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PATENT PROTECTION, OPTIMAL LICENSING, AND INNOVATION WITH ENDOGENOUS ENTRY

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How does patent policy affect innovation when patent licensing is crucial for firms? To address this question, the present study incorporates voluntary patent licensing between an innovator and followers, as discussed in the literature of industrial organization, into a dynamic general equilibrium model. Unlike previous studies, both the licensing fee and the number of licensees are endogenously determined by the innovator's maximization and the free-entry condition. Using this model, we show that strong patent protection does not always enhance innovation, economic growth, and welfare. Furthermore, the extended analysis provides the policy implication that the effect of patent policy depends on how difficult further innovation is without patent licensing of the current leading technology.

Keywords: Innovation, Patent Protection, Optimal Patent Licensing, Endogenous Market Structure

1. INTRODUCTION

Patent licensing between firms is a crucial process for technological diffusion and innovation in modern knowledge-intensive industries. If a firm in such markets were not licensed in the leading technology, the firm would be hard-pressed not only to gain from the sale of products but also to develop a new technology. In fact, Wang et al. (2013) find a "learning-by-licensing" process among licensees by showing that licensed firms perform better in a subsequent innovation than non-licensed firms do. From the perspective of patented firms, licensing has growing importance as a way to earn revenue from intellectual property. Robbins (2009) estimates that US firms' licensing revenue has been rising rapidly year after year. She also reports that the knowledge-intensive industries in which rapid technological advances have occurred, such as electronics, chemicals, and pharmaceuticals,

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are the largest recipients of licensing revenue. This fact naturally requires us to explore technological advances with patent licensing between firms.

How does patent policy affect innovation when patent licensing is crucial for firms? Licensing has been subjected to separate theoretical analyses in endogenous growth theory and industrial organization (IO). In endogenous growth theory, some existing studies have investigated this question in R&D-based growth models with patent licensing. In particular, O'donoghue and Zweimüller (2004), Chu (2009), Chu et al. (2012), and Chu and Pan (2013) analyze the effect of "patent blocking" on innovation by considering the case of compulsory patent licensing involving a new innovator and previous innovator(s) in which the new technology infringes the patent of the previous holder.[1] For example, in Chu et al. (2012), a new innovator in a vertical innovation must transfer a fraction $(0 < s < 1)$ of the profit to the previous innovator as a license fee. The authors assume that s is exogenously determined by implicit bargaining between the firms and regard s as the strength of patent protection. Then, the stronger is patent protection, the larger is s, which works to block the subsequent innovation. Using the setup, Chu et al. (2012) show that the pro-patent policy stimulates horizontal innovation but hinders vertical innovation. On the one hand, as with their study, most of the literature on patent blocking simply assumes exogenous profit division between firms.[2]

On the other hand, numerous studies in IO have analyzed the optimal licensing strategy for a patented firm. For example, as an application of non-cooperative game theory, Kamien and Tauman (1986), Rockett (1990), Muto (1993), Sen and Tauman (2007), Sen and Stamatopoulos (2009), Kitagawa et al. (2014), Chen (2017), and Sen and Tauman (2018) investigate how a patented firm should charge a license fee for profit maximization in several situations. Kamien and Tauman (1986), a seminal study, develop a three-stage model in which an innovator determines the fixed licensing fee or per-unit output royalty in the first stage, other firms individually decide whether to contract the license or not in the second stage, and all of them engage in Cournot competition in the third stage. The authors show that the optimal mode of licensing contract for a patented firm is a fixed fee, independent of the amount of production of a licensee, rather than a royalty. In addition, there are many IO studies in which the licensing fee is endogenously determined. For example, Mukherjee and Mukherjee (2008) assume that the licensing fee is determined through Nash bargaining between the incumbent and entrant.

The present study provides new insights into the effect of patent policy on the optimal licensing fee, the innovation rate, and welfare by developing a unified framework that incorporates the feature of endogenous licensing fee in IO models into an R&D-based growth model. Unlike Kamien and Tauman (1986), who suppose a fixed number of firms in an industry, this study considers an endogenous market structure (EMS) in which the number of firms is determined by a free-entry condition. Moreover, this study explicitly considers subsequent R&D undertaken by licensees. This setup is clearly different from existing studies in

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both the IO and growth literature, which assume that non-patented firms enter the licensing agreement to produce patented goods, not to conduct R&D for quality improvement.

Using the unified framework, this study demonstrates that strengthening patent protection does not necessarily enhance innovation, as some studies empirically show (e.g., Sakakibara and Branstetter 2001; Qian 2007). This conclusion is drawn from two opposite effects of patent licensing on innovation. In the model, stronger patent protection makes it difficult for non-licensed firms to imitate an innovative good without infringement of the patent, and then the innovator can charge a higher licensing fee. This outcome increases the innovator's total profit, which is the sum of profit in Cournot competition and license revenue, which in turn stimulates the incentive to innovate. However, under a certain condition, the policy simultaneously decreases the number of licensees that undertake R&D activities, since these licensees are charged a high fee, which negatively affects innovation. Therefore, innovation may show an inverted U-shaped relationship with the strength of patent protection.

This study makes several contributions to the existing literature. First, this study complements the results obtained in the existing studies on growth theory by shedding light on *voluntary* patent licensing. Although many studies consider compulsory patent licensing between innovators, this is just one among numerous forms of patent licensing in practice. Instead, this study focuses on voluntary patent licensing between the innovator and other firms that will produce or improve the technology. Thus, this study can highlight another aspect of the effect of patent policy on innovation. Second, the present study provides a *microfoundation* for the common assumption in existing studies that stronger patent protection increases the patent holder's share of profit. While previous studies implicitly assume that strong patent protection increases the patent holder's power of negotiation, the present study demonstrates an equivalent effect by constructing the aforementioned three-stage game.

In addition, the present study is related to other studies that analyze the link between patent protection and innovation-driven growth in EMS models.[3] For example, Chu et al. (2016) develop a hybrid model of variety expansion and quality improvement in which the number of firms is endogenously determined. Their model shows that strict patent protection enhances economic growth in the short run but hinders it in the long run, because the larger is the number of entrants, the lower is the value of quality improvement innovation. By contrast, the present study finds that strong patent protection has a non-monotonic effect on growth even in the long run. Although similar results are obtained in many theoretical studies, to the best of our knowledge, no other study shows this effect in a model with EMS and endogenous licensing fee.[4]

The rest of the paper is organized as follows. After constructing the basic model in Section 2, we solve the equilibrium in Section 3. Then, we investigate how patent protection affects the optimal licensing fee, the innovation rate, and welfare in Section 4. An extended analysis is provided in Section 5. Mathematical proofs and derivations are given in the Appendices.

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2. THE BASIC MODEL

The model is based on the quality-ladder model of Grossman and Helpman (1991). Although the problem for households is unchanged, the setup of industries is substantially redesigned.

2.1. Households

We consider an economy consisting of L identical and infinitely living households. There is no population growth. Each household supplies a unit of labor inelastically and earns a wage in every period. The households' intertemporal utility function is as follows:

$$U_t = \int_0^\infty \exp(-\rho t) \ln c_t dt, \tag{1}$$

where ρ is the subjective discount rate and c_t is an index of consumption at time t. The economy has a continuum of industries indexed by $i \in [0, 1]$ and the households consume final goods across all industries. The period utility is

$$\ln c_t = \int_0^1 \ln \left(\sum_{k=0}^{\tilde{k}(i)} \lambda^k x_{kt}^d(i) \right) di, \tag{2}$$

where $x_{kt}^d(i)$ is the consumption of a good whose quality is λ^k in industry i at time t, and $\tilde{k}(i)$ means that innovation has occurred $\tilde{k}(i) + 1$ times in industry i. In the industry, there are $\tilde{k}(i) + 1$ generations of goods ($k = 0, 1, \ldots, \tilde{k}(i)$), and according to the additive specification in the above-mentioned period utility, they are perfect substitutes for households. The quality of each good is represented as an integer k power of $\lambda > 1$, which means that the quality of the new good is λ times higher than the previous one.

The budget constraint of each household is

$$\dot{a}_t = r_t a_t + w_t - e_t,$$

where a_t, r_t, w_t, and e_t are the per capita asset holding, interest rate, wage rate, and per capita expenditure given by

$$e_t = \int_0^1 \ln \left(\sum_{k=0}^{\tilde{k}(i)} p_{\tilde{k}}(i) x_{kt}^d(i) \right) di,$$

where $p_{\tilde{k}}$ is the price of the good whose quality is $\tilde{k}(i)$. The notations often omit t for simplicity when there is no concern about confusion.

