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Electrostatic nonlinear supersolitons in dusty plasmas

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Using the Sagdeev pseudopotential method, the properties of negative dust-acoustic supersolitons are investigated in a plasma comprising cold negative dust and Boltzmann and Cairns non-thermal positive ions. Supersolitons have electric field profiles with additional wiggles on traditional bipolar structures, and their existence requires pseudopotentials having two sub-wells before a root is reached. Once the existence ranges are determined in parameter space, typical pseudopotential, supersoliton and electric field profiles can be easily generated. Their characteristics are very robust and yield dust and dust-ion-acoustic supersolitons in a wide range of compositional models and parameter ranges.

1. Introduction

Supersolitons are a recently discovered form of electrostatic solitons in plasmas, which differ from their traditional cousins by their electric field signatures. These show additional wiggles on standard bipolar structures, even though the soliton profiles could look very similar. The terminology was coined by Dubinov and Kolotkov (2012), who considered a five-species dusty plasma, but discussed only one single set of parameter values. It was later shown that supersolitons were not an artefact of complicated plasmas, but could already be generated in several three-species models (Hellberg et al. 2013; Verheest et al. 2013a, b, c). It is obvious that the more species one includes, the easier it becomes for supersolitons to occur, as exemplified by the four-species model investigated by Maharaj et al. (2013).

Supersolitons have typical characteristics, some of which, remarkably, were already present in earlier papers (White et al. 1972; Verheest 2009; Baluku et al. 2010; Verheest 2011; Das et al. 2012), but were not recognized as being special. Although White et al. (1972) clearly indicated that an additional soliton range can exist, separated from the standard range by a discontinuous jump in amplitude, there seems to have been no further exploration of this, mainly because the electric field signatures were not investigated, and their wiggles are not seen. There are indications that such modified bipolar structures have been observed in Cluster observations (Pickett et al. 2004). Even when supersolitons have rather small amplitudes and/or are barely superacoustic, they cannot be described by reductive perturbation theory but need a fully nonlinear treatment via the Sagdeev (1996) pseudopotential technique.

Dusty plasmas comprise charged dust grains, besides the ubiquitous electrons and ions, and occur commonly in space, industrial and laboratory situations (Shukla and

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Eliasson 2009). As the dust charge-to-mass ratio is much smaller than that of ions and electrons, the dust grains introduce a timescale that is much longer, leading to a low frequency mode in which dust inertia plays a key role. In a seminal paper, Rao et al. (1990) discussed both linear and nonlinear behaviour of dust-acoustic (DA) wave, a low frequency analogue of the ion-acoustic mode, the restoring force being provided by mobile electrons and/or ions and the inertia by the dust grain mass. There are also modifications of ion-acoustic modes in dusty plasmas, called the dust-ion-acoustic (DIA) mode (Shukla and Silin 1992), with a charge imbalance between electrons and ions.

In this paper I revisit a plasma comprising cold negative dust in the presence of two positive ion species: a cooler one with a Boltzmann distribution, and a hotter one with a non-thermal Cairns distribution (Cairns et al. 1995), all electrons assumed accreted unto the dust. This model was discussed previously (Verheest 2011), with emphasis on ordinary DA solitons and double layers, and possible coexistence of positive and negative solutions. Coexistence means that there are regions of parameter space in which either positive or negative potential solitons may occur, depending on initial disturbances. However, in the present paper the focus will be solely on DA supersolitons.

Analogous studies have addressed DA supersolitons in plasmas with two Boltzmann or kappa superthermal ion species (Hellberg et al. 2013), for all kappa down to very hard superthermal spectra, or DIA supersolitons in plasmas with stationary negative dust, cold positive ions and non-thermal Cairns electrons (Verheest et al. 2013b).

