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A NOTE ON NONLINEAR FISCAL REGIMES AND INTEREST RATE POLICY

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Much empirical evidence finds that governments react to fiscal imbalances in a nonlinear way, through an increasing marginal response of primary surpluses to changes in debt. This paper shows that nonlinear fiscal regimes alter equilibria under active and passive monetary–fiscal policies. The Fisher equation combined with nonlinear fiscal policies leads to multiple steady states. Under passive interest rate rules, even if the steady state in which fiscal policy is active is locally saddlepath stable, there exist infinite equilibrium paths originating in the neighborhood of that steady state that converge into a high-debt trap. Under active interest rate rules, even if the steady state at which fiscal policy is active is locally unstable, there exists a saddle connection with the high-debt equilibrium along which inflation is uniquely determined.

Keywords: Nonlinear Fiscal Rules, Interest Rate Policy, Multiple Equilibria, Global Dynamics

1. INTRODUCTION

A large body of empirical studies show that fiscal authorities react to public debt accumulation in a nonlinear way.[1] Governments tend to enhance the adoption of corrective actions as fiscal imbalances worsen, through an *increasing* marginal response of primary surpluses to changes in debt. What are the implications for monetary–fiscal policy interactions?

This paper shows that nonlinear fiscal regimes give rise to steady-state multiplicity and alter dynamic equilibria under "active" and "passive" monetary–fiscal policies.[2] The long-run Fisher equation, combined with a fiscal policy stance displaying convex nonlinearity in the surplus–debt relationship, yields two types of steady states: a low-debt steady state in which fiscal policy is active and a high-debt steady state at which fiscal policy is passive. It is demonstrated that the two steady states are dynamically connected under either a passive or an active

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monetary policy stance. Under a passive interest rate policy, even if the steady state in which fiscal policy is active is locally a unique stable equilibrium [Leeper (1991); Woodford (2003)], there exist an infinite number of equilibrium paths originating in the neighborhood of that steady state that converge to a high-debt trap. Under an active interest rate policy, even if the steady state in which fiscal policy is active is locally unstable [Leeper (1991); Woodford (2003)], there exists a saddle connection with the high-debt equilibrium along which inflation is uniquely determined. Such dynamic equilibria are obscured if one restricts attention to local dynamics around a particular steady state.

Therefore, a nonlinear fiscal policy stance per se renders global dynamic properties of interest rate rules fundamentally different from local dynamic properties. This is, relatedly, critical for questions of determinacy. Specifically, two major implications emerge. First, if governments respond to debt accumulation nonlinearly, inflation turns out to be pinned down only by an active monetary policy stance, even near a steady state in which fiscal policy is active. Second, the passive monetary, active fiscal local regime does not ensure equilibrium determinacy, as in the context of standard theoretical formulations of monetary–fiscal interactions focused on local analysis around a single steady state. Consequently, in terms of inflation control and equilibrium uniqueness, our theoretical findings strengthen the analytical arguments in favor of the adoption of active interest rate policies.

In general, the analysis conducted in this paper proposes an additional perspective for the study of monetary–fiscal interactions, for it emphasizes that the typical assumption of linearity in fiscal policy making is not immaterial to deriving the stabilizing properties of policy regimes. Incorporating the occurrence of nonlinearities into governments' actions, in line with empirical evidence, appears to be critical for a global characterization of macroeconomic dynamics under interest rate rules.

The paper is organized as follows. Section 2 presents the monetary model with nonlinear fiscal policy rules. Section 3 analyzes equilibrium dynamics and states the main results. Section 4 discusses the model's extensions, which are formalized in the Appendix, and provides concluding comments.

2. THE MODEL

Consider a continuous-time environment à la Benhabib et al. (2001). There are a large number of identical infinitely lived households with the utility function given by

$$\int_0^\infty e^{-rt} u[c(t), m(t)]dt, \tag{1}$$

where $r > 0$ denotes the rate of time preference, $c(t)$ consumption, and $m(t)$ real money balances at instant of time t; $u(\cdot\,,\cdot)$ is strictly increasing and strictly concave in $c(t)$ and $m(t)$, which are Edgeworth complements ($u_{cm} > 0$). The

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representative household's instantaneous budget constraint is

$$\dot{a}(t) = [R(t) - \pi(t)]\, a(t) - R(t)m(t) + y(t) - c(t) - \tau(t), \tag{2}$$

where $a(t)$ denotes real financial wealth, consisting of interest-bearing government bonds and money balances, $y(t)$ an endowment of perishable goods, $\tau(t)$ lump-sum taxes net of public transfers, $R(t)$ the nominal interest rate on bonds, and $\pi(t)$ the inflation rate. Ponzi games are precluded, so that $\lim_{t\to\infty} e^{-\int_0^t [R(j)-\pi(j)]dj} a(t) \geq 0$. Let $\lambda(t)$ be the co-state variable. Hence, optimality implies that

$$u_c[c(t), m(t)] = \lambda(t), \tag{3}$$

$$u_m[c(t), m(t)] = \lambda(t) R(t), \tag{4}$$

$$\dot{\lambda}(t) = \lambda(t)\,[r + \pi(t) - R(t)], \tag{5}$$

$$\lim_{t\to\infty} e^{-\int_0^t [R(j)-\pi(j)]dj} a(t) = 0. \tag{6}$$

Next, consider the public sector. The monetary authority adopts an interest rate feedback rule of the form

$$R(t) = r + \pi^* + \psi[\pi(t) - \pi^*], \tag{7}$$

where π^* denotes the target inflation rate and ψ a positive parameter. Monetary policy is "active" if $\psi > 1$ and is "passive" if $\psi < 1$. The government's instantaneous budget constraint is given by

$$\dot{a}(t) = [R(t) - \pi(t)]\, a(t) - s(t), \tag{8}$$

where $s(t) = \tau(t) + R(t)m(t)$ is the primary surplus inclusive of interest savings from the issue of money.[3] We depart from the literature in the description of the fiscal policy stance. We assume that the fiscal authority adjusts the primary surplus according to a nonlinear feedback policy,

$$s(t) = \sigma[a(t)], \tag{9}$$

where the function $\sigma(\cdot)$ is continuous and positive and satisfies $\sigma', \sigma'' > 0$.