Following Grossman and Helpman (1991), we solve the utility maximization problem in two steps: static problem and dynamic problem. First, given instantaneous expenditure level e, we maximize the period utility function $\ln c$. Under the logarithmic utility function, households spend their budget equally across the

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product line $i \in [0, 1]$. Moreover, for each line, they choose the latest good that has the lowest quality-adjusted price. Therefore, the individual demand in industry i is $x_{\tilde{k}}^d(i) = e/p_{\tilde{k}}(i)$ and $x_k^d(i) = 0$ for $k = 0, 1, \ldots, \tilde{k} - 1$.

Second, we solve the dynamic maximization problem. Each household decides the expenditure e in each period so as to maximize intertemporal utility function, U_t, subject to the intertemporal budget constraint. Their indirect period utility function is given by $\ln c_t = \ln e_t - \ln P_t$, where P_t is the ideal price index associated with the consumption index c_t, which is defined as

$$\ln P_t = \int_0^1 \ln \left(\frac{p_{\tilde{k}t}(i)}{\lambda^{\tilde{k}t(i)}} \right) di. \tag{3}$$

Given the aggregate price index, households spend to maximize their intertemporal utility. From the maximization result, the household's optimal time path for spending is represented by $\dot{e}_t/e_t = r_t - \rho$. Following Grossman and Helpman (1991) and subsequent studies, we treat aggregate expenditure $E_t \equiv e_t L$ as the *numéraire* by normalizing the price index in each period so that $E_t = P_t \cdot c_t L = 1$. This means that the price index falls over time at the growth rate of the consumption index. From this, we obtain $E_t = 1$ and $r_t = \rho$. Hereafter, the notations omit i in cases in which there is no risk of misunderstanding.

2.2. Industries: A Three-stage Game

Suppose that there is a successful innovator of the highest quality good and many potential firms in an industry. We consider a three-stage game that is a variant of Mankiw and Whinston (1986).[5] In the first stage, the innovator decides the licensing fee for its patent and informs the potential entrants about the fee. We assume that all licensees must pay a fixed licensing fee $F > 0$ in each period.[6,7] In the second stage, potential firms individually decide (i) whether to be a licensee; (ii) whether to enter the goods market; and (iii) whether to conduct R&D investment for further innovation. Non-licensed firms can also enter the industry by imitating the innovator's good without infringing the patent. We call such non-licensed firms "imitators." Finally, in the third stage, all firms engage in Cournot competition.[8] After the game, the innovator can obtain both the profit by production and the licensing revenue in every period unless further innovation occurs in the industry.

Figure 1 briefly illustrates the structure of the model. In an industry, the leader (innovator) and the followers (licensees and imitators) produce the latest good. If a following firm succeeds in developing a superior good, then it becomes the new leader. Thereafter, the old leader and the old followers play a new three-stage game with the new leader, and each firm becomes a licensee of the new leader or an imitator.[9] As a result, all firms produce the latest good; therefore, the situation returns to the same as before. Thus, the quality level in an industry rises through the cycle of innovation and technological diffusion through patent licensing or imitation.

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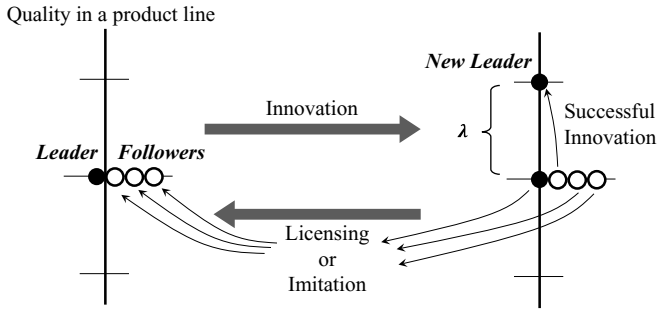


FIGURE 1. The model structure. The quality-ladder on the left-hand side illustrates a situation in which each follower enters to the industry as a licensee or an imitator through the three-stage game. After one of the followers succeeds in R&D, it becomes the new leader, which has the patent of the latest good whose quality is λ times higher than that of the previous good, which is illustrated on the right-hand side. Then, in a new three-stage game, the old leader and old followers re-enter the market through the licensing contract or imitation. Note that this figure does not mean that the model has two different states, because the entry in the three-stage game occurs instantaneously.

We assume that there is no sunk entry cost, because unfortunately, we cannot analytically solve the equilibrium under a positive entry cost. While controlling entry cost is an important industrial deregulation policy, as discussed in the literature of market deregulation (e.g., Chugh and Ghironi 2018; Hamano and Zanetti 2017; Hamano and Zanetti 2018), our model cannot study the economic effects in exchange for analytical tractability. Instead, the present study focuses on the patent licensing fee, which also works to affect firms' entry and exit decisions.

2.2.1. Cournot equilibrium. We solve the three-stage game with backward induction. Before we derive the Cournot equilibrium in an industry in the third stage of the game, let us explain the difference in production technology between firms. We assume that the productivity of imitators is lower than that of the innovator and licensees. While the innovator and licensees can produce one state-of-the-art good by devoting one unit of labor to its production, imitators must employ $\chi > 1$ units of labor to produce one unit of the same quality good, where $\chi \in (1, \lambda)$ indicates the degree of patent breadth that is the extent to which patent holders can legally prevent imitators from copying their patented technologies. For example, the old innovator can perfectly imitate the production technology of the new innovator under the narrowest breadth ($\chi = 1$). By contrast, under the broadest breadth ($\chi = \lambda$), the old innovator cannot improve its production technology by the imitation, since the new good is perfectly protected.[10]

Consider an industry in which an innovator, ℓ licensees, and m imitators produce the same good. The inverse demand function for goods in the industry is $p = 1/X$ where $X \equiv x^d L$ is the aggregate demand for goods in the industry. In the equilibrium, X equals the aggregate output in the industry. Given the inverse

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demand function and wage rate of one unit of labor, denoted by w, producer j maximizes its own profit, $\pi(j)$. The profit maximization problem is

$$\max_{x(j)} \pi(j) = \frac{1}{X} \cdot x(j) - c(j) \cdot w \cdot x(j), \tag{4}$$

where $x(j)$ is the output and $c(j)$ is the production cost. We have $c(j) = 1$ when producer j is the innovator or a licensee and $c(j) = \chi$ when it is an imitator. By solving this problem, we obtain the output of producer j as follows:

$$\begin{aligned} \frac{\partial \pi(j)}{\partial x(j)} = 0 &\Leftrightarrow \frac{1}{X} - \frac{x(j)}{X^2} - c(j) \cdot w = 0 \\ &\Leftrightarrow x(j) = X - c(j) \cdot w \cdot X^2. \end{aligned} \tag{5}$$

Let x_i, x_ℓ, and x_m denote the output of the innovator, a licensee, and an imitator, respectively. Assume that all licensees are symmetric, and so are all imitators. Then, the aggregate output in the industry is written as

$$X = x_i + \ell \cdot x_\ell + m \cdot x_m. \tag{6}$$

By using (5) and (6), we can derive the industry's aggregate output in the Cournot equilibrium as follows:

$$\begin{aligned} X = X - wX^2 + \ell \cdot \left(X - wX^2\right) + m \cdot \left(X - \chi w X^2\right) \\ \Leftrightarrow X = \left(\frac{\ell + m}{1 + \ell + m\chi}\right) \frac{1}{w}. \end{aligned} \tag{7}$$

Then, the price in the Cournot equilibrium is

$$p = \left(\frac{1 + \ell + m\chi}{\ell + m}\right) w. \tag{8}$$

The ratio of output between the two firms is equal to that of their markup $p - c(j) \cdot w$ from (5) and $X = 1/p$. Then, we have

$$x_i = x_\ell \quad \text{and} \tag{9}$$

$$x_m = \left[\frac{1 - \ell(\chi - 1)}{1 + m\chi}\right] x_i. \tag{10}$$

Using (6), (9), and (10), we obtain the equilibrium output of each producer as follows:

$$x_i = x_\ell = \frac{(\ell + m)[1 + m(\chi - 1)]}{(1 + \ell + m\chi)^2} \left(\frac{1}{w}\right) \quad \text{and} \tag{11}$$

$$x_m = \max\left[0, \frac{(\ell + m)[1 - \ell(\chi - 1)]}{(1 + \ell + m\chi)^2} \left(\frac{1}{w}\right)\right]. \tag{12}$$

In the Cournot equilibrium, there is no imitator in the market when the number of licensees is sufficiently high ($\ell \geq \bar{\ell}$), because the equilibrium price is smaller

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than the unit cost of imitators, where $\bar{\ell} \equiv 1/(\chi - 1)$ is the minimum number of licensees required to exclude imitators.