2. Theoretical treatment and numerical discussions

The cold dust fluid equations of continuity and momentum and the two ion densities yield the overall charge density,

$$\rho(\varphi, M) = f\exp\left(-\frac{\varphi}{\tau}\right) + (1-f)(1+\beta\varphi+\beta\varphi^2)\exp(-\varphi) - \left(1+\frac{2\varphi}{M^2}\right)^{-1/2}. \quad (2.1)$$

Here φ is the electrostatic potential, measured in terms of T_h/e, where T_h is the temperature (in energy units) that the hot ions would have in the absence of non-thermal effects. In an inertial frame where the undisturbed plasma is at rest, the soliton velocity is V, but the treatment uses a frame co-moving with the nonlinear structure and a Mach number $M = V/V_{da}$ in terms of a DA velocity defined through $V_{da}^2 = Z_d T_h/m_d$, with Z_d/m_d being the absolute value of the dust charge-per-mass ratio. Further compositional parameters are the cool-to-hot ion temperature ratio $\tau = T_c/T_h < 1$, the non-thermality parameter β (Cairns et al. 1995) and the fractional cool ion density f.

Integration of Poisson's equation leads to an energy integral

$$\frac{1}{2}\left(\frac{d\varphi}{dx}\right)^2 + S(\varphi, M) = 0, \quad (2.2)$$

with the Sagdeev pseudopotential given by (Verheest 2011)

$$S(\varphi, M) = M^2\left(1-\sqrt{1+\frac{2\varphi}{M^2}}\right) + f\tau\left[1-\exp\left(-\frac{\varphi}{\tau}\right)\right] + (1-f)\left[1+3\beta-(1+3\beta+3\beta\varphi+\beta\varphi^2)\exp(-\varphi)\right]. \quad (2.3)$$

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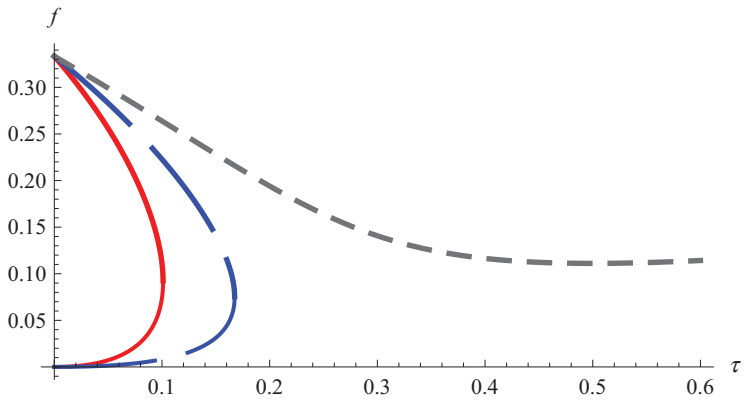


FIGURE 1. (Color online) Variation of the critical density fraction, f_c, with τ along the curve $S'''(0, M_s) = 0$, for $\beta = 0$ (solid curve), 0.3 (dashed curve) and 0.5 (dotted curve).

To have solitary waves, one needs the undisturbed conditions ($\varphi = 0$) to correspond a local maximum of $S(\varphi, M)$, at least on one side, but for supersolitons the conditions are that $S(\varphi, M)$ have two distinct sub-wells on the same side (Verheest et al. 2013a), and this is equivalent to obtaining at least some $S(\varphi, M)$ with three roots of the same sign. On the negative polarity side, φ will be limited by $\varphi_c = -M^2/2$, the condition for the dust density to be real. On the positive side, $S(\varphi, M) \to -\infty$ for $\varphi \to +\infty$, which, coupled with the required convexity conditions at $\varphi = 0$, means that if positive roots exist, they must come in pairs. Up to now, models with four roots of one polarity have not been encountered, thus the positive soliton range is limited by double layers and no roots beyond: there can be no positive supersolitons. Hence, the focus will only be on negative supersolitons, as the usual positive solitons have been discussed before (Verheest 2011).