2.1. Equilibrium

Equilibrium in the goods market requires that $c(t) = y(t)$. Assuming that the endowment is constant over time, $y(t) = y$ for each $t \in [0, \infty)$, equations (3) and (4) imply that $\lambda(t) = L[R(t)]$, with $L' = \lambda/(u_{mm}/u_{cm} - R) < 0$. Using (5) and (7), inflation dynamics is given by

$$\dot{\pi}(t) = -\frac{L\{r + \pi^* + \psi[\pi(t) - \pi^*]\}}{\psi L'\{r + \pi^* + \psi[\pi(t) - \pi^*]\}} \left\{(\psi - 1)\,[\pi(t) - \pi^*]\right\}. \tag{10}$$

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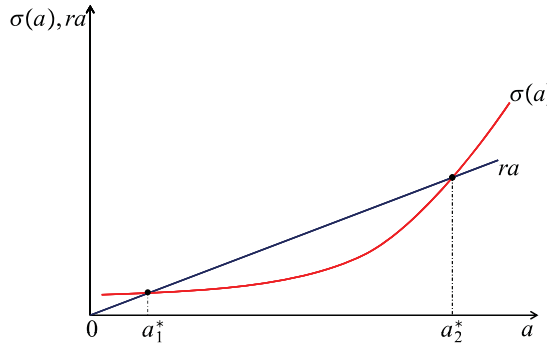


FIGURE 1. Multiple steady states under a nonlinear fiscal regime.

Substituting (9) and (7) into (8), the law of motion of government liabilities can be expressed as

$$\dot{a}(t) = \left\{r + (\psi - 1)\left[\pi(t) - \pi^*\right]\right\} a(t) - \sigma[a(t)]. \tag{11}$$

2.2. Steady States and Active–Passive Fiscal Policies

Setting $\dot{\pi}(t), \dot{a}(t) = 0$ in (10) and (11) yields $\pi = \pi^*$ and $\sigma(a) = ra$. Because the fiscal policy reaction function $\sigma(\cdot)$ is positive and obeys $\sigma', \sigma'' > 0$, the steady-state relation $\sigma(a) = ra$ has, in general, two solutions, $a_1^*, a_2^* > 0$. Figure 1 displays the two steady-state equilibria for government liabilities, which occur at the intersections of the functions $\sigma(a)$ and ra. We have set $a_1^* < a_2^*$, implying that $\sigma'(a_1^*) < r$ and $\sigma'(a_2^*) > r$. So in the context of Leeper's (1991) terminology, fiscal policy is "active" in the neighborhood of (π^*, a_1^*), because from (11) $\partial\dot{a}(t)/\partial a(t)|_{(\pi^*, a_1^*)} = r - \sigma'(a_1^*) > 0$, i.e., government liabilities per se tend to explode, and is "passive" in the neighborhood of (π^*, a_2^*), because from (11) $\partial\dot{a}(t)/\partial a(t)|_{(\pi^*, a_2^*)} = r - \sigma'(a_2^*) < 0$, i.e., government liabilities per se tend to be stable.

3. EQUILIBRIUM DYNAMICS

We now examine local and global equilibrium dynamics. Consider first the dynamic effects of a passive interest rate rule ($\psi < 1$). The following proposition holds.

PROPOSITION 1. *Suppose that fiscal policy is nonlinear ($\sigma', \sigma'' > 0$) and monetary policy is passive ($\psi < 1$). Then, from the equilibrium system (10) and (11): (a) locally, the steady state (π^*, a_1^*) is a saddlepoint and the steady state (π^*, a_2^*) is a sink; (b) globally, there exist infinite equilibrium paths originating in the neighborhood of (π^*, a_1^*) and converging asymptotically to (π^*, a_2^*); the*

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saddle manifold associated to (π^*, a_1^*) *is the boundary of the basin of attraction of* (π^*, a_2^*).