The equilibrium profit of the innovator and licensee are

$$\pi_i = \pi_\ell = \left(1 - \frac{\ell + m}{1 + \ell + m\chi}\right)^2. \tag{13}$$

The equilibrium profit of the imitator is

$$\pi_m = \begin{cases} \left[\frac{1-\ell(\chi-1)}{1+\ell+m\chi}\right]^2 & \text{when } \ell < 1/(\chi - 1) \\ 0 & \text{when } \ell \geq 1/(\chi - 1) \end{cases} \tag{14}$$

2.2.2. The entry and R&D processes. Before we consider the entry decision of the licensees, let us describe the R&D process in the model. We simply regard R&D investment as a binary choice, and its success follows a Poisson process.[11] By employing a worker, a firm can draw a lottery that may succeed in creating a high-quality good with a small probability. We assume that the licensees' probability of R&D success is given by $a > 0$ while the non-licensees' probability is $b > 0$. Following Wang et al. (2013), we assume $a > b$, because patent licensing improves R&D productivity. The success or failure of R&D emerges after the game. In the basic model, as a benchmark, we focus on a situation in which no non-licensee conducts R&D in the equilibrium by assuming that b is sufficiently close to zero. This assumption is relaxed in an extended model in Section 4. Note that the successful innovator does not conduct additional R&D owing to Arrow's replacement effect. Under imperfect protection ($\chi < \lambda$), the innovator cannot increase its profit even if it succeeds in further R&D, because other firms catch up by imitating the new good again. Therefore, all R&D is undertaken by licensees in the basic model.

In the second stage, the potential entrants individually decide (i) whether to be a licensee; (ii) whether to enter the industry; and (iii) whether to conduct R&D investment for further innovation. The decision process and the entrants' payoff are depicted in Figure 2. Although the decisions lead to $2^3 = 8$ results, some of them are not chosen in the equilibrium. For example, the third path (Accept, Not, Invest) in the decision tree in Figure 2 is not chosen. As we show later, $\pi_\ell > 0$ holds in the equilibrium. Then, all firms prefer the first path (Accept, Entry, Invest) to the third path. The fourth path (Accept, Not, Not) is also not chosen, because it provides less than zero payoff. Furthermore, in the basic model, we assume that b is zero. Then, all firms prefer the sixth path (Not, Entry, Not) to the fifth path (Not, Entry, Invest). Therefore, the fifth path is not chosen in the basic model. The seventh path (Not, Not, Invest) is also not chosen, because it provides less than zero payoff.

Let us consider the licensees' decision on whether to conduct R&D investment or only produce the good. Licensees that undertake R&D are referred to as "innovative licensees" in this model, and we call the licensees that only produce the goods "non-innovative licensees." Let $R \in [0, 1]$ denote the fraction of innovative

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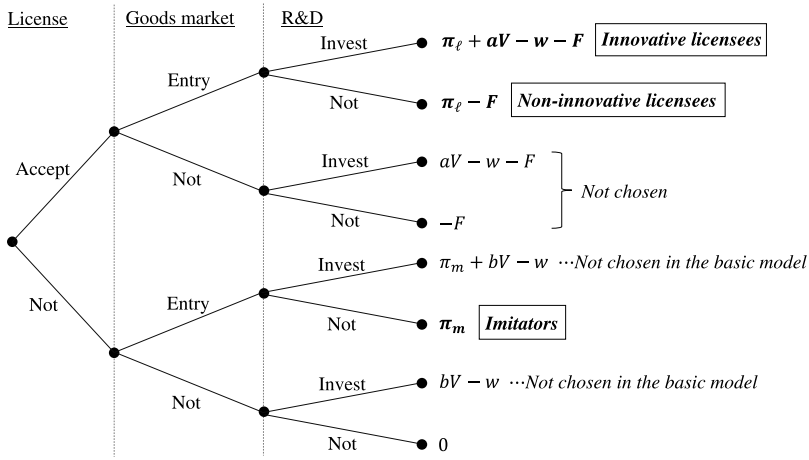


FIGURE 2. The licensing, entry, and R&D decisions in the second stage of the game. A potential entrant decides whether to accept the license, enter the goods market, and undertake R&D investment. The decisions lead to eight results, and the payoff for each is written at the end of the tree.

licensees. Then, the number of innovative licenses is $R\ell$. The free-entry condition for non-innovative licensee is

$$\pi_\ell \leq F \text{ equality holds when } (1-R)\ell > 0. \tag{15}$$

Similarly, the free-entry condition for innovative licensee is

$$\pi_\ell + a\bar{V} - w \leq F \text{ equality holds when } R\ell > 0, \tag{16}$$

where $\bar{V}$ is the *future* firm value that an innovative licensee can gain if it succeeds in R&D. Conditions (15) and (16) are equivalent when $a\bar{V} = w$ holds.

The entry decision of imitators is quite simple. On the one hand, since imitators do not pay license fee F, they enter the market infinitely as long as $\pi_m > 0$. Therefore, from (14), the number of imitators goes to infinity ($m \to \infty$) when $\ell < \bar{\ell}$. On the other hand, when $\ell \geq \bar{\ell}$, no imitator enters the market ($m = 0$), since by (12), they cannot produce a positive amount of goods. The entry decision of imitators is summarized as follows:

$$m = \begin{cases} \infty & \ell < 1/(\chi - 1) \\ 0 & \ell \geq 1/(\chi - 1) \end{cases} \tag{17}$$

Let us consider how many licensees are innovative. The licensees' decision obviously depends on $\bar{V}$, w, m, and F. However, we later observe that the innovator decides F so as to invite some licensees into the market ($\ell = \bar{\ell}$) and to exclude all imitators from the market ($m = 0$) in the first stage of the game. Hence, we here use the result in advance. Then, we obtain the following lemma.

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Lemma 1. *Assume that* $m = 0$ *and* $\ell > 0$ *hold. Then, all licensees are innovative* ($R = 1$) *if and only if* $a\bar{V} > w$ *holds. When* $a\bar{V} = w$ *holds, the number of innovative licensees* R *is indeterminate in* $[0, 1)$.

Proof. See Appendix A. □

This lemma shows that there are two possible cases for the ratio of innovative licensees, R. For convenience, we label each case according to the number of types of licensees that exist in the market. We call the situation in which all licensees are innovative ($R = 1$) "case 1," since there is only one type of licensee in the market. Similarly, we call the situation in which non-innovative licensees and innovative licensees coexist ($0 < R < 1$) "case 2," since there are two types of licensees in the market.

2.2.3. The optimal licensing fee. The innovator earns profit by production and from licensing revenue in each period. For simplicity, we assume that licensing contracts are terminated when one of the licensees succeeds in R&D because it makes the licensed technology obsolete.[12] Since all firms engage in Cournot competition, the previous innovator can remain in the industry after a further innovation occurs.[13] Appendix B shows that after the innovation occurs, the previous innovator becomes a licensee of the successful firm or an imitator and produces the new good, instead of continuing to produce the old good. Since the firm value of licensees and that of imitators are zero under free entry, the innovator loses V when further innovation occurs.

The innovator's expected present value of profit V is derived by the following Bellman equation:

$$V = \max_{F} \left[(\pi_i(\ell, m) + \ell F)dt + \exp(-rdt)(1 - aR\ell dt)V \right]. \tag{18}$$

By approximating ($\exp(-rdt) \simeq 1 - rdt$ and $(dt)^2 \simeq 0$), this equation can be rewritten as

$$rV = \max_{F} \left[\pi_i(\ell, m) + \ell F - aR\ell V \right]. \tag{19}$$

In the first stage of the game, the innovator chooses F so as to maximize the right-hand side (RHS) of the above equation, taking R, $\bar{V}$, and w as given. We assume that the innovator cannot distinguish whether a licensee is going to undertake R&D under the contract because of information asymmetry. Therefore, the innovator cannot practice price discrimination between innovative and non-innovative licensees and regards R as uncontrollable. However, the innovator can decrease the number of innovative licensees ($R\ell$) by choosing F, because ℓ depends on the licensing fee.

