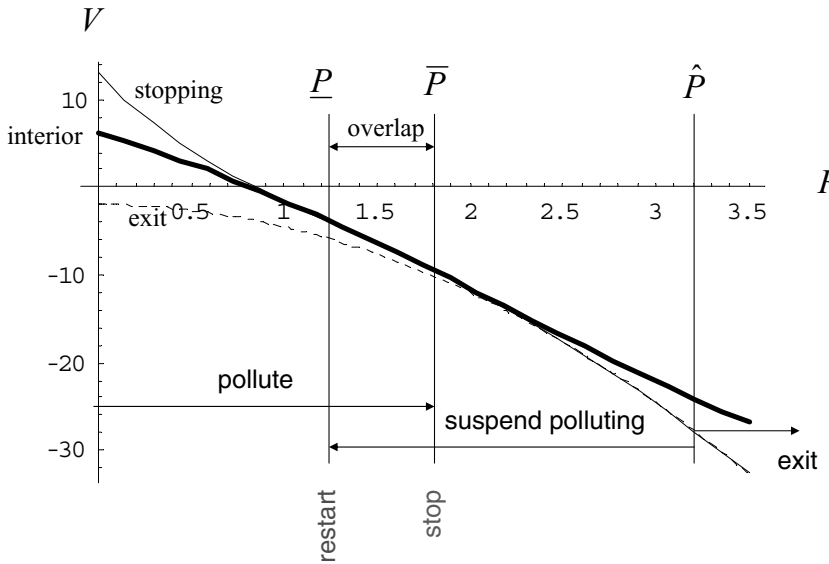


Figure 1. *Actions, their domain and the corresponding value functions for the reference parameters in (15)*

via numerical examples in the following section. In addition, simplified versions are studied that allow for an explicit analytical solution.

4. A numerical example

The following purely numerical example

$$u = 1, K = 0.1, \gamma = 0.01, c = 1/4, \bar{x} = 0.1, r = 0.05, \sigma = 0.2, \tag{15}$$

can be interpreted in the following way: the flow of benefits (u) is normalized to 1$ per unit of time (say a year), which provides an evaluation of the corresponding cost items (in relative terms, percentages, or in cents), e.g. restarting (K) costs 10 cents (once) and keeping the technology on the shelves for future uses (γ) costs 1 cent per unit of time; the cost parameter c states that damage equals benefit (of 1$) at the pollution stock of $P = 2$ and given the pollution flow of $\bar{x} = 0.1$ yields the expectation of 20 periods of polluting to reach $P = 2$ for starting at $P_0 = 0$; the discount rate equals 5% per annum and the standard deviation is 0.2 (which corresponds to 10% of the pollution stock level of $P = 2$).

Figure 1 plots the value functions and the corresponding thresholds, i.e., where the different parts are joined, and the corresponding optimal actions. Optimal stopping is at the pollution level $\bar{P} = 1.82$. This is below the above-mentioned break even and gives the country an expected horizon of 18 years of polluting. Restarting requires a substantially lower pollution stock, $\underline{P} = 1.26$, due to the assumption of a significant (but not really large, 10% of the annual benefit) entry fee. A consequence of such overlaps is that cross-country comparisons of pollution intensities are problematic given

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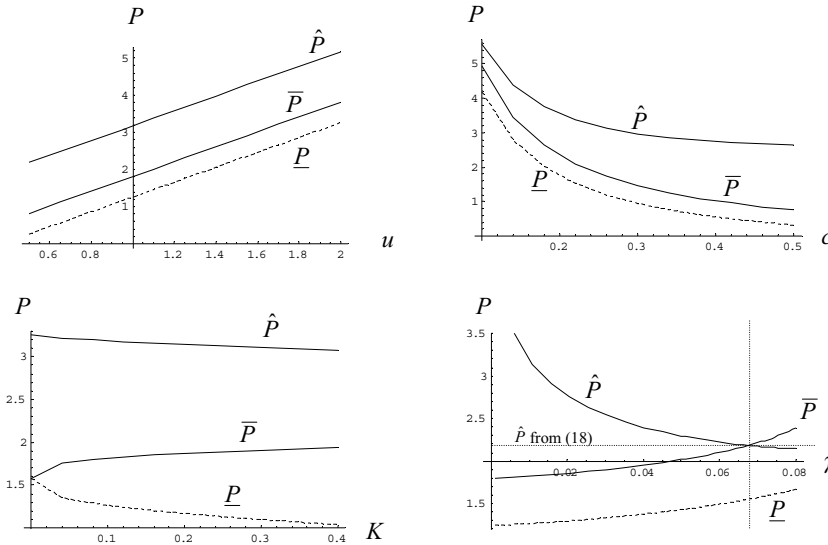


