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New low-frequency waves and negative mass instability in dusty plasmas

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Abstract. Low-frequency dusty plasma waves with frequencies much smaller than the frequency of charging collisions of plasma particles with dust particles are considered taking into account elastic and charging collisions of plasma particles with dust and neutrals. The usual dust sound waves with an upper frequency equal to the dust plasma frequency are found to be present only for wavelengths much smaller than the plasma particle effective mean free path due to the effective collision frequency. The effectice collision frequency is found to be inversely proportional to the square root of the product of the charging frequency and the frequency of particle momentum losses, involving processes due to elastic plasma particle–dust collisions and collisions with neutrals. It is shown that when the wavelength of the wave is much larger than the mean free path for effective collisions, the properties of the waves are different from those considered previously. A negative mass instability is found in this domain of frequencies when the effective mean free path of ions is larger than the effective mean free path of electrons. In the absence of neutrals, this appears to be possible only if the temperature of ions exceeds the electron temperature. This can occur in laboratory experiments and space plasmas but not in plasma-etching experiments. In the absence of instability, a new dust oscillation, a dust charging mode, is found, whose frequency is almost constant over a certain range of wave numbers. It is inversely proportional to the dust mass and charging frequency of the dust. A new dust electron sound wave is found for frequencies less than the frequency of the dust charging mode. The velocity of the dust electron sound wave is determined by the electron temperature but not the ion temperature, as for the usual dust sound waves, with the electron temperature substantially exceeding the ion temperature.

1. Introduction

The main feature of dusty plasmas is rapid dust charging followed by continuous electron and ion recombination on dust particles, which, in the absence of the photoelectric effect, gives rise to negatively charged dust grains. The dust grain will be considered here to be much smaller in size than the Debye shielding radius in

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which the limiting orbit theory approach is shown to describe the charging process (Havnes and Tsytovich 1993; Khodataev et al. 1996). The charging time is very small and is inversely proportional to the charging frequency ν_{ch}, and may be estimated from Havnes and Tsytovich (1993)

$$\nu_{ch} \approx \omega_{pi} \frac{a}{d_i}, \tag{1}$$

where $\omega_{pi} = \sqrt{4\pi n_i e^2/m_i}$ is the ion plasma frequency (singly ionized ions assumed), a is the size of the grain (assumed spherical), $d_i = v_{Ti}/\omega_{pi}$ is the ion Debye radius and $v_{Ti} = \sqrt{T_i/m_i}$ is the ion mean thermal velocity. The charging process occurs on a much shorter timescale, with the corresponding frequency much larger than that given above by a factor of $\sqrt{m_i/m_e}$. The final stage of charging is determined by this expression. The charging is accompanied by the recombination of electrons and ions on dust particles and, in the final stage, when the dust particle charge reaches its equilibrium value, the recombination rate becomes constant. This constant rate should be compensated by a constant ionization rate (production of electron–ion pairs) under constant plasma density conditions.

A dusty plasma is an open system for which the general statement for equilibrium systems that the dielectric permittivity $\epsilon_{\vec{k},\omega}$ for $\omega = 0$ cannot be zero is not valid. This opens the possibility for the attraction of dust particles having the same sign as proposed in Tsytovich (1994). The physical cause of attraction is the force of bombardment of electrons and ions that leads to dust charging. When two dust particles are close to each other, one of these shields the bombardment from the side of the other. The bombardment force, as has been shown by numerical simulation (Khodataev et al. 1996) can exceed the electrostatic forces. The attraction force is inversely proportional to the square of the interdust distance. As predicted by limiting orbit theory (Bernstein and Rabinovich 1959; Tsytovich et al. 1996), and confirmed by numerical simulation (Khodataev et al. 1996), a long-range electrostatic repulsive force inversely proportional to the cube of the interdust distance also appears. In the present paper, we restrict ourselves to a range of wave vectors much larger than that of the order of the inverse length for which the long-range dust repulsion and dust attraction is larger than the dust inertia. The new low-frequency waves and the negative mass instability considered in this paper correspond to frequencies larger than those related to both dust attraction and long-range repulsion.