In the licensing fee decision, the innovator faces a trade-off between earning licensing revenue and the risk of further innovation by licensees. On the one hand, if the innovator were to charge a sufficiently low licensing fee, it could collect revenue from its licensees but would eventually be replaced by some of

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the innovative licensees. On the other hand, if the innovator were to charge an extremely high licensing fee that prevented the entry of licensees, it could not earn licensing revenue but could earn profit from production forever. The latter case is uninteresting, because the further innovation never occurs. To avoid this outcome, we assume that $a < \rho$ holds. In other words, the risk of further innovation is small and the innovator is sufficiently myopic. Under this parameter assumption, Appendix C demonstrates the following result.

PROPOSITION 1. *Assume that $a < \rho$ holds. Then, the optimal licensing fee for the innovator is $F^* \equiv [1 - 1/\chi]^2 + a\bar{V} - w$ in case 1 and $F^* \equiv [1 - 1/\chi]^2$ in case 2. In both cases, the number of licensees is $\ell = \bar{\ell}$ and there is no imitator ($m = 0$).*

Thus, the innovator chooses the licensing fee F^* that excludes the imitators in the market. The intuition is as follows. From the viewpoint of the innovator, the imitators are harmful, since they do not pay the license fee and push down the market price. Therefore, the innovator attempts to drive the imitators out of the market by attracting licensees instead into the market. This *exclusion of imitators* is a reason for patent licensing by the innovator.

Furthermore, the innovator sets the licensing fee F so that the number of licensees ℓ equals $\bar{\ell}$, the minimum number required to exclude imitators from the market. Basically, there is a negative relationship between F and ℓ, as shown in Appendix C. By this trade-off, the innovator has incentive to charge high F to decrease ℓ as much as possible. There are three reasons for this motive. First, the innovator can raise profit from Cournot competition by decreasing ℓ from (13). Second, the innovator can increase the expected survival time by decreasing ℓ from (19). Third, the innovator can earn high licensing revenue ($F \cdot \ell$), an increasing function of F, as shown in Appendix C. Consequently, the innovator raises F as long as some imitators are in the market, and such threshold value is F^* in Proposition 1.

3. THE EQUILIBRIUM

In this section, we solve the equilibrium in the economy.[14] In the labor market, aggregate labor demand must equal labor supply:

$$x_i + \ell x_\ell + R\ell = L. \tag{20}$$

Furthermore, $r = \rho$ and $V = \bar{V}$ hold in the equilibrium.

***Case 1: All licensees are innovative* ($R = 1$)**

In case 1, $R = 1$ and $\ell^* = 1/(\chi - 1)$. Given this, V and w are determined in the equilibrium. From (11) and (20), we can derive

$$w^* = \left(1 - \frac{1}{\chi}\right)\left(\frac{1}{L(\chi - 1) - 1}\right). \tag{21}$$

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We assume that the labor supply is sufficiently large so that $w^* > 0$ holds. By using (19), (21), and F^*, we can also derive the equilibrium value of V as follows:

$$V^* = \frac{1}{\rho}\left[\left(1 - \frac{1}{\chi}\right) - \frac{1}{\chi}\left(\frac{1}{L(\chi - 1) - 1}\right)\right]. \tag{22}$$

Case 2: Coexistence **($0 < R < 1$)**

This case occurs when $aV = w$ from Lemma 1. Given this, V and R are determined in the equilibrium. From (11), (20), and $aV = w$, we can derive

$$V = \frac{1}{a\chi(L - R/(\chi - 1))}. \tag{23}$$

From (19), we also have

$$V = \frac{1 - 1/\chi}{\rho + aR/(\chi - 1)}. \tag{24}$$

Solving (23) and (24), the equilibrium value of R is

$$R^* = \left(1 - \frac{1}{\chi}\right)\left[L(\chi - 1) - \frac{\rho}{a}\right]. \tag{25}$$

This value is positive when $L(\chi - 1) > \rho/a$ holds. If $L(\chi - 1) > \rho/a$ does not hold, all licensees are non-innovative ($R = 0$). The critical value of χ at which $R = 0$ holds is defined as follows:

$$\chi_0 \equiv \frac{\rho}{aL} + 1. \tag{26}$$

In other words, to allow potential firms to have an incentive to obtain the license, the patent protection must be stronger than χ_0. Therefore, we assume that $\chi > \chi_0$. Furthermore, to make the analysis interesting, we assume that $\chi_0 < \lambda$ holds. Similarly, from (25), another critical value of χ at which $R = 1$ holds can be calculated as

$$\chi_1 \equiv \frac{(\rho/a + 1) + \left((\rho/a + 1)^2 + 4L\right)^{1/2}}{2L} + 1. \tag{27}$$

If $\chi_1 < \lambda$ holds, case 1 arises in the region of $\chi \in [\chi_1, \lambda)$. Similarly, case 2 ($0 < R < 1$) arises in the region of $\chi \in (\chi_0, \chi_1)$.

3.1. Strengthening Patent Policy

This subsection analyzes the effect of strengthening patent protection ($\chi \uparrow$) on the innovation rate and welfare.

3.1.1. The effect on innovation. The economic growth rate $g^* = \dot{C}/C$ is calculated as $g^* = aR^*\ell^* \ln \lambda$, where $aR^*\ell^*$ is the innovation rate. Therefore, we can also interpret the results in the subsection as growth effects.

By using $\ell^* = 1/(\chi - 1)$ and (25), we obtain the following proposition.

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PROPOSITION 2. *The relationship between the strength of patent protection and innovation is an inverted U-shape when χ_1 is smaller than λ. On the other hand, strengthening patent protection enhances innovation when χ_1 is larger than λ.*

Proof. Suppose that $\chi_1 < \lambda$. From the previous discussion, we know that case 1 arises in $\chi \in [\chi_1, \lambda)$ and case 2 arises in $\chi \in (\chi_0, \chi_1)$. For convenience, we first consider the policy effect in case 2. By using $\ell^* = 1/(\chi - 1)$ and (25), we obtain

$$R^*\ell^* = \left(1 - \frac{1}{\chi}\right) L - \frac{\rho}{a\chi}. \tag{28}$$

This is an increasing function of χ, which means that strengthening patent protection stimulates innovation in case 2. Second, we consider the policy effect in case 1. In this case, since all licensees are innovative ($R = 1$), we have $R^*\ell^* = 1/(\chi - 1)$. This is a decreasing function of χ, which means that strengthening patent protection stifles innovation. As a result, the relationship between the strength of patent protection and innovation is an inverted U-shape when $\chi_1 < \lambda$.

Clearly, when χ_1 is larger than λ, case 1 never occurs and only case 2 arises. Therefore, strengthening patent protection enhances innovation in this case. □

To understand the intuition of Proposition 2, we separately consider the effect on ℓ and R. Strengthening patent protection reduces the number of licensees from $\ell^* = 1/(\chi - 1)$.[15] This is because strengthening patent protection raises the innovator's optimal licensing fee F^*. To understand this, recall that the minimum number of licensees required to exclude imitators, $\bar{\ell} = 1/(\chi - 1)$, is small under strong patent protection, and there is a trade-off between F and ℓ. This implies that the innovator can charge a higher F to increase its profit and licensing revenue.[16] On the other hand, strengthening patent protection increases the ratio of innovative licensees ($R \uparrow$). From (13), the reduction in ℓ increases the Cournot profit π_i that a successful innovator can earn and also the value of innovation, V. This naturally stimulates the surviving licensees' incentive to innovate, and thus, R increases.

In case 2 ($R < 1$), strengthening patent protection increases $R\ell$, because the positive effect always dominates the negative effect. However, in case 1 ($R = 1$), there is no positive effect, because all licensees are already innovative. Therefore, strengthening patent protection decreases $R\ell$ in case 1. Since case 1 does not occur when $\chi_1 > \lambda$ holds, the relationship between patent protection and innovation is always positive, as shown in Figure 3. However, when $\chi_1 \leq \lambda$, patent protection and innovation have an inverted U-shaped relationship, as shown in Figure 4.

Thus, the effect of strengthening patent protection on innovation depends on whether χ_1 is less than λ or not. To understand this implication, let us return to equations (24), (25), (27), and $aV = w$. They show that a large L decreases both w and χ_1, which implies that an economy with a small wage rate is likely to be a case of $\chi_1 < \lambda$. This result has the implication that a small-income economy is likely to have an inverted U-shaped relationship between patent protection and

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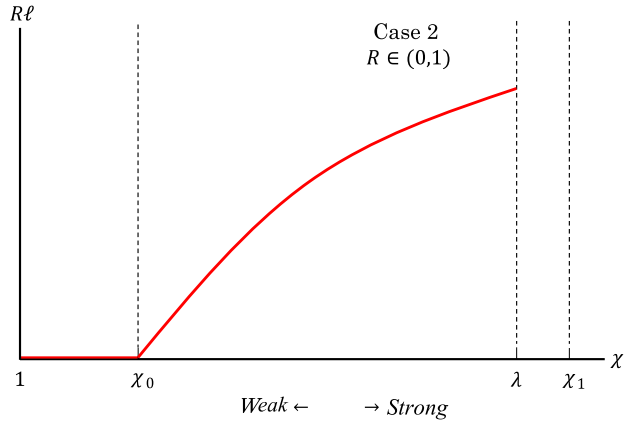


FIGURE 3. The effect of strengthening patent protection on innovation when χ_1 is larger than λ.