Figure 2. *Sensitivity I of thresholds due to variations of the (reference) parameters (15)*

such hystereses effects, since it is optimal to continue polluting in one instance (if polluting at the moment) but not in the other case (if the polluting technology is at the moment not in use). Optimal scrapping should be delayed until pollution exceeds the rather high level $\hat{P} = 3.19$; nothing is emitted since $P > \bar{P} = 1.82$ and possibly already for $P > \underline{P} = 1.26$, thus the probability of such a shutdown is not very high.

Figures 2 and 3 vary each of the parameters in isolation showing the following:

- Higher external costs (increasing c) reduce all three thresholds $\underline{P}$, $\bar{P}$, $\hat{P}$ and the sensitivity is in this (decreasing) order: Higher benefits (raising u but for unchanged pollution flows $\bar{x}$) increase all thresholds and the opposite holds for higher emissions $\bar{x}$ (because holding u simultaneously constant amounts to a reduction of benefits per unit emission). Of these three thresholds, $\underline{P}$ is the most sensitive one: doubling the benefit roughly triples the threshold $\underline{P}$, doubling the external costs reduces the threshold to one-fifth. Least sensitive is in both cases the shutdown decision $\hat{P}$, which is increased by only 50 per cent for a doubling of the benefit and reduced by less than 20 per cent for twice the external cost. Since the sensitivity of $\bar{P}$ is in between, the overlap and the stopping domain increase with respect to external costs but decrease with higher benefits.
- Higher entry costs (increasing K) widen the stopping domain, but primarily the overlap: $\underline{P}$ declines, $\bar{P}$ increases, and $\hat{P}$ decreases (but is actually fairly insensitive). The economic reason is that paying a higher entry fee only makes sense if the expected duration of (beneficial) operating is sufficiently long. In contrast, larger maintenance costs (γ)

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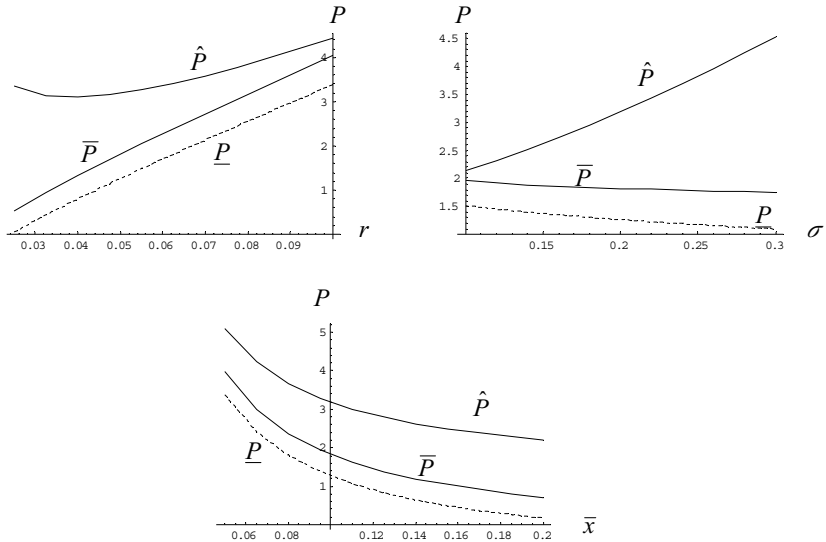


Figure 3. *Sensitivity II of thresholds due to variations of the (reference) parameters (15)*

reduce the exit threshold $\hat{P}$ substantially (clearly, higher costs for holding the option of a later re-entry render this option less profitable), increase $\underline{P}$ as well as $\bar{P}$, but have little impact on the size of the overlap (the difference between stopping and re-entry) because that depends primarily on the entry fee K.