The convexity condition $S''(0, M) \leqslant 0$ leads to a minimum value for M, given by

$$M^2 \geqslant M_s^2 = [f/\tau + (1 - f)(1 - \beta)]^{-1}\,, \tag{2.4}$$

where primes denote derivations of $S(\varphi, M)$ with respect to φ. The lower bound M_s corresponds to the (normalized) global DA velocity, and thus M/M_s represents the true Mach number for the given plasma composition. Theoretically, we know that the sign of

$$S'''(0, M_s) = f/\tau^2 + (1 - f) - 3\,[f/\tau + (1 - f)(1 - \beta)]^2 \tag{2.5}$$

determines the polarity of the KdV-like solutions (Baluku et al. 2010), but these might have large amplitudes if M can be increased sufficiently above M_s. Indeed, 'KdV-like' only means that their amplitude becomes infinitesimally small as $M \to M_s$ and vanish at $M = M_s$ itself.

The curves along which $S'''(0, M_s) = 0$ are drawn in Fig. 1 for some values of β, in the space of $\{\tau, f\}$. In the concave part of the curves for $\beta = 0$ and $\beta = 0.3$, $S'''(0, M_s) > 0$, while in the convex part, $S'''(0, M_s) < 0$. For $\beta < 0.423$, the two values coalesce at a maximal τ, whereas beyond $\beta = 0.423$ there is only one switch-over curve that can be continued all the way up to $\tau = 1$. Below the curve for $\beta = 0.5$ (a commonly used value, already encountered in the original paper by Cairns et al. 1995) $S'''(0, M_s) > 0$, while above it $S'''(0, M_s) < 0$. For negative supersolitons one needs to pick fairly high values of f when $\beta > 0.423$, but these have not been

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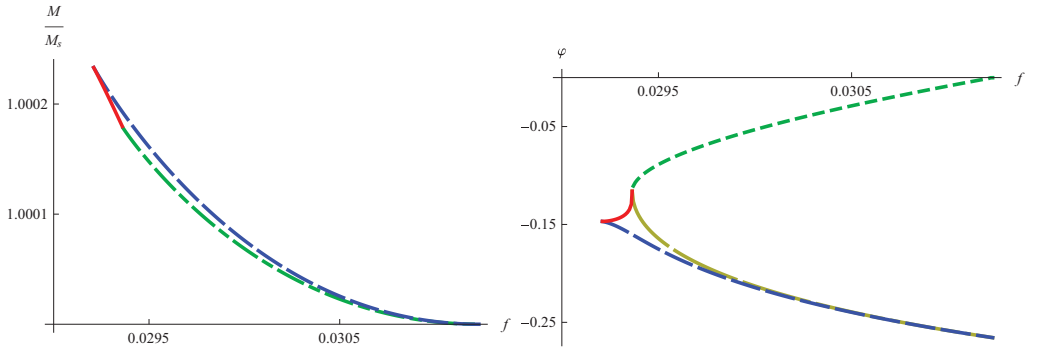


FIGURE 2. (Color online) Left panel: Negative supersolitons are found above the green dotted curve (occurrence of negative double layers) or the red solid curve (emergence of two sub-wells) and below the blue dashed curve (merging of two sub-wells) for $\beta = 0.3$ and $\tau = 0.15$. Right panel: In terms of amplitudes, the curves have the same coding as in the left panel, with the addition of a dark yellow long-dashed curve, corresponding to the third root of the double layer pseudopotentials and giving the minimum supersoliton amplitude in absolute value.

found in this range, only standard solitons limited by infinite dust compression or by exceeding the validity of model assumptions.

Figure 2 shows that for typical parameter values, $\beta = 0.3$ and $\tau = 0.15$, negative double layers occur in the range $0.02936 < f < 0.03123$, and their pseudopotentials have a third root, which is larger in absolute value. The lower limit $f = 0.02936$ indicates the merging of three negative roots into a triple root, an example of this is illustrated in Fig. 5, and at the upper limit $f = 0.03123$, the double layer amplitudes vanish. For $f > 0.03123$, the polarities change and there would initially be a range of positive double layers (Verheest 2011) outside the scope of the present analysis. I have checked that the second range of negative KdV-like solitons, for $f > 0.14090$, does not sustain supersolitons.