Proof. (a) Let $J_{(\pi^*,a_1^*)}$ be the Jacobian of (10) and (11) evaluated at (π^*, a_1^*). We have $\det J_{(\pi^*,a_1^*)} = \frac{-L(r+\pi^*)(\psi-1)[r-\sigma'(a_1^*)]}{\psi L'(r+\pi^*)} < 0$, because $L'(r+\pi^*) < 0$, $\psi < 1$, and $\sigma'(a_1^*) < r$. Thus, the steady state (π^*, a_1^*) is a saddlepoint, with the stable arm $\pi(t) = \pi^* + \frac{-L(r+\pi^*)(\psi-1)/\psi L'(r+\pi^*)-[r-\sigma'(a_1^*)]}{(\psi-1)a_1^*}[a(t) - a_1^*]$. Let $J_{(\pi^*,a_2^*)}$ be the Jacobian of (10) and (11) evaluated at (π^*, a_2^*). We have $\mathrm{tr}J_{(\pi^*,a_2^*)} = -\frac{L(r+\pi^*)(\psi-1)}{\psi L'(r+\pi^*)} + [r - \sigma'(a_2^*)] < 0$ and $\det J_{(\pi^*,a_2^*)} = \frac{-L(r+\pi^*)(\psi-1)[r-\sigma'(a_2^*)]}{\psi L'(r+\pi^*)} > 0$, because $L'(r+\pi^*) < 0$, $\psi < 1$, and $\sigma'(a_2^*) > r$. Thus, the steady state (π^*, a_2^*) is a sink. (b) From (10) and (11), the two isoclines $\dot{\pi}(t) = 0$ and $\dot{a}(t) = 0$ are $\pi(t) = \pi^*$ and $\pi(t) = \pi^* + \frac{\{\sigma[a(t)]/a(t)\}-r}{(\psi-1)}$, respectively. We have $\frac{d\pi(t)}{da(t)}|_{\dot{a}(t)=0} = \frac{\sigma'[a(t)]-\{\sigma[a(t)]/a(t)\}}{(\psi-1)a(t)}$, which is positive at (π^*, a_1^*), where $\sigma'(a_1^*) < [\sigma(a_1^*)/a_1^*] = r$; equal to zero for $\sigma'[a(t)] = \sigma[a(t)]/a(t)$; and negative at (π^*, a_2^*), where $\sigma'(a_2^*) > [\sigma(a_2^*)/a_2^*] = r$. Thus, in the phase plane $[\pi(t), a(t)]$, the isocline $\dot{\pi}(t) = 0$ is horizontal, the isocline $\dot{a}(t) = 0$ is inverted U-shaped, and the two isoclines intersect at (π^*, a_1^*) and (π^*, a_2^*). From (10) and (11), $\dot{\pi}(t) > (<)0$ if $\pi(t) < (>)\pi^*$ and $\dot{a}(t) > (<)0$ if $\pi(t) < (>)\pi^* + \frac{\{\sigma[a(t)]/a(t)\}-r}{(\psi-1)}$. Figure 2 shows the global dynamics. The stable arm of the saddlepoint passing through (π^*, a_1^*), SS, has a positive slope, given by $\frac{-L(r+\pi^*)(\psi-1)/\psi L'(r+\pi^*)-[r-\sigma'(a_1^*)]}{(\psi-1)a_1^*}$, which is greater than the slope of the isocline $\dot{a}(t) = 0$ evaluated at (π^*, a_1^*), given by $\frac{\sigma'(a_1^*)-r}{(\psi-1)a_1^*}$. Hence, in the neighborhood of the steady state (π^*, a_1^*), for a given initial condition $a(0)$, there exist an infinite number of equilibrium initial values $\pi(0) < \pi(0)_S = \pi^* + \frac{-L(r+\pi^*)(\psi-1)/\psi L'(r+\pi^*)-[r-\sigma'(a_1^*)]}{(\psi-1)a_1^*}[a(0) - a_1^*]$, for example $\pi(0)_1$ and $\pi(0)_2$ in Figure 2, such that $[\pi(t), a(t)]$ will converge asymptotically to the steady state (π^*, a_2^*). The saddle manifold associated with (π^*, a_1^*) is thus the boundary of the basin of attraction of (π^*, a_2^*). ■

Leeper (1991) and Woodford (2003) demonstrate that an active fiscal, passive monetary regime yields local determinacy. In Figure 2, this result corresponds to the existence of a saddlepath associated with the steady state (π^*, a_1^*). If one restricts attention to local dynamics around (π^*, a_1^*), for a given initial condition $a(0) \neq a_1^*$, there in fact exists a unique value of inflation, $\pi(0)_S = \pi^* + \frac{-L(r+\pi^*)(\psi-1)/\psi L'(r+\pi^*)-[r-\sigma'(a_1^*)]}{(\psi-1)a_1^*}[a(0) - a_1^*]$, such that $[\pi(t), a(t)]$ will converge to the steady state (π^*, a_1^*). Nevertheless, (π^*, a_1^*) is not the only steady-state equilibrium if the conduct of fiscal policy is nonlinear. Proposition 1 shows that, when the fiscal regime is nonlinear, even if the steady state (π^*, a_1^*) at which fiscal policy is active is locally a unique stable equilibrium, there exist an infinite number of equilibrium paths originating in the neighborhood of (π^*, a_1^*) that converge to the high-debt steady state (π^*, a_2^*). Inflation no longer needs to "jump" on the saddlepath to ensure global stability. All initial values $\pi(0)$ below the saddlepath are equilibrium values that make

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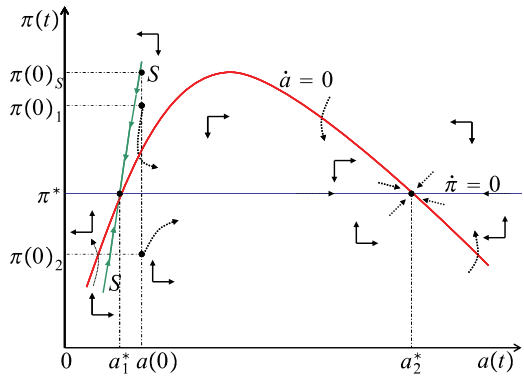


FIGURE 2. Dynamic behavior of $[\pi(t), a(t)]$ with nonlinear fiscal regime and passive monetary rule.

$[\pi(t), a(t)]$ converge to (π^*, a_2^*).[4] This implies that the dynamic system is globally indeterminate.

See now the dynamic effects of an active interest rate rule ($\psi > 1$). The next proposition holds.

PROPOSITION 2. *Suppose that fiscal policy is nonlinear ($\sigma', \sigma'' > 0$) and monetary policy is active ($\psi > 1$). Then, from the equilibrium system (10) and (11): (a) locally, the steady state (π^*, a_1^*) is a source and the steady state (π^*, a_2^*) is a saddlepoint; (b) globally, there exists a saddle connection joining the two steady states (π^*, a_1^*) and (π^*, a_2^*), along which inflation is uniquely determined ($\pi = \pi^*$).*