- $\hat{P}$ can fall below $\bar{P}$ with the consequence that an immediate exit is optimal rather than holding on to this technology given the high associated costs, see figure 2 (for $\gamma > 0.068\ldots$). Yet if exit rather than stopping is advisable, then the re-entry option is also obsolete and the value of $\hat{P}$ is calculated on wrong presumptions. The proper solution for this irreversible exit without any (temporary) stopping is (even analytically) derived in section 4 as one of the two special cases (but is already shown in figure 2 for the variation in γ).
- Higher discount rates (r) increase the entry and stopping levels ($\underline{P}$ and $\bar{P}$), while a higher uncertainty (an increase in σ) lowers them. The exit level $\hat{P}$ is non-monotonic in discounting and shows little sensitivity. In contrast, the shutdown decision $\hat{P}$ is sensitive to the degree of uncertainty and increases with respect to this parameter, because higher uncertainty increases also the probability that the stock of pollution is drastically reduced.

5. Special cases

5.1. Irreversible exit

The first special case concerns an irreversible exit, i.e. a direct transition from polluting to an irreversible shutdown without the detour of keeping the

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dirty technology on hold without using it. This is not only a simplification but is in fact optimal for high values of γ, i.e. if $\hat{P} < \bar{P}$, as already discussed in the previous section. An alternative justification is that any shutdown impedes future usage due to high re-start costs; this is the implicit assumption in Pindyck (2000). Whatever the reason (a high γ or high restart costs), it is assumed that the restart option is not available, so that the optimal solution consists of joining the interior (8) with the exit value function (11) via value matching and smooth pasting

$$V(\widehat{P}_+) = \left[\frac{u}{r} - \frac{c\overline{P}^2}{2r} - \frac{c\sigma^2}{2r^2} - \frac{c\overline{x}\overline{P}}{r^2} - \frac{c\overline{x}^2}{r^3}\right] + c_1 e^{\alpha_1 \overline{P}}$$

$$= \left[-\frac{c\widehat{P}^2}{2r} - \frac{c\sigma^2}{2r^2}\right] = V(\widehat{P}_-), \tag{16}$$

$$V'(\widehat{P}_+) = \left[-\frac{c\overline{P}}{r} - \frac{c\overline{x}}{r^2}\right] + c_1\alpha_1 e^{\alpha_1 \overline{P}} = -\frac{c\widehat{P}}{r} = V'(\widehat{P}_-). \tag{17}$$

The two equations allow for a closed-form solution (already obtained in Pindyck, 2000)

$$\widehat{P} = \frac{ru}{\gamma\overline{x}} - \frac{\overline{x}}{r} + \frac{1}{\alpha_1} \tag{18}$$

and differentiating (18) shows that *higher discount rates, larger benefits from emissions, and, surprisingly, higher uncertainty (a larger σ) increase the threshold, but higher external costs and higher emissions lower the threshold $\hat{P}$.*

5.2. $K = \gamma = 0$

Absent the costs for entry, K, and for shelving, γ, the two thresholds $\underline{P}$ and $\bar{P}$ collapse into a single switching point, say $\tilde{P} = \underline{P} = \bar{P}$. Furthermore, an irreversible exit makes no sense given no costs for holding the option of future pollution. This eliminates also $\hat{P}$ and the analysis can be restricted to a single threshold, as in the previous subsection. Finally it reduces the valuation of the exit option when pollution is only suspended so that $k_1 = 0$. Summarizing, accounting for the absence of costs ($\gamma = 0$) yields the following solution of the value function in the stopping domain

$$V(P) = \left[-\frac{1}{2}\frac{cP^2}{r} - \frac{1}{2}\frac{c\sigma^2}{r^2}\right] + k_2 e^{-\frac{\sqrt{2r}}{\sigma}P},\ k_2 > 0 \quad \text{for } P > \widetilde{P}.$$