The third root of a pseudopotential having a double layer serves as a lower limit for supersoliton amplitudes, reasoning in absolute values. A closer perusal of Fig. 2 reveals a tiny range $0.02921 < f < 0.02936$ below the triple point value, where supersolitons are still possible, but their existence domain is limited by the emergence or coalescence of two sub-wells, which are one of the essential characteristics of this type of solitons.

Once the existence ranges are established, as in Fig. 2, it is straightforward to pick a value of f and draw corresponding pseudopotentials, as many as one wishes, which then differ for a given plasma composition by the variation of M. This is illustrated in Fig. 3, for the compositional parameter values: $f = 0.02950$, $\beta = 0.3$ and $\tau = 0.15$. Other variations in compositional parameters have been investigated and yield qualitatively similar diagrams (not shown here). In the left panel of Fig. 3, the pseudopotentials have been presented in a traditional manner, yielding for increasing M a standard soliton (blue dotted curve), a double layer (green dashed curve) and a supersoliton (red solid curve). Further increase of M would soon result in the merger of the two sub-wells, caused by the coalescence of the two local extrema closest to $\varphi = 0$. Once this happens, one is again in the domain of usual solitons, which may be limited in amplitude by infinite dust compression (Verheest 2011) or exceeding the model assumptions.

In the right panel of Fig. 3, the corresponding hodographs are given, plotting $d\varphi/dx$ vs. φ. Here the discontinuous jump in amplitude from the double layer to

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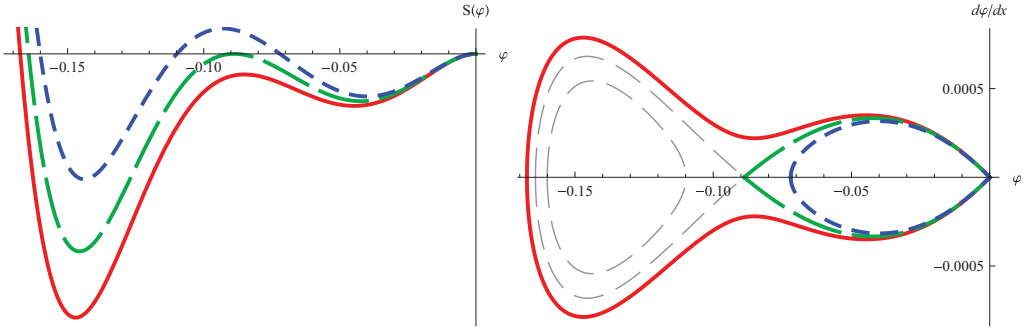


FIGURE 3. (Color online) Left panel: Pseudopotentials for $f = 0.02950$, $\beta = 0.3$ and $\tau = 0.09$, yielding a standard soliton (blue dotted curve, $M/M_s = 1.000144$), a double layer (green dashed curve, $M/M_s = 1.000148$) and a supersoliton (red solid curve, $M/M_s = 1.000151$), respectively. There being no positive supersolitons, the positive φ domain has been omitted. Right panel: Hodographs, plotting $d\varphi/dx$ as functions of φ. Thin dashed curves in gray indicate ranges which are not accessible from the undisturbed conditions.

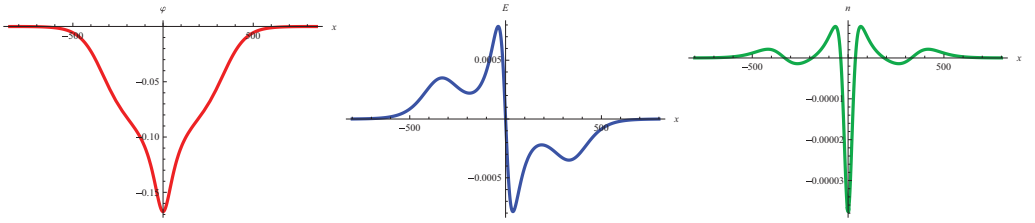


FIGURE 4. (Color online) From left to right: Soliton profile, electric field E and total charge density ρ for the supersoliton corresponding to the red solid pseudopotential curve given in Fig. 3.