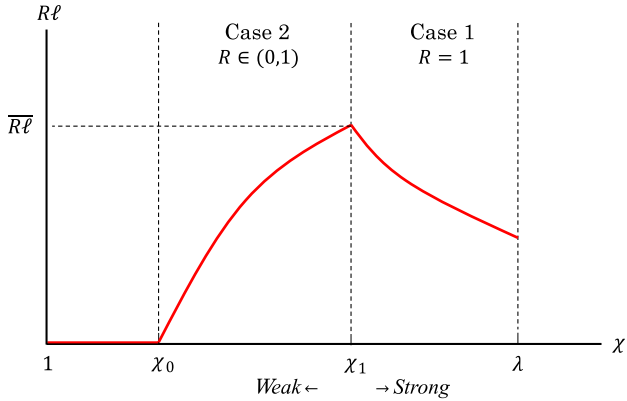


FIGURE 4. The effect of strengthening patent protection on innovation when χ_1 is smaller than λ.

innovation, as shown in Figure 4, while a high-income economy is likely to have a positive relationship, as shown in Figure 3. Some empirical studies yield a similar result. For example, Thompson and Rushing (1996) find that patent protection is likely to be positively related to economic growth in high-income countries, but observe no similar relationship in low-income countries.

3.1.2. The effect on welfare. We start to investigate the welfare effect of strengthening patent protection. To do so, we evaluate welfare in the case in which the economy starts in the steady state. From equation (2) and the labor market equilibrium condition, we have $\ln c_t = g^* \cdot t + \ln(L - R\ell) - \ln L$. By using this and integrating the lifetime-utility function (1) with respect to time, we obtain the

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household's welfare as follows:

$$W = \frac{1}{\rho}\left[\frac{aR^*\ell^* \ln \lambda}{\rho} + \ln(L - R^*\ell^*) - \ln L\right]. \quad (29)$$

By differentiating this with respect to $R^*\ell^*$, we obtain

$$\frac{\partial W}{\partial R^*\ell^*} > 0 \Leftrightarrow R^*\ell^* < L - \frac{\rho}{a \ln \lambda}. \quad (30)$$

When this inequality holds, the welfare effect of strengthening patent protection has the same sign as the effect on innovation. In this case, patent protection and welfare too have either a positive or inverted U-shaped relationship, and innovation-maximizing patent protection maximizes welfare. However, when this inequality does not hold, patent protection and welfare have either a negative or U-shaped relationship. In this case, the innovation-maximizing patent protection minimizes welfare. The following proposition provides a sufficient condition for the welfare effect of strengthening patent protection to have the same sign as the effect on innovation.

PROPOSITION 3. *The welfare effect of strengthening patent protection is qualitatively the same as the effect on innovation if*

$$L > \frac{\rho}{a}\left(\frac{\lambda}{\ln \lambda} - 1\right).$$

Proof. From the previous discussion, the upper bound of $R^*\ell^*$ is given at $\min \chi_1, \lambda$ in both cases (see Figures 3 and 4). The upper bound is maximized when $\chi_1 = \lambda$, and the value is $(1 - 1/\lambda)L - \rho/(a\lambda)$. If this is strictly lower than $L - \rho/(a \ln \lambda)$, the condition of (30) is always satisfied. By rewriting the inequality, we derive the above-mentioned sufficient condition. □

Thus, when L, λ, and a are large and ρ is small, the strengthening patent protection tends to improve welfare. Intuitively, the welfare effect of strengthening patent protection is determined by (i) static distortion by higher price and (ii) dynamic gain from innovation. A large L decreases the wage rate in the labor market equilibrium, and reduces the market price of goods. Then, the static distortion is mitigated and is dominated by the dynamic gain. Likewise, a small ρ allows the households to be more patient, and then they regard the dynamic gain as more important, while a large a and λ naturally magnify the dynamic gain of innovation.

4. DISCUSSION

4.1. The Socially Desirable Patent Protection

We analyze the social optimality of the market equilibrium and the socially desirable patent protection policy. Throughout this subsection, we assume that $\chi_1 < \lambda$ so as to discuss both cases 1 and 2.

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By the previous discussion, the equilibrium number of innovative licensees is $R^*\ell^* = 1/(\chi - 1)$ in case 1 and $R^*\ell^* = (1 - 1/\chi)L - \rho/(a\chi)$ in case 2. Meanwhile, the welfare shown in (29) is maximized at $R\ell^W \equiv L - \rho/(a \ln \lambda)$. Since $R^*\ell^* = R\ell^W$ does not generally hold, the innovation rate in the equilibrium may be either insufficient or excessive depending on the parameters. This is due to many market failures in this model. For example, as in Grossman and Helpman (1991), there is a business-stealing effect that leads firms to conduct more R&D than is socially optimal. On the other hand, there is a positive externality, since firms invest in improving quality without taking into account the welfare of the household. In addition, the leader, when deciding the patent licensing fee, does not take into account the followers' incentive to innovate.

As shown in Figure 4, the government can achieve any $R\ell \in [0, \overline{R\ell}]$ by implementing patent protection policy, where $\overline{R\ell}$ is the upper bound given when $\chi = \chi_1$. This suggests that the government conditionally achieves the socially desirable allocation by patent protection policy. For example, on the one hand, if $R\ell^W \leq \overline{R\ell}$, the government can achieve $R\ell = R\ell^W$ by setting χ at an appropriate level. On the other hand, if $\overline{R\ell} < R\ell^W$, the first-best allocation cannot be attained by the patent protection policy.

4.2. Another Patent Policy: Royalty Cap

Next, we analyze the effect of another patent policy on innovation and welfare. As the policy instrument, we consider a royalty cap that is the upper limit on the patent licensing fee that the innovator can legally charge. In reality, antitrust law often regards an exploitative licensing fee charged by patent holders as an abuse of dominant position and strictly prohibits it.

Let $\bar{F}$ be the maximum licensing fee that the leader can legally charge. Clearly, such regulation is meaningless if the optimal licensing fee is lower than $\bar{F}$. To make the analysis interesting, we investigate the policy effect under $\bar{F} < F^*$. Unfortunately, we cannot analytically solve the equilibrium in case 1 under the royalty cap although we can show its existence. Therefore, we focus on the policy effect in case 2.

Let us solve the equilibrium value of ℓ. As shown in Appendix C, the leader's objective function is increasing in F when $F < F^*$. This implies that the leader charges $\bar{F}$ as a result of maximization. Then, the number of licensees is $\ell^* = 1/\bar{F}^{1/2} - 1$, which is larger than ℓ in the basic model. This means that the royalty cap stimulates the entry of firm ($\ell \uparrow$) by decreasing the patent licensing fee.

Next, we solve the equilibrium value of R in case 2. From (11), (19), (20), and $aV = w$, we obtain

$$R^* = \bar{F}^{1/2}\left(\frac{L}{1/\bar{F}^{1/2} - 1} - \frac{\rho}{a}\right).$$

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This is strictly lower than R^* in the basic model (25). Therefore, the royalty cap decreases the ratio of innovative licensees ($R \downarrow$). The intuition is as follows: since the license fee becomes low, many licensees enter the market. Then, Cournot competition decreases the innovator's profit, and the reward from innovation V also shrinks. Moreover, in the labor market, the many entrants increase the R&D cost because their labor demand pushes up the equilibrium wage rate.

Finally, the number of innovative licensees is given by

$$R^*\ell^* = \bar{F}^{1/2}\left[L - \frac{\rho}{a}\left(\frac{1}{\bar{F}^{1/2}} - 1\right)\right].$$

To investigate the effect of the royalty cap, let us compare this with $R^*\ell^*$ in the basic model (28). Then, we observe that the royalty cap decreases $R^*\ell^*$. The reason is that many new non-innovative licensees that are attracted by the reduction of the license fee crowd out the innovative licensees. This outcome suggests that a policy aimed at diffusing the patented technology to following firms might not stimulate the incentive to innovate.