In spite of all these simplifications – elimination of two thresholds and of a coefficient – the solution faces an additional problem since the corresponding value matching and smooth pasting conditions are eliminated so that only two (of the original six) equations, one for value matching (properly simplified)

$$\left[\frac{u}{r} - \frac{c\overline{x}\widetilde{P}}{r^2} - \frac{c\overline{x}^2}{r^3}\right] + c_1 e^{\alpha_1 \widetilde{P}} = k_2 e^{\beta_2 \widetilde{P}}, \tag{19}$$

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and another for smooth pasting

$$-\frac{c\overline{x}}{r^2}+c_1\alpha_1 e^{\alpha_1\widetilde{P}}=k_2\beta_2 e^{\beta_2\widetilde{P}}, \tag{20}$$

and three unknowns, c_1, k_2, and $\tilde{P}$, remain. This lack of a boundary condition is due to costless switching such that the two options, to stop and to re-start, coincide at $\tilde{P}$ and explicit maximization will provide the necessary third condition. Using the two equations (19) and (20) to solve for c_1 and k_2 and substituting into the interior part of the value function yields

$$V(P;\widetilde{P})=\left[\frac{u}{r}-\frac{cP^2}{2r}-\frac{c\sigma^2}{2r^2}-\frac{c\overline{x}P}{r^2}-\frac{c\overline{x}^2}{r^3}\right] + \frac{\beta_2}{\alpha_1-\beta_2}e^{\alpha_1(P-\widetilde{P})}\left(\frac{u}{r}-\frac{c\overline{x}\widetilde{P}}{r^2}-\frac{c\overline{x}^2}{r^3}+\frac{c\overline{x}}{\beta_2 r^2}\right). \tag{21}$$

As indicated, the missing boundary condition will be obtained by maximizing the value with respect to the remaining parameter $\tilde{P}$, i.e. $\max V_{\tilde{P}}(P;\ \tilde{P})$. This implies the first-order condition

$$V_{\widetilde{P}}=\frac{e^{\alpha_1(P-\widetilde{P})}[r\beta_2 c\overline{x}+\alpha_1(rc\overline{x}+\beta_2(r^2u-rc\overline{x}\widetilde{P}-c\overline{x}^2))]}{(\beta_2-\alpha_1)r^3}=0, \tag{22}$$

which solved for $\tilde{P}$ yields

$$\widetilde{P}=\frac{ru}{c\overline{x}}-\frac{\overline{x}}{r}+\frac{1}{\alpha_1}+\frac{1}{\beta_2}. \tag{23}$$

The so-obtained stopping level $\tilde{P}$ maximizes the value, because

$$V_{\widetilde{P}\widetilde{P}}=\frac{e^{\alpha_1(P-\widetilde{P})}\alpha_1}{(\alpha_1-\beta_2)r^3}\{r\beta_2 c\overline{x}-[r\beta_2 c\overline{x}+\alpha_1(rc\overline{x}+\beta_2(r^2u-rc\overline{x}\widetilde{P}-c\overline{x}^2))]\}<0,$$

since the term between the squared brackets equals $V_{\tilde{P}}=0$, $(\alpha_2-\beta_2)>0$ and $\beta_2<0$.

The derivation can be summarized as follows: Reversible stopping implies more conservationist stopping, $\tilde{P}<\hat{P}$ (which follows directly from comparing (18) with (23) and accounting for $\beta_2<0$). The qualitative effects from parameter variations (which follow from differentiating (23)) are identical to the irreversible case with one exception: higher uncertainty lowers (in line with economic intuition) the threshold for reversible stopping, but increases it in the case of an irreversible exit. This difference is substantial for the parameters of the reference example: $\tilde{P}=1.55$ versus $\hat{P}=2.18$. Figure 4 shows and compares the different consequences of uncertainty on the two threshold levels $\tilde{P}$ and $\hat{P}$. The economic reason for this much less conservationist policy in case of irreversible stopping is that continuing to pollute is the only way to keep the option of future and potentially beneficial uses open.