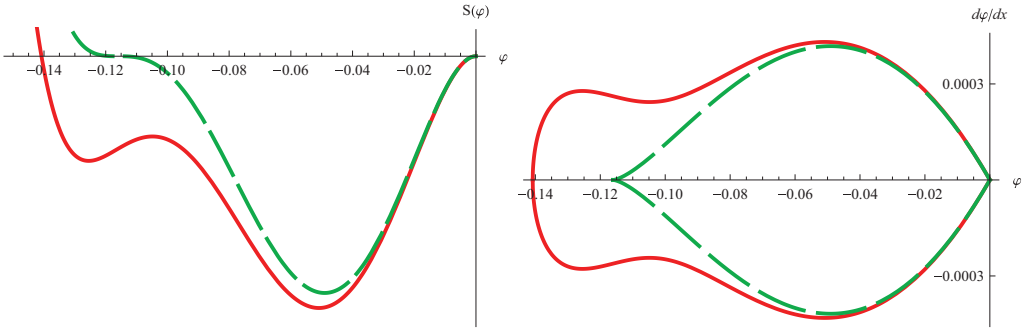


FIGURE 5. (Color online) Left panel: Pseudopotentials for $f = 0.02936$, $\beta = 0.3$ and $\tau = 0.15$, yielding a triple layer (green dashed curve, $M/M_s = 1.000178$) and a supersoliton (red solid curve, $M/M_s = 1.0000180$). Right panel: Hodographs, plotting $d\varphi/dx$ as functions of φ.

supersoliton is obvious, as are the parts of pseudopotentials which cannot be accessed from the undisturbed conditions. Such hodographs were already present in earlier work (Baluku et al. 2010), but their information was not fully used, and the importance of hodographs was stressed by Dubinov and Kolotkov (2012).

The next steps are to compute a supersoliton profile and its corresponding electric field E, as illustrated in the left and middle panels of Fig. 4. In addition, it is

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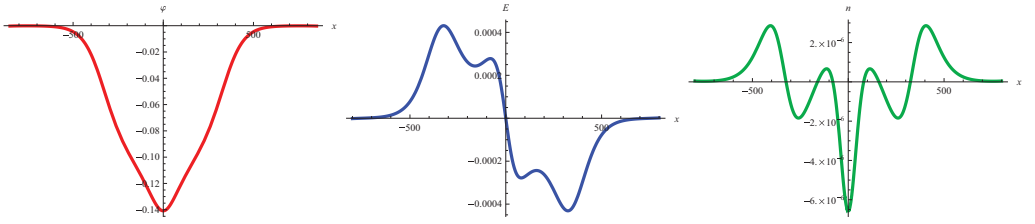


FIGURE 6. (Color online) From left to right: Soliton profile, electric field and total charge density for the supersoliton corresponding to the red solid curve in Fig. 5.

instructive to include here the global charge density, $\rho(\varphi, M)$, as given by (2.1). This is a seven-layer structure, whereas for an ordinary soliton it is a three-layer structure.

An analogous discussion is given for $f = 0.02936$, where the pseudopotential has a negative triple root. It is drawn in Fig. 5, together with a pseudopotential having a supersoliton. The relative positions of deeper and shallower bulges have been interchanged in Figs. 3 and 5, which will influence the height of wiggles in electric fields, as indicated by a comparison between Figs. 4 and 6.

3. Conclusions

To summarize, I have dealt with DA supersolitons in a three-species dusty plasma, where the negative dust provides inertia, and the two hot ion species provide pressure. The existence range is associated with that of (negative) double layers, in the regime where these have a third root, which all of a sudden becomes accessible when M is increased beyond the double-layer value. However, the supersoliton range (in the model treated here) is not limited in amplitude by the constraint imposed by the reality of dust density, but by the coalescence of two sub-wells. This occurs much before the dust reality or other limits come in, which are of importance for ordinary solitons beyond the supersoliton range. In this sense, the supersolitons are sandwiched between two ranges of solitons. Other typical supersoliton characteristics are the wiggles on the electric field, and an overall charge density with seven instead of three charge layers. Finally, a comparison of different graphs in this paper with those obtained in earlier papers (Dubinov and Kolotkov 2012; Hellberg et al. 2013; Maharaj et al. 2013; Verheest et al. 2013a, b, c) clearly shows that typical supersoliton characteristics are very robust and supersolitons can be generated in a wide range of compositional models and parameter ranges, and are not just an artefact of very complicated plasma compositions.