Can the government improve welfare by such regulation of the patent licensing fee? The answer is, conditionally, yes. Since the royalty cap discourages R&D activity, if firms conduct R&D beyond the socially optimal level $R\ell^W$ when there is no royalty cap, the policy could improve welfare. By contrast, if R&D activity is socially insufficient when there is no royalty cap, the policy might worsen welfare. Thus, the policy effect depends on the relation between $R^*\ell^*$ and $R\ell^W$.

5. EXTENSION: INNOVATION BY NON-LICENSEES

Up to now, our focus has been on the case in which only licensed firms are innovative. However, this section discusses a more general case in which non-licensed firms can also be innovative in the equilibrium by assuming that their own probability of R&D success b is sufficiently large. We still assume that $a > b$ holds. The rest of the setup is the same as in the basic model.

Let us denote the number of innovative non-licensees by n. Then, the innovator's Bellman equation (19) is rewritten as

$$rV = \max_F \left[\pi_i(\ell, m) + \ell F - (aR\ell + bn)V\right]. \tag{31}$$

As in the basic model, the innovator chooses F so as to maximize the RHS of (31), taking R, n, $\bar{V}$, and w as given.

This extension does not affect the Cournot equilibrium in the third stage of the game, because each firm's decision in the Cournot competition is independent of its own R&D activity. Therefore, all results in Section 2.2.1 still hold. Furthermore, as in the basic model, the innovator chooses the licensing fee that excludes all imitators (non-licensed firms) from the goods market in the first stage. Therefore, $\pi_m = 0$ holds in the equilibrium, as in the basic model. As a result, in the extended model, each non-licensed firm decides only whether to be innovative or non-innovative in the second stage of the game. These firms' free-entry condition is

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$$bV \leq w \text{ equality holds when } n > 0. \tag{32}$$

Even in the extended model, all non-licensed firms may be non-innovative when the reward for R&D success V is small owing to a low patent protection level. To consider this, suppose that case 2 applies to the economy ($\chi < \chi_1$). Then, since $aV = w$ holds, as discussed in the basic model, the free-entry condition (32) becomes $bV < w$. This means that there is no innovative non-licensee in case 2. Intuitively, licensees are likely to be more innovative than non-licensees because the licensees' probability of R&D success is relatively high ($a > b$). When $n = 0$ holds, the Bellman equation (31) returns to (19) in the basic model. Then, the optimal licensing fee equals F^* obtained in the basic model. As a result, the extension does not change the results in the basic model in the case 2 economy. In the extended model, we do not focus on this uninteresting situation.

By contrast, some non-licensed firms may be innovative when the reward for R&D success V is large owing to a high patent protection level. Appendix D shows that some non-licensees are innovative in the equilibrium when the patent protection is higher than the critical value χ_b. In this situation, both licensees and non-licensees are innovative because $\chi_1 < \chi_b$ holds, as shown in Appendix D. Throughout the following analysis, we assume $\chi_b < \chi < \lambda$ to generate non-licensed innovative firms in the equilibrium.

For the innovator's maximization, Appendix E shows that the optimal license fee is still $F^* = [1 - 1/\chi]^2 + a\bar{V} - w$ derived in the basic model. This result implies that the number of licensees becomes $\ell^* = 1/(\chi - 1)$ again. By the assumption, we obtain $R = 1$ and $bV = w$, since all licensees are innovative now and some non-licensees are innovative. Thus, n and V are determined in the equilibrium. From the labor market equilibrium condition, we obtain

$$\begin{aligned} & x_i + \ell x_\ell + R\ell + n = L \\ \Leftrightarrow V = \frac{1}{b}\left(1 - \frac{1}{\chi}\right)&\left[\frac{1}{(L-n)(\chi-1)-1}\right]. \end{aligned} \tag{33}$$

Furthermore, from the Bellman equation and F^*, we obtain

$$\begin{aligned} V &= \frac{(1-1/\chi)^2 + \left[(1-1/\chi)^2 + aV - w\right]/(\chi-1)}{\rho + a/(\chi-1) + bn} \\ &= \left(1 - \frac{1}{\chi}\right)\frac{1}{\rho + b[n + 1/(\chi-1)]}. \end{aligned} \tag{34}$$

Solving these two equations, we obtain the number of innovative non-licensees in the equilibrium as follows:

$$n^* = L\left(1 - \frac{1}{\chi}\right) - \left(\frac{\rho}{b\chi} + \frac{1}{\chi - 1}\right). \tag{35}$$

The strengthening patent protection increases the number of innovative non-licensees, because (35) is an increasing function of χ. This is caused by three

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reasons. First, strengthening patent protection increases the gross profit of the innovator ($\pi_\ell + \ell F$). It also increases the firm value of the innovator, which naturally stimulates the incentive to innovate. Second, the strengthening patent protection decreases the R&D cost. As shown in the basic model, the number of licensees declines, since the innovator charges a higher license fee. Then, the licensees' labor demand falls, reducing the wage rate in the labor market equilibrium. Third, the non-licensees do not pay the higher license fees caused by the strengthening patent protection.

While the strengthening patent protection increases innovation by non-licensees, the policy simultaneously excludes innovative licensees ($R\ell^* \downarrow$) as in the basic model. Therefore, the total effect is ambiguous. The total innovation rate is given by

$$aR\ell^* + bn^* = \left(\frac{a-b}{\chi - 1}\right) + \left[Lb\left(1 - \frac{1}{\chi}\right) - \frac{\rho}{\chi}\right]. \tag{36}$$

In other words, the total effect depends on b. For example, the total effect is positive in the extreme case of $b \to a$. The result implies that, even if strengthening patent protection were to decrease the number of innovative licensees, the policy might enhance total innovation when b is close to a. The logic is very simple. As discussed above, strengthening patent protection increases the number of innovative non-licensees ($n \uparrow$). The effect is naturally amplified when the coefficient b in the total innovation rate is high. However, the total effect can be negative or ambiguous when b is not high. For example, Figure 5 presents a numerical example that exhibits a very complex relationship between the total innovation rate and patent protection.

The analysis provides the following policy implication: the effect of a patent policy depends on how difficult further innovation would be without patent licensing of the current leading technology. In some industries, often the latest technology is the "standard-essential patent." In this case, further innovation may be very difficult for non-licensed firms (b is very small), and thus, overly strong patent protection might not encourage innovation in such industries, as shown in the basic model.

6. CONCLUSION

We analyzed the effect of patent policy on licensing fees, innovation, and welfare in an endogenous growth model with a strategic environment involving an innovator and entrants. In this model, the innovator can charge a patent licensing fee so as to maximize its profit. Thereafter, the entrants individually decide whether to become licensees. In other words, this study endogenizes both (i) the licensing fee and (ii) the number of firms in the goods market. These features are not shown in previous studies. Using this setup, we demonstrate that strengthening

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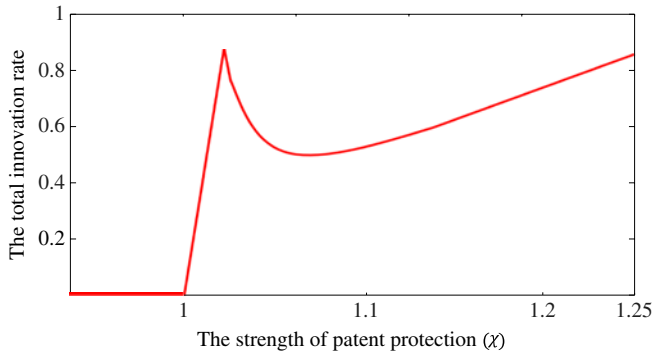


FIGURE 5. A numerical example of the relationship between patent protection and innovation. The parameters are $\lambda = 1.25$, $a = 0.02$, $b = 0.002$, $\rho = 0.03$, and $L = 2000$. The threshold values are $\chi_0 = 1.00075$, $\chi_1 = 1.023$, and $\chi_b = 1.02675$.

patent protection does not always stimulate innovation. Although the policy makes the licensees more innovative, since the reward for innovation increases, the policy also discourages the entrant's incentive to become a licensee, by raising the license fee. The mechanism and intuition of the latter effect are as follows. Strengthening patent protection decreases the profit of firms that are not licensed. This implies that the value of the outside option decreases from the perspective of the entrant, which considers whether or not to become a licensee. Then, since the entrant would like to be a licensee even if the license fee is high, the innovator charges a high license fee to maximize its profit, decreasing the number of licensees. As a result, the model shows an inverted U-shaped relationship between patent protection and innovation under a certain condition.