Proof. (a) Let $J_{(\pi^*, a_1^*)}$ be the Jacobian of (10) and (11) evaluated at (π^*, a_1^*). We have $\text{tr } J_{(\pi^*, a_1^*)} = -\frac{L(r+\pi^*)(\psi-1)}{\psi L'(r+\pi^*)} + [r - \sigma'(a_1^*)] > 0$ and $\det J_{(\pi^*, a_1^*)} = \frac{-L(r+\pi^*)(\psi-1)[r-\sigma'(a_1^*)]}{\psi L'(r+\pi^*)} > 0$, because $L'(r + \pi^*) < 0$, $\psi > 1$, and $\sigma'(a_1^*) < r$. Thus, the steady state (π^*, a_1^*) is a source. Let $J_{(\pi^*, a_2^*)}$ be the Jacobian of (10) and (11) evaluated at (π^*, a_2^*). We have $\det J_{(\pi^*, a_2^*)} = \frac{-L(r+\pi^*)(\psi-1)[r-\sigma'(a_2^*)]}{\psi L'(r+\pi^*)} < 0$, because $L'(r + \pi^*) < 0$, $\psi > 1$, and $\sigma'(a_2^*) > r$. Thus, the steady state (π^*, a_2^*) is a saddlepoint, with the stable arm $\pi(t) = \pi^*$. (b) From (10) and (11), the two isoclines $\dot{\pi}(t) = 0$ and $\dot{a}(t) = 0$ are $\pi(t) = \pi^*$ and $\pi(t) = \pi^* + \frac{\{\sigma[a(t)]/a(t)\} - r}{(\psi-1)}$, respectively. We have $\frac{d\pi(t)}{da(t)}|_{\dot{a}(t)=0} = \frac{\sigma'[a(t)] - \{\sigma[a(t)]/a(t)\}}{(\psi-1)a(t)}$, which is negative at (π^*, a_1^*), where $\sigma'(a_1^*) < [\sigma(a_1^*)/a_1^*] = r$; equal to zero for $\sigma'[a(t)] = \{\sigma[a(t)]/a(t)\}$; and positive at (π^*, a_2^*), where $\sigma'(a_2^*) > [\sigma(a_2^*)/a_2^*] = r$. Thus, in the phase plane $[\pi(t), a(t)]$, the isocline $\dot{\pi}(t) = 0$ is horizontal, the isocline $\dot{a}(t) = 0$ is U-shaped, and the two isoclines intersect at (π^*, a_1^*) and (π^*, a_2^*). From (10) and (11), $\dot{\pi}(t) > (<) 0$ if $\pi(t) > (<) \pi^*$ and $\dot{a}(t) > (<) 0$ if $\pi(t) > (<) \pi^* + \frac{\sigma[a(t)] - ra(t)}{(\psi-1)a(t)}$. Figure 3 shows the global dynamics. There exists a unique trajectory originating in the neighborhood of the steady state (π^*, a_1^*) and converging to the

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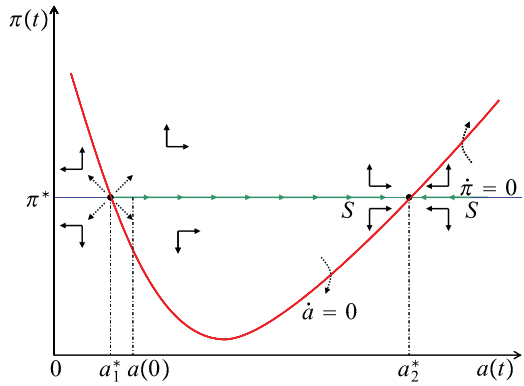


FIGURE 3. Dynamic behavior of $[\pi(t), a(t)]$ with nonlinear fiscal regime and active monetary rule.

other steady state (π^*, a_2^*). The saddle connection joining the two steady states is given by the isocline $\dot{\pi}(t) = 0$ $(\pi = \pi^*)$, which is also the stable arm of the saddlepath-stable steady state (π^*, a_2^*), SS. ■

If monetary policy is active, the steady state (π^*, a_1^*) at which fiscal policy is active now delivers local instability, consistent with the results in Leeper (1991) and Woodford (2003). Therefore, when attention is restricted to local dynamics around (π^*, a_1^*), no stable equilibria exist. However, Proposition 2 shows that, in the case in which the fiscal policy stance is nonlinear, even if the steady state (π^*, a_1^*) is locally unstable, there exists a saddle connection with the high-debt steady state (π^*, a_2^*). As emerges from Figure 3, the saddle connection corresponds to the isocline $\dot{\pi}(t) = 0$, $\pi = \pi^*$. Thus, under a nonlinear fiscal regime, the dynamic system is globally determinate and inflation is pinned down to the target rate even around the steady state at which fiscal policy is active.

4. CONCLUSIONS

Much empirical evidence supports the hypothesis of an increasing marginal reaction of primary budget surpluses to public debt. The consequences for macroeconomic dynamics in monetary models have been unexplored. The contribution of the present paper is to show that this type of nonlinearity in the conduct of fiscal policy is an independent source of multiplicity of steady-state equilibria and critically influences the dynamic interaction between monetary and fiscal policy.

The basic reason is that the Fisher equation, in conjunction with a nonlinear fiscal regime characterized by an increasing marginal response of primary surpluses to government liabilities, necessarily leads to two steady-state solutions. In the neighborhood of the low-debt steady state, fiscal policy is active, because the marginal adjustment of the primary surplus is lower than the steady-state real interest rate. In the neighborhood of the high-debt steady state, fiscal policy is

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passive, because the marginal adjustment of the primary surplus is higher than the steady-state real interest rate.

The central point is that the two steady states are dynamically connected under either passive or active interest rate policies. This modifies the equilibrium dynamics induced by feedback monetary and fiscal policies from what is commonly argued by the literature. Specifically, under a passive monetary policy stance, the steady state at which fiscal policy is active displays local equilibrium determinacy, but there exist an infinite number of self-fulfilling trajectories originating arbitrarily close to that steady state and spiraling up into a high-debt trap. Under an active monetary policy stance, one the other hand, the steady state at which fiscal policy is active displays local instability, but there exists a saddle connection with the high-debt equilibrium along which inflation is uniquely pinned down. Although both types of monetary policies lead to the high-debt equilibrium, the foregoing findings support the use of active interest rate rules in terms of inflation control.