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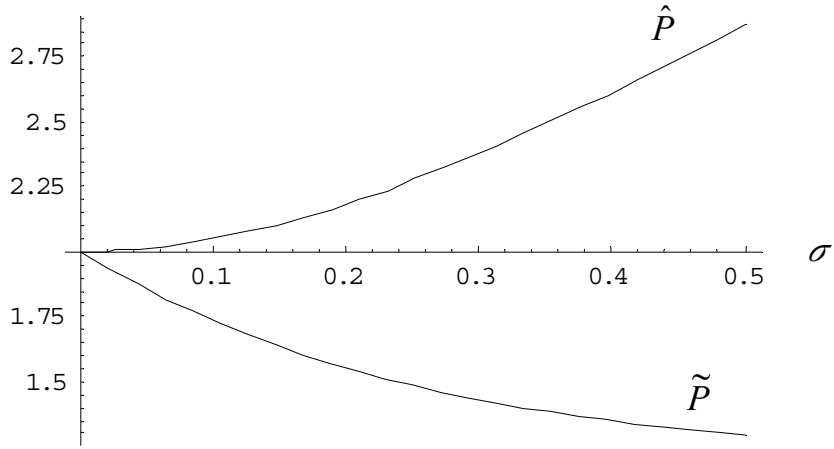


Figure 4. *Comparison of the special case ($K = \gamma = 0$): reversible and irreversible stopping versus uncertainty otherwise the reference parameters from (15)*

6. Summary

This paper investigated (socially) optimal strategies of pollution as means for economic development (or formulated differently when to engage in environmental protection) under uncertainty allowing for different kinds of stopping: a complete exit from a polluting technology or a temporary stopping, which allows, at a cost, for future pollution if the environment (at large, or a particular river, lake, etc.) recovers.

Technically speaking optimal stopping (and starting and exiting) levels were derived by applying the real options approach. This requires the joining of three different value functions and in the end numerics, yet simplified versions were solved analytically. Option values are attached to the three possibilities: to stop emissions temporarily, to exit entirely and to re-start polluting (for an 'entry fee'). The possibility to shelve a polluting technology that could be still beneficial in the future allows for more conservationist strategies compared with the case in which any stopping is irreversible (as assumed in the reviewed literature). Such an irreversible stopping – scrapping a technology for all the future – is rational only at high pollution stocks, i.e. the domain of keeping but not using a polluting technology is large. And indeed, discussions about a restart of 'polluting' after a period of banning is observable in developing countries, such as in the case of the elephant and DDT,[1] and also in developed countries in the case of whales. However, the option to stop rather than to exit can be void if the costs of holding but not using a technology or of re-entering are very large. The costs associated with a re-start lead to hystereses effects

[1] DDT (Dichlordiphenyltrichlorethan) was originally prepared in 1873, but it was not until 1939 that Paul Muller of Geigy Pharmaceutical discovered the effectiveness of DDT as an insecticide and he was awarded the Nobel Prize in medicine and physiology in 1948. It was banned in the US in 1972.

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(already emphasized in Dixit (1989) in a different context) such that the policy does not only depend on the current state of the system (the stock of pollution), but also on the history in the sense of whether polluting is 'on' or 'off'. This makes cross-country comparisons about the efficiency of national (or regional) pollution strategies difficult, unless one accounts for this difference.

Further research can depart in many directions, e.g. one may consider more general processes (geometric Brownian noise, depreciation, etc.), separating costs from the stock of pollution (i.e. considering two states and more), more general controls (i.e. continuous choice of the flow of pollution, $x(t) \in [0, \infty)$), or even strategic interactions.

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