NOTES

1. Instead, Iwaisako and Futagami (2003) analyze compulsory patent licensing between the innovator and non-innovative firms. The authors interpret the royalty rate to indicate the level of patent protection.

2. Chu and Pan (2013) is an exception. Although they consider a similar profit-division rule to Chu et al. (2012), by which the fraction of profit transferred to the previous innovator, denoted by $0 < s < 1$, is endogenously determined, they assume that $s = \beta/z$, where $\beta \in [0, z]$ is the strength of patent protection and z is the size of quality improvement, determined by the R&D firm.

3. Suzuki (2017) develops a growth model with EMS and demonstrates that strict patent protection can affect innovation both positively and negatively, depending on the level of market competition. In that model, all followers must incur an exogenous operating cost in each period, which works as an effective entry cost under the free-entry condition. Suzuki (2017) does not explicitly specify what the fixed operating cost is, whereas the present study considers it as the licensing fee of the leader's patent. See also Etro (2007) and Etro (2009) for variants of the EMS model and its applications.

4. See Furukawa (2007), Futagami and Iwaisako (2007), Horii and Iwaisako (2007), Cysne and Turchick (2012), and Suzuki (2015).

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5. Mukherjee and Mukherjee (2008) consider a similar three-stage game. In their model, the entrant decides on entry in stage 1. Then, the incumbent and the entrant decide on licensing in stage 2, and they engage in Cournot competition in stage 3. While their model assumes that the potential entrant is a single firm, the present model considers that the number of entrants is endogenously determined by the free-entry condition.

6. Such a mode of payment is called "fixed sum royalty." In practice, there are various forms of payment in patent licensing, such as percentage royalty, per-output royalty, minimum royalty, maximum royalty, fixed-sum royalty, paid-up royalty, and milestone royalty. Furthermore, a new form of licensing contract is often made by mixing two types. Of course, we cannot consider all styles, and thus, this study assumes that firms pay the licensing fee only in the form of a fixed royalty.

7. This setup contrasts with an exogenous profit-division rule in existing studies. In practice, a licensing contract with profit-division between firms is rare because the concept of "profit" is not clear. From the perspective of the licensor, it is difficult to determine whether expenditure incurred by the licensee, including the cost of the licensed technology, is necessary to produce the goods. A licensor that is concerned about the moral hazard of licensees would not like to sign such a contract.

8. This setup deviates from the original model of Grossman and Helpman (1991), because they assume that all firms engage in Bertrand competition. In our model, firms have no incentive to be licensed under Bertrand competition, since all licensed firms earn zero profit despite paying the licensing fee. To undertake an analysis of patent licensing interesting, we adopt Cournot competition in our model.

9. Appendix B shows that the old leader and the old followers never keep producing the old good after the leader's replacement, because they can increase the profit by imitating the new good.

10. For the discussion, see Appendix B. Iwaisako (2016) sets patent breadth that is essentially equivalent to our setup.

11. Horii and Iwaisako (2007) and Marsiglio and Tolotti (2015) also consider a binary choice of research.

12. Similarly, Chu et al. (2012) assume that only the most advanced technology is patented. Furthermore, we assume no "grant-back" clauses, which would require the licensees to assign any improvements that originate from the licensed technology to the licensor. For the analyses of innovation and licensing with grant-back clauses, see Choi (2002).

13. This is in contrast to the model of Grossman and Helpman (1991), in which all firms engage in Bertrand competition. In their model, the old innovator must exit the industry after further innovation, since the new innovator practices a limit-pricing strategy.

14. In this section, we solve the equilibrium by using the labor market-clearing condition. However, we can also solve the equilibrium by using the asset market equilibrium condition $A = V$ and the household's budget constraint.

15. This result seems to be consistent with Arora and Ceccagnoli (2006), who find that patent protection decreases licensing propensity when firms are able to commercialize technology themselves. By contrast, when firms lack specialized complementary assets required to commercialize new technologies, patent protection increases the licensing propensity. Our model corresponds to the former case, since the innovator can commercialize its technology by itself.

16. This positive relationship between χ and F is a contribution of this study. While some existing studies assume that stronger patent protection increases the license fee, the present study endogenously demonstrates it by constructing the three-stage game.

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APPENDIX A: PROOF OF LEMMA 1

In this Appendix, we prove that (i) $R = 1$ holds when $a\bar{V} > w$, and (ii) R is indeterminate in $[0, 1)$ when $a\bar{V} = w$, under the assumption of $m = 0$ and $\ell > 0$. Before the proof, let us consider the entry decision of licensees under $m = 0$. In this case, the equilibrium profit of the licensee is $\pi_\ell = [1/(1+\ell)]^2$. Then, from (16), innovative licensees can enter the market as long as ℓ is less than $1/(F - a\bar{V} + w)^{1/2} - 1$. On the other hand, from (15), a non-innovative licensee can enter only if ℓ is less than $1/F^{1/2} - 1$. Thus, there is a difference between each licensee's threshold value of ℓ that enables its entry.

First, we solve the equilibrium of entry decision of licensees under $a\bar{V} > w$. In this case, $1/F^{1/2} - 1 < 1/(F - a\bar{V} + w)^{1/2} - 1$ holds. By the previous discussion, both types of licensees enter the market as long as $\ell < 1/F^{1/2} - 1$. However, after the number of licensees exceeds $1/F^{1/2} - 1$, non-innovative licensees exit while innovative licensees can still enter. As a result, in the equilibrium, the number of innovative licensees is $\ell = 1/(F - a\bar{V} + w)^{1/2} - 1$, and the number of non-innovative licensees is zero ($R = 1$). Intuitively, the entry incentive is stronger for innovative licensees than for non-innovative licensees. The reason is that R&D is a very profitable investment under $a\bar{V} > w$, and attracts only innovative licensees, while it is of no concern to non-innovative licensees. Conversely, $a\bar{V} > w$ holds when $R = 1$ because the free-entry condition for innovative licensee (16) is binding while (15) is not. As a result, all licensees are innovative ($R = 1$) if and only if $a\bar{V} > w$ holds.

Second, we solve the equilibrium of entry decision under $a\bar{V} = w$. In this case, the free-entry conditions for all licensees, (15) and (16), are equivalent. Intuitively, $a\bar{V} = w$ implies that the expected benefit from innovation equals the R&D cost, and thus, licensees are indifferent to the amount of R&D activity. Therefore, non-innovative licensees and innovative licensees can coexist in the market. In this case, R is determined in $[0, 1)$. □

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APPENDIX B: THE BEHAVIOR OF OLD INNOVATORS

This Appendix shows that, after the innovation occurs, the old innovator becomes a licensee of the successful firm or an imitator and produces the new good, instead of continuing to produce the old good.

To consider this, let us focus on the difference of productivity between the old innovator and the imitator of the new innovator. The old innovator can produce λ units of the "second-latest" good (quality $\lambda^{\tilde{k}-1}$) by employing λ units of labor. On the other hand, if the old innovator imitates the new innovator's good, it can produce one unit of the latest good (quality $\lambda^{\tilde{k}}$) by employing χ units of labor. From the perspective of households' period utility (2), λ units of the second-latest good and one unit of the latest good are exactly the same. This means that the old innovator can decrease its marginal cost and increase profit by imitation as long as $\chi < \lambda$ holds. Therefore, the old innovator never keeps producing the old good, and becomes a licensee of the successful firm or an imitator after the innovation occurs. The same argument can be applied to old followers as long as $\chi < \lambda$ holds. Consequently, no firm keeps producing the old good. Furthermore, from this discussion, we can understand why the old innovator cannot improve its production technology at all under $\chi = \lambda$.

APPENDIX C: PROOF OF PROPOSITION 1

To derive the optimal licensing fee for the innovator, this Appendix investigates the shape of the RHS of Bellman equation (19). In this proof, for simplicity, we assume that the economy is already in equilibrium.

CASE 1: ALL LICENSEES ARE INNOVATIVE ($R = 1$)

Suppose that $a\bar{V} > w$ holds. Then, from Lemma 1, all licensees are innovative ($R = 1$). We divide F into three regions: $(0, a\bar{V} - w]$, $(a\bar{V} - w, (1 - 1/\chi)^2 + a\bar{V} - w]$, and $((1 - 1/\chi)^2 + a\bar{V} - w, \infty)$. Then, we investigate the shape of Bellman equation (19) in each region.