The setup we have analyzed delivers the key points of the paper in a direct way. In the Appendix, we incorporate three relevant extensions to address results' robustness. We examine global dynamics under interest rate feedback policies also reacting to output.[5] Relatedly, we move from an endowment to a production economy environment. In this context, we first maintain the assumption of flexible prices and then introduce sticky prices. The main finding is that, if interest rate responses to output are excessively aggressive, global indeterminacy tends to prevail even under an active monetary policy. However, we show that the essence of the results obtained in the main text is not altered if one adopts the perspective of the "generalized Taylor principle" [Woodford (2003)], which in synthesis internalizes the effect of inflation on output within the interest rate rule. That is, we demonstrate that equilibrium uniqueness along the saddle connection occurs only when the equilibrium "cumulative" increase in the nominal interest rate in response to an increase in inflation is more than one for one.

In conclusion, the analytical results derived in this paper suggest that considering potential nonlinearities in governments' budgetary policies may be essential for a global characterization of monetary–fiscal policy interactions. Further extensions of the present framework to incorporate, for example, the zero-lower-bound problem on nominal interest rates, distortionary taxation, sovereign risk, and/or agents' learning, may be the object of further research.

NOTES

1. See, for example, Bohn (1998), Sarno (2001), Arestis et al. (2004), Bajo-Rubio et al. (2004, 2006), Arghyrou and Luintel (2007), Chortareas et al. (2008), Considine and Gallagher (2008), Cipollini et al. (2009), Legrenzi and Milas (2012a, 2012b), and Arghyrou and Fan (2013).

2. Fiscal policy is "passive" ("active") in Leeper's (1991) terminology, or "locally Ricardian" ("locally non-Ricardian") in Woodford's (2003) terminology, if the primary surplus set by the fiscal authority yields local stability (instability) of government liabilities for all stable paths of the other endogenous variables, such as inflation and output, in the neighborhood of a steady state. Monetary policy is "active" ("passive") in Leeper's (1991) terminology, or satisfies (does not satisfy) the "Taylor

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(1993) principle" in Woodford's (2003) terminology, if the nominal interest rate set by the monetary authority increases more (less) than one for one with respect to an increase in the inflation rate. See Canzoneri et al. (2011) for a literature review on monetary–fiscal policy interactions. See Benassy (2008) and Rossi (2014) for further issues arising in models with overlapping generations and rule-of-thumb consumers, respectively.

3. Public consumption is set equal to zero, for simplicity.

4. Inflation follows a nonmonotonic path if $\pi(0)$ lies in the region between the saddlepath and the isocline $\dot{a}(t) = 0$.

5. Woodford (2003) and Surico (2008) show how reacting to output may affect local determinacy in New Keynesian models.

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APPENDIX: THE MODEL'S EXTENSIONS

To address the results' robustness, this Appendix extends our baseline framework to incorporate a monetary policy response to endogenous output movements, under both flexible and sticky prices.

Consider first a flexible-price economy with identical household-firm units. Production uses labor, $h(t)$, via the technology $y(t) = f[h(t)]$, where $f' > 0$, $f'' < 0$. The household's utility function is now given by $\int_0^\infty e^{-rt}\{u[c(t), m(t)] - v[h(t)]\}dt$, where $v', v'' > 0$. Optimality yields (3)–(6) together with

$$v'[h(t)] = \lambda(t) f'[h(t)]. \tag{A.1}$$

We now assume interest rate feedback policies of the form

$$R(t) = r + \pi^* + \psi[\pi(t) - \pi^*] + \omega[y(t) - y^*]/y^*, \tag{A.2}$$

where y^* is the steady-state level of output and ω is a positive parameter. At equilibrium, conditions (3), (4), and (A.1) imply that $y(t) = Y[R(t)]$, with $Y' = u_{cm} f'/\Delta < 0$, and $\lambda(t) = L[R(t)]$, with $L' = u_{cm}(v'' - \lambda f'')/f'\Delta < 0$, where $\Delta \equiv -f'(u_{cc}u_{mm} - u_{cm}^2) - [(v'' - \lambda f'')/f'](u_{cm}R - u_{mm}) < 0$. From (A.2), it follows that $R(t) = \Psi[\pi(t)]$, with $\Psi' = \psi/(1 - \omega Y'/y^*) > 0$. We thus obtain the system

$$\dot{\pi}(t) = -\frac{L\{\Psi[\pi(t)]\}}{\Psi'[\pi(t)]\, L'\{\Psi[\pi(t)]\}}\left[(\psi - 1)[\pi(t) - \pi^*] + \omega\left(Y\{\Psi[\pi(t)]\} - y^*\right)/y^*\right], \tag{A.3}$$

$$\dot{a}(t) = \left[r + (\psi - 1)[\pi(t) - \pi^*] + \omega\left(Y\{\Psi[\pi(t)]\} - y^*\right)/y^*\right]a(t) - \sigma[a(t)]. \tag{A.4}$$

Hence, the following propositions apply.

PROPOSITION 3. *Suppose that fiscal policy is nonlinear ($\sigma', \sigma'' > 0$) and monetary policy is passive ($\psi < 1$). Then, from the equilibrium system (A.3) and (A.4), results (a) and (b) of Proposition 1 hold.*

Proof. Notice that $\psi < 1$ implies $\Psi' = \frac{\psi}{1-\omega Y'/y^*} < 1$, because $Y' < 0$. Hence, because $\det J_{(\pi^*, a_1^*)} = \frac{-L(r+\pi^*)[\Psi'(\pi^*)-1][r-\sigma'(a_1^*)]}{\Psi'(\pi^*)L'(r+\pi^*)} < 0$, the steady state (π^*, a_1^*) is a saddlepoint, with the stable arm $\pi(t) = \pi^* + \frac{-L(r+\pi^*)[\Psi'(\pi^*)-1]/\Psi'(\pi^*)L'(r+\pi^*)-[r-\sigma'(a_1^*)]}{[\Psi'(\pi^*)-1]a_1^*}[a(t) - a_1^*]$. Because $\mathrm{tr}J_{(\pi^*, a_2^*)} = -\frac{L(r+\pi^*)[\Psi'(\pi^*)-1]}{\Psi'(\pi^*)L'(r+\pi^*)} + [r - \sigma'(a_2^*)] < 0$ and $\det J_{(\pi^*, a_2^*)} = \frac{-L(r+\pi^*)[\Psi'(\pi^*)-1]}{\Psi'(\pi^*)L'(r+\pi^*)} \times [r -$