REFERENCES

Baluku, T. K., Hellberg, M. A. and Verheest, F. 2010 New light on ion acoustic solitary waves in a plasma with two temperature electrons. *Europhys. Lett.* **91**, 15001.

Cairns, R. A., Mamun, A. A., Bingham, R., Boström, R., Dendy, R. O., Nairn C. M. C. and Shukla, P. K. 1995 Electrostatic solitary structures in non-thermal plasmas. *Geophys. Res. Lett.* **22**, 2709–2712.

Das, A., Bandyopadhyay, A. and Das, K. P. 2012 Dust ion-acoustic solitary structures in non-thermal dusty plasma. *J. Plasma Phys.* **78**, 149–164.

Dubinov, A. E. and Kolotkov, D. Yu. 2012 Ion-acoustic super solitary waves in dusty multispecies plasmas. *IEEE Trans. Plasma Sci.* **40**, 1429–1433.

Hellberg, M. A., Baluku, T. K., Verheest, F. and Kourakis, I. 2013 Dust-acoustic supersolitons in a three-species dusty plasma with kappa distributions. *J. Plasma Phys.* **79**, 1039–1043.

https://doi.org/10.1017/S0022377814000282 Published online by Cambridge University Press

Maharaj, S. K., Bharuthram, R., Singh, S. V. and Lakhina, G. S. 2013 Existence domains of dust-acoustic solitons and supersolitons. *Phys. Plasmas* **20**, 083705.

Pickett, J. S., Chen, L.-J., Kahler, S. W., Santolík, O., Gurnett, D. A., Tsurutani, B. T. and Balogh, A. 2004 Isolated electrostatic structures observed throughout the Cluster orbit: relationship to magnetic field strength. *Ann. Geophys.* **22**, 2515–2523.

Rao, N. N., Shukla, P. K. and Yu, M. Y. 1990 Dust-acoustic waves in dusty plasmas. *Planet. Space Sci.* **38**, 543–546.

Sagdeev, R. Z. 1966 Cooperative phenomena and shock waves in collisionless plasmas. In *Reviews of Plasma Physics* Vol. 4 (ed. M. A. Leontovich). New York, NY: Consultants Bureau, 1966, pp. 23–91.

Shukla, P. K. and Eliasson, B. Y. 2009 Colloquium: fundamentals of dust-plasma interactions. *Rev. Mod. Phys.* **81**, 25–44.

Shukla, P. K. and Silin, V. P. 1992 Dust ion-acoustic wave. *Phys. Scr.* **45**, 508.

Verheest, F. 2009 Nonlinear acoustic waves in nonthermal plasmas with negative and positive dust. *Phys. Plasmas* **16**, 013704.

Verheest, F. 2011 Dust-acoustic solitary modes in plasmas with isothermal and nonthermal ions: polarity switches and coexistence domains. *Phys. Plasmas* **18**, 083701.

Verheest, F., Hellberg, M. A. and Kourakis, I. 2013a Electrostatic supersolitons in three-species plasmas. *Phys. Plasmas* **20**, 012302.

Verheest, F., Hellberg, M. A. and Kourakis, I. 2013b Dust-ion-acoustic supersolitons in dusty plasmas with nonthermal electrons. *Phys. Rev. E* **87**, 043107.

Verheest, F., Hellberg, M. A. and Kourakis, I. 2013c Ion-acoustic supersolitons in plasmas with two-temperature electrons: Boltzmann and kappa distributions. *Phys. Plasmas* **20**, 082309.

White, R. B., Fried, B. D. and Coroniti, F. V. 1972 Structure of ion acoustic solitons and shock waves in a two-component plasma. *Phys. Fluids* **15**, 1484–1490.

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