First, suppose that the innovator chooses a licensing fee in $0 < F \leq a\bar{V} - w$. From (16), the licensees continue to enter infinitely even if $\pi_\ell = 0$, and then, from (15), no imitator enters the market. Consequently, $m = 0$ and $\ell \to \infty$ hold as a result of the entry game. However, $R\ell \to \infty$ never occurs in labor market equilibrium, because labor supply is infinite. Therefore, in equilibrium, the innovator never chooses F below $a\bar{V} - w$. Thus, we do not need to analyze the range.

Second, suppose that the innovator chooses a licensing fee in $a\bar{V} - w < F \leq (1 - 1/\chi)^2 + a\bar{V} - w$. Then, from (13), (16), and the proof of Lemma 1, the entry decision of licensees given m and F is summarized as follows:

$$\ell = \begin{cases} 1/(F - a\bar{V} + w)^{1/2} - 1 & \text{when } m = 0 \\ \infty & \text{when } m \to \infty \text{ and } F < (1 - 1/\chi)^2 + a\bar{V} - w \\ [0, \infty) & \text{when } m \to \infty \text{ and } F = (1 - 1/\chi)^2 + a\bar{V} - w. \end{cases} \quad \textbf{(C.1)}$$

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The equilibrium of the entry stage is derived from this and (17) in the manner of solving two reaction functions. When $F < (1 - 1/(\lambda\chi))^2 + a\bar{V} - w$, the number of licensees is necessarily higher than $1/(\chi - 1)$. Therefore, the equilibrium is $m = 0$ and $\ell = 1/(F - a\bar{V} + w)^{1/2} - 1$. Then, the innovator's gross profit is

$$\pi_i + \ell \cdot F = \left(\frac{1}{1+\ell}\right)^2 + F\left(\frac{1}{(F - a\bar{V} + w)^{1/2}} - 1\right), \tag{C.2}$$

which is increasing in F, since the number of licensees ℓ is decreasing in F. Consequently, the RHS of equation (19) is increasing in F. This means that the innovator raises F as long as $a\bar{V} - w < F < [1 - 1/\chi]^2 + a\bar{V} - w$. When the innovator chooses $F^* \equiv [1 - 1/\chi]^2 + a\bar{V} - w$, there are two equilibria: (i) $m = 0$ and $\ell = 1/(\chi - 1)$ and (ii) $m \to \infty$ and $\ell = 0$. To avoid the complexity of multiple equilibria, we assume that (i) is selected as the equilibrium in the second stage of the game. By using $\bar{V} = V$ and w^* in (21) in advance,[17] the RHS of equation (19) in equilibrium can be derived as follows:

$$\left(1 - \frac{1}{\chi}\right)^2 + \left(\frac{1}{\chi}\right)\left[\left(1 - \frac{1}{\chi}\right) - \frac{1}{L(\chi - 1) - 1}\right]. \tag{C.3}$$

Third, suppose that the innovator chooses a licensing fee strictly higher than F^*. Then, the entry decision of licensees is as follows:

$$\ell = \begin{cases} 1/(F - a\bar{V} + w)^{1/2} - 1 & \text{when } m = 0 \\ 0 & \text{when } m \to \infty. \end{cases} \tag{C.4}$$

Then, the number of licensees is necessarily strictly smaller than $1/(\chi - 1)$ by $F = F^*$. Thereafter, imitators infinitely enter the market by (17), and all licensees finally exit the market by (C.4). As a result, $m \to \infty$ and $\ell = 0$ hold in equilibrium. In this case, the RHS of equation (19) in equilibrium becomes $(1 - 1/\chi)^2$. This implies that the innovator chooses F^* in the equilibrium if the second term in (C.3) is strictly positive. The condition is

$$\left(1 - \frac{1}{\chi}\right) - \frac{1}{L(\chi - 1) - 1} > 0, \tag{C.5}$$

which is increasing in χ. Therefore, if this condition is satisfied when $\chi = \chi_1$, then it is always satisfied in case 1 ($\forall \chi \in [\chi_1, \lambda]$). From (25), we obtain

$$\left(1 - \frac{1}{\chi_1}\right) = \frac{1}{L(\chi_1 - 1) - \rho/a}. \tag{C.6}$$

Therefore, if $a < \rho$ holds, condition (C.5) is necessarily satisfied in case 1, and the relationship between the RHS of (19) and F is depicted as Figure C1. As a result, when $a\bar{V} > w$, the optimal fee in equilibrium is F^*.

CASE 2: COEXISTENCE ($0 < R < 1$)

Next, we consider the case of $a\bar{V} = w$. Then, from Lemma 1, the ratio of innovative licensees is $R < 1$. Following the same procedure as in case 1, we solve the optimal licensing fee in case 2.

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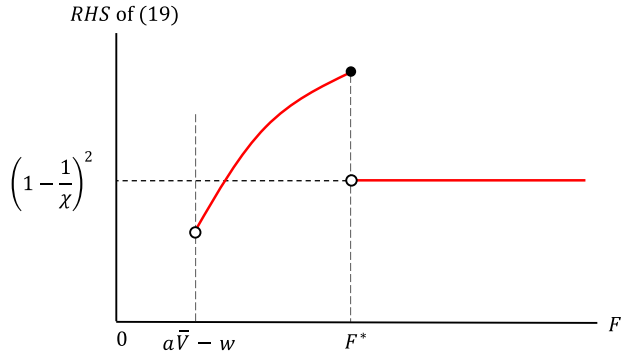


FIGURE C1. The RHS of (19) and the license fee in case 1.

First, suppose that the innovator chooses a licensing fee in $0 < F \leq (1-1/\chi)^2$. The entry decision of the licensees is the same as (C.1) for case 1:

$$\ell = \begin{cases} 1/F^{1/2} - 1 & \text{when } m = 0 \\ \infty & \text{when } m \to \infty \text{ and } F < (1-1/\chi)^2 \\ [0, \infty) & \text{when } m \to \infty \text{ and } F = (1-1/\chi)^2 . \end{cases} \tag{C.7}$$

Using (17) and (C.7), we can solve the equilibrium of the entry stage. When $F < (1-1/\chi)^2$, as in case 1, the number of licensees is larger than $1/(\chi-1)$ regardless of the number of imitators. Then, from (17), there is no imitator in the market ($m = 0$). As a result, $m = 0$ and $\ell = 1/F^{1/2} - 1$ hold in equilibrium. The RHS of equation (19) becomes

$$F^{1/2} - aR\left(\frac{1}{F^{1/2}-1}\right)V. \tag{C.8}$$

Since this is an increasing function of F, the innovator raises F as long as $0 < F < (1-1/\chi)^2$. When the innovator chooses $F^* = (1-1/\chi)^2$, there are two equilibria: (i) $m = 0$ and $\ell = 1/(\chi-1)$ and (ii) $m \to \infty$ and $\ell = 0$. As in case 1, we assume that the first one is chosen as the equilibrium in this case. Then, the RHS of equation (19) becomes

$$\left(1-\frac{1}{\chi}\right) + \frac{aRV}{\chi-1}. \tag{C.9}$$

Second, suppose that the licensing fee is $F > (1-1/\chi)^2$. Then, by (16) and (17), $m \to \infty$ and $\ell = 0$ hold in the equilibrium of the game. In this case, the RHS of equation (19) becomes $(1-1/\chi)^2$, and is necessarily lower than (C.9). Therefore, when $a\bar{V} = w$, the optimal fee is $F^* = (1-1/\chi)^2$.

APPENDIX D: DERIVATION OF χ_b

The threshold value χ_b is given by substituting $n = 0$ into (35) and solving the equation. Then, we obtain

$$\chi_b = \frac{1}{\lambda}\left[\frac{(\rho/b+1) + \left((\rho/b+1)^2 + 4L\right)^{1/2}}{2L} + 1\right]. \tag{D.1}$$

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This is strictly higher than χ_1 in (27), since $a > b$ holds. We assume that $\chi_b < \lambda$ holds in the extended model. In other words, we assume that $\lambda < \chi_b$ holds in the basic model by considering a sufficiently low b.

APPENDIX E: OPTIMAL F IN THE EXTENDED MODEL

Here, we prove that the optimal license fee is still F^* derived in the basic model. To do so, let us compare Bellman equation (31) to that in the basic model (19). The RHS of (31) is of a form that has subtracted only bnV, independent from F in the maximization, from the RHS of (19). This implies that the shape of the graph of (31) is the same as that of (19), as illustrated in Figure C1 in Appendix C. Therefore, the RHS of (31) attains the maximum at $F^* \equiv [1 - 1/\chi]^2 + a\bar{V} - w$, as in the basic model.

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