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$\sigma'(a_2^*)] > 0$, the steady state (π^*, a_2^*) is a sink. The isoclines $\dot{\pi}(t) = 0$ and $\dot{a}(t) = 0$ are $\pi(t) = \pi^*$ and $(\psi-1)[\pi(t)-\pi^*]+\omega(Y\{\Psi[\pi(t)]\}-y^*)/y^* = \{\sigma[a(t)]/a(t)\}-r$. We have $\frac{d\pi(t)}{da(t)}|_{\dot{a}(t)=0} = \frac{\sigma'[a(t)]-\{\sigma[a(t)]/a(t)\}}{\{\Psi'[\pi(t)]-1\}a(t)}$, which is positive at (π^*, a_1^*), equal to zero for $\sigma'[a(t)] = \sigma[a(t)]/a(t)$, and negative at (π^*, a_2^*). In the phase plane $[\pi(t), a(t)]$, the isocline $\dot{\pi}(t) = 0$ is horizontal, the isocline $\dot{a}(t) = 0$ is inverted U-shaped, and the two isoclines intersect at (π^*, a_1^*) and (π^*, a_2^*). From (A.3) and (A.4), $\dot{\pi}(t) > (<)0$ if $\pi(t) < (>)\pi^*$ and $\dot{a}(t) > (<)0$ if $(\psi - 1)[\pi(t) - \pi^*] + \omega(Y\{\Psi[\pi(t)]\} - y^*)/y^* > (<)\{\sigma[a(t)]/a(t)\} - r$. Qualitatively, global dynamics is that displayed in Figure 2. Quantitatively, because $\Psi' < \psi$ for any $\omega > 0$, both the stable arm passing through (π^*, a_1^*) and the isocline $\dot{a}(t) = 0$ are flatter with respect to the baseline framework. ■

PROPOSITION 4. *Suppose that fiscal policy is nonlinear ($\sigma', \sigma'' > 0$) and monetary policy is active ($\psi > 1$). Then, from the equilibrium system (A.3) and (A.4): (i) if reaction to output is relatively high, such that $\omega > -\frac{(\psi-1)}{Y'/y^*}$, implying that $\Psi' < 1$, results (a) and (b) of Proposition 1 hold; (ii) if reaction to output is relatively low, such that $\omega < -\frac{(\psi-1)}{Y'/y^*}$, implying that $\Psi' > 1$, results (a) and (b) of Proposition 2 hold.*

Proof. (i) If $\psi > 1$, but $\omega > -\frac{(\psi-1)}{Y'/y^*}$, we have $\Psi' < 1$. This implies that local and global dynamic properties follow those shown in the proof of Proposition 3. (ii) If $\psi > 1$ and $\omega < -\frac{(\psi-1)}{Y'/y^*}$, we also have $\Psi' > 1$. In this case, tr $J_{(\pi^*,a_1^*)} > 0$ and det$J_{(\pi^*,a_1^*)} > 0$, so that the steady state (π^*, a_1^*) is a source, and det $J_{(\pi^*,a_2^*)} < 0$, so that the steady state (π^*, a_2^*) is a saddlepoint, with the stable arm $\pi(t) = \pi^*$. The isocline $\dot{a}(t) = 0$ is now U-shaped and steeper than in the baseline framework. We have $\dot{\pi}(t) > (<)0$ if $\pi(t) > (<)\pi^*$ and $\dot{a}(t) > (<)0$ if $(\psi - 1)[\pi(t) - \pi^*] + \omega(Y\{\Psi[\pi(t)]\} - y^*)/y^* > (<)\{\sigma[a(t)]/a(t)\} - r$. Qualitatively, global dynamics is that displayed in Figure 3. ■

Proposition 4 shows that, even under an active monetary policy, there exist multiple equilibrium paths converging to the high-debt equilibrium when interest rate responses to output are excessively aggressive, such that $\omega > -\frac{(\psi-1)}{Y'/y^*}$. However, the essence of the results obtained in Section 3 is not altered if one adopts the perspective of the "generalized Taylor principle" [Woodford (2003)], which accounts for the effect of inflation on output within the interest rate rule. In this setup, increases in inflation and thus in the nominal interest rate dampen the level of output via a real balance effect à la Brock (1974). If $\omega > -\frac{(\psi-1)}{Y'/y^*}$, the policy concern over output stabilization implies that, at equilibrium, the "cumulative" increase in the nominal interest rate is lower than the increase in inflation, that is, $\Psi' < 1$; and, for this reason, the economy falls in the global indeterminacy depicted in Figure 2. In contrast, if $\omega < -\frac{(\psi-1)}{Y'/y^*}$, the generalized Taylor principle is verified, that is, $\Psi' > 1$, and hence global determinacy applies along the saddle connection depicted in Figure 3.

Consider the case of sticky prices. Each household-firm unit j produces a differentiated good $y^j(t)$ via the production function $y^j(t) = f[h^j(t)]$, and faces a demand function of the form $y^d(t)d[P^j(t)/P(t)]$, where $y^d(t)$ denotes aggregate demand, $P^j(t)$ the product j's price, and $P(t)$ the price level, and $d(\cdot)$ satisfies $d' < 0$, $d(1) = 1$, and $d'(1) = -1$. Lifetime utility is of the form $\int_0^\infty e^{-rt}\{u[c^j(t), m^j(t)] - v[h^j(t)] - \frac{\rho}{2}[\frac{\dot{P}^j(t)}{P^j(t)} - \pi^*]^2\}dt$, where ρ is a positive parameter, consistent with Rotemberg (1982). The budget constraint in real terms is now $\dot{a}^j(t) = [R(t)-\pi(t)]a^j(t) - R(t)m^j(t) + \frac{P^j(t)}{P(t)}f[h^j(t)] - c^j(t) - \tau(t)$. Let $\delta^j(t)$ be the multiplier associated with the constraint that output is demand-determined, $f[h^j(t)] = y^d(t)d[P^j(t)/P(t)]$, and set $\dot{\pi}^j(t) \equiv \dot{P}^j(t)/P^j(t)$. Optimality gives equations

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(3)–(6) indexed by j and

$$v'\left[h^j(t)\right] = \left[\lambda^j(t)\frac{P^j(t)}{P(t)} - \delta^j(t)\right] f'\left[h^j(t)\right], \tag{A.5}$$

$$\dot{\pi}^j(t) = r\left[\pi^j(t) - \pi^*\right] - \frac{1}{\rho}\left\{\lambda^j(t)\frac{P^j(t)}{P(t)} f\left[h^j(t)\right] + \delta^j(t)\frac{P^j(t)}{P(t)} y^d(t) d'\left[\frac{P^j(t)}{P(t)}\right]\right\}. \tag{A.6}$$

In the symmetric equilibrium, equations (3) and (4) imply that $y(t) = Y[\lambda(t), R(t)]$, with $Y_\lambda = -(u_{cm}R - u_{mm})/(u_{cc}u_{mm} - u_{cm}^2) < 0$ and $Y_R = -\lambda/(u_{cc}u_{mm} - u_{cm}^2) < 0$. From (A.2), it follows that $R(t) = \Psi[\pi(t), \lambda(t)]$, with $\Psi_\pi = \psi/(1 - \omega Y_R/y^*) > 0$ and $\Psi_\lambda = \omega Y_\lambda/(1 - \omega Y_R/y^*) < 0$. In this case we obtain the system

$$\dot{\lambda}(t) = -\lambda(t)\left[(\psi - 1)\left[\pi(t) - \pi^*\right] + \omega\left(Y\left\{\lambda(t), \Psi\left[\pi(t), \lambda(t)\right]\right\} - y^*\right)/y^*\right], \tag{A.7}$$

$$\dot{\pi}(t) = r\left[\pi(t) - \pi^*\right] - \frac{\lambda(t) Y\left\{\lambda(t), \Psi\left[\pi(t), \lambda(t)\right]\right\}}{\rho} \times \left\{1 + \varepsilon - \frac{\varepsilon v'\left[f^{-1}\left(Y\left\{\lambda(t), \Psi\left[\pi(t), \lambda(t)\right]\right\}\right)\right]}{\lambda(t) f'\left[f^{-1}\left(Y\left\{\lambda(t), \Psi\left[\pi(t), \lambda(t)\right]\right\}\right)\right]}\right\}, \tag{A.8}$$

$$\dot{a}(t) = \left[r + (\psi - 1)\left[\pi(t) - \pi^*\right] + \omega\left(Y\left\{\lambda(t), \Psi\left[\pi(t), \lambda(t)\right]\right\} - y^*\right)/y^*\right] \times a(t) - \sigma[a(t)], \tag{A.9}$$

where $\varepsilon \equiv d'(1) < -1$. Consequently, the next propositions apply.

PROPOSITION 5. *Suppose that fiscal policy is nonlinear ($\sigma', \sigma'' > 0$) and monetary policy is passive ($\psi < 1$). Then, from the equilibrium system (A.7)–(A.9): (i) If reaction to output is relatively low, implying that $\frac{dR}{d\pi} = \Psi_\pi + \Psi_\lambda \frac{d\lambda}{d\pi} < 1$, (a) locally, the steady state $(\lambda^*, \pi^*, a_1^*)$ is a saddlepoint with a one-dimensional stable space and the steady state $(\lambda^*, \pi^*, a_2^*)$ is a saddlepoint with a two-dimensional stable space; (b) globally, there exist infinite equilibrium paths originating in the neighborhood of $(\lambda^*, \pi^*, a_1^*)$ and converging asymptotically to $(\lambda^*, \pi^*, a_2^*)$; the saddle manifold associated with $(\lambda^*, \pi^*, a_1^*)$ is the boundary of the basin of attraction of $(\lambda^*, \pi^*, a_2^*)$. (ii) If reaction to output is relatively high, implying that $\frac{dR}{d\pi} = \Psi_\pi + \Psi_\lambda \frac{d\lambda}{d\pi} > 1$, (a) locally, the steady state $(\lambda^*, \pi^*, a_1^*)$ is a source and the steady state $(\lambda^*, \pi^*, a_2^*)$ is a saddlepoint with a one-dimensional stable space; (b) globally, there exists a saddle connection joining the two steady states $(\lambda^*, \pi^*, a_1^*)$ and $(\lambda^*, \pi^*, a_2^*)$, along which $[\lambda(t), \pi(t)]$ are uniquely determined ($\lambda = \lambda^*, \pi = \pi^*$).*

Proof. (i) (a) The Jacobian of (A.7)–(A.9), $J_{(\lambda^*, \pi^*, a_i^*)}$, $i = 1, 2$, is block diagonal. One eigenvalue is $r - \sigma'(a_i^*)$ and the remaining two eigenvalues are obtained from the submatrix

$$K = \begin{bmatrix} -\lambda\Psi_\lambda & -\lambda(\Psi_\pi - 1) \\ k_{21} & k_{22} \end{bmatrix}, \tag{A.10}$$

where $k_{21} = \frac{\varepsilon y^*}{\rho f'}\left[(v'' - \frac{v' f''}{f'})\frac{Y_\lambda}{(1 - \omega Y_R/y^*) f'} - \frac{v'}{\lambda^*}\right] > 0$ and $k_{22} = r + \frac{\varepsilon y^*}{\rho (f')^2}(v'' - \frac{v' f''}{f'}) Y_R \Psi_\pi > 0$. Notice that $\psi < 1$ implies $\Psi_\pi = \frac{\psi}{1 - \omega Y_R/y^*} < 1$. Because $\det K = \lambda[-\Psi_\lambda k_{22} + k_{21}(\Psi_\pi - 1)] < 0$, because $\frac{dR}{d\pi}|_{(\lambda^*, \pi^*)} = \Psi_\pi - \Psi_\lambda \frac{k_{22}}{k_{21}} < 1$, $\mathrm{tr} K = k_{11} + k_{22} > 0$, $r - \sigma'(a_1^*) > 0$, and $r - \sigma'(a_2^*) < 0$, the steady states $(\lambda^*, \pi^*, a_1^*)$ and $(\lambda^*, \pi^*, a_2^*)$ are saddlepoints with one- and two-dimensional stable spaces, respectively. Let μ_1 be the negative eigenvalue

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associated with (A.10). Then the saddlepath solution around $(\lambda^*, \pi^*, a_1^*)$ yields $\lambda(t) = \lambda^* - \frac{\lambda^*(\Psi_\pi - 1)}{\mu_1 + \lambda^* \Psi_\lambda}[\pi(t) - \pi^*]$ and $\pi(t) = \pi^* + \frac{\mu_1 - [r - \sigma'(a_1^*)]}{(\Psi_\pi - 1)a_1^*[1 - \lambda^* \Psi_\lambda/(\mu_1 + \lambda^* \Psi_\lambda)]}(a(t) - a_1^*)$, where $a(t) = a_1^* + [a(0) - a_1^*]e^{\mu_1 t}$. (b) From (A.9), $\frac{d\dot{a}(t)}{d\pi(t)} = (\Psi_\pi + \Psi_\lambda \frac{d\lambda}{d\pi} - 1)a(t) < 0$. This implies that, in the neighborhood of $(\lambda^*, \pi^*, a_1^*)$, for a given initial condition $a(0)$, there exist an infinite number of equilibrium initial values $\pi(0) < \pi(0)_S = \pi^* + \frac{\mu_1 - [r - \sigma'(a_1^*)]}{(\Psi_\pi - 1)a_1^*[1 - \lambda^* \Psi_\lambda/(\mu_1 + \lambda^* \Psi_\lambda)]}[a(0) - a_1^*]$, such that $[\lambda(t), \pi(t), a(t)]$ will converge asymptotically to $(\lambda^*, \pi^*, a_2^*)$ along the saddlepath associated with the submatrix (A.10), given by $\lambda(t) = \lambda^* - \frac{\lambda^*(\Psi_\pi - 1)}{\mu_1 + \lambda^* \Psi_\lambda}[\pi(t) - \pi^*]$ around both $(\lambda^*, \pi^*, a_1^*)$ and $(\lambda^*, \pi^*, a_2^*)$. The saddle manifold associated with $(\lambda^*, \pi^*, a_1^*)$ is thus the boundary of the basin of attraction of $(\lambda^*, \pi^*, a_2^*)$. (ii) (a) Because $\det K > 0$, because $\frac{dR}{d\pi}|_{(\lambda^*, \pi^*)} > 1$, $\mathrm{tr} K > 0$, $r - \sigma'(a_1^*) > 0$, and $r - \sigma'(a_2^*) < 0$, the steady states $(\lambda^*, \pi^*, a_1^*)$ and $(\lambda^*, \pi^*, a_2^*)$ are a source and a saddlepoint with a one-dimensional stable space, respectively. The saddlepath solution around $(\lambda^*, \pi^*, a_2^*)$ is given by $\lambda(t) = \lambda^*$, $\pi(t) = \pi^*$, and $a(t) = a_2^* + [a(0) - a_2^*]e^{[r - \sigma'(a_2^*)]t}$. (b) Since $\frac{d\dot{a}(t)}{da(t)}|_{(\lambda^*, \pi^*)} = r - \sigma'[a(t)]$ is positive at a_1^* and negative at a_2^*, there is a saddle connection joining the two steady states $(\lambda^*, \pi^*, a_1^*)$ and $(\lambda^*, \pi^*, a_2^*)$, along $\lambda(t) = \lambda^*$ and $\pi(t) = \pi^*$, which are also the stable arms of the saddlepoint $(\lambda^*, \pi^*, a_2^*)$. ■

PROPOSITION 6. *Suppose that fiscal policy is nonlinear ($\sigma', \sigma'' > 0$) and monetary policy is active ($\psi > 1$). Then, from the equilibrium system (A.7)–(A.9): (i) if $\frac{dR}{d\pi} = \Psi_\pi + \Psi_\lambda \frac{d\lambda}{d\pi} < 1$, results (a) and (b) of Proposition 5(i) hold; (ii) if $\frac{dR}{d\pi} = \Psi_\pi + \Psi_\lambda \frac{d\lambda}{d\pi} > 1$, results (a) and (b) of Proposition 5(ii) hold.*

Proof. The same procedures developed in the proof of Proposition 5 apply. In addition, because the aggregate supply schedule (A.8) implies that $\frac{d\lambda}{d\pi}|_{(\lambda^*, \pi^*)} = -\frac{k_{22}}{k_{21}} < 0$, in this case a sufficient condition for the existence of the saddle connection joining the two steady states $(\lambda^*, \pi^*, a_1^*)$ and $(\lambda^*, \pi^*, a_2^*)$ is $\omega < -\frac{(\psi - 1)}{Y_R/y^*}$, which yields $\Psi_\pi > 1$ and occurs when reaction to output is relatively low. ■

Hence, from Propositions 5 and 6, the presence of price stickiness in the framework does not affect the root of our results under the perspective of the generalized Taylor principle. Indeed, consistent with the foregoing setup, global determinacy along the saddle connection is verified when the equilibrium cumulative increase in the nominal interest rate in response to an increase in inflation is more than one for one.

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