The aim of the present paper is to investigate the plasma dispersion properties in the low-frequency domain, for frequencies much smaller than the charging frequency, where the charging substantially changes the plasma dispersion but the dust attraction and long-range electrostatic repulsion are still unimportant. We also investigate the possibility of the existence of a negative mass instability in this range of frequencies. The goal is to find new wave modes and the criteria for instability. We find a new dust charging mode and a new electron dust sound wave mode for frequencies much larger than those for which dust attraction occurs. The term 'negative mass instability' in this range of frequencies is used in the sense of an effective negative mass of the dust particles. In the usual response of dust particles, the dispersion relation contains (in the dielectric permittivity), and for the case where only the dust inertia is taken into account, the term $(-\omega_{pd}^2/\omega^2)$. Here $\omega_{pd}^2 = 4\pi n_d Z_d^2/m_d$ is the square of the dust plasma frequency, m_d is the mass of dust particles, n_d is their density and Z_d is their charge in units of electron charges.

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The negative mass of dust particles means $\omega_{pd}^2 < 0$. We intend to show that, in the low-frequency limit, for frequencies much less than the frequency corresponding to particle recombination on dust, the dust response has a form $-(\omega_{pd}^2/\omega^2)F(k^2)$, where the factor $F(k^2)$ can be negative for sufficiently low values of k^2 and in this sense leads to an effective negative mass analogous to the usual gravitational instability.

The paper is organized as follows. We first write a general dispersion relation for a dusty plasma in the low-frequency limit. Then we find new dust modes and the necessary conditions for the development of the negative mass instability, its threshold and critical wavenumber. The properties of a new dust charging acoustic mode for conditions under which the negative mass instability does not develop are discussed. The role of the attractive force between dust particles is estimated and applications to laboratory experiments, as well as new problems, are discussed.

2. Basic equations

To fix notation, let us define all frequencies, including recombination and charging frequencies, which depend on the dust density. Let P represents the charge on a dust particle relative to the electron charge per unit volume

$$P = \frac{n_d Z_d}{n_e}. \tag{1}$$

Here n_d and n_e represent the dust and electron density, respectively. For simplicity we will assume that the plasma ions are singly ionized and write the charge neutrality condition in the form

$$n_i = n_e(1 + P). \tag{2}$$

In etching processes P is equal to or larger than 1, while in space plasmas (planetary rings, for example) $P \ll 1$. P of the order of 1 can also occur in astrophysics (comet tails). The recombination frequency is of the order $\bar{\nu} \approx P\nu_{ch}$. The rate of plasma particle reaction on the charging processes is highest for $\nu \ll \bar{\nu}$, where $1/\nu$ is the characteristic time of the dynamical processes.

We will consider the dispersion properties of a dusty plasma using the limiting orbit charging process for frequencies much less than both ν_{ch} and $\bar{\nu}$. The effective 'dust hydrodynamics' will be used to describe the plasma particle motion for collision frequencies of plasma particles with dust particles much larger than the binary plasma collisions. The local plasma particle distribution originates from collisions with dust and the neutral component, but not binary collisions. The charge of an individual dust particle, given by limiting orbit approach, is determined by the equation

$$exp(-z) = \frac{(1 + P)}{\sqrt{\tau}}\sqrt{\frac{m_e}{m_i}}(\tau + z), \tag{3}$$

where $z = Z_d e^2/(aT_e)$, a is their radius and $\tau = T_i/T_e$ is the ratio of the ion to electron temperatures. In etching experiments, $\tau \approx 10^{-2}$, while in space plasmas $\tau \approx 1$, or even as large as 10. The charging frequency in limiting orbit theory is given by

$$\nu_{ch} = \frac{\omega_{pi}^2 a}{\sqrt{2\pi v_{Ti}}}(1 + \tau + z), \tag{4}$$

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and is of the order of 1 for $\tau \approx 1$ and $z \approx 1$. For most low-temperature plasmas where dust is present, a neutral gas component is also present. To describe its density, n_n, we will use a dimensionless value $N = n_n/n_e$. In most etching experiments $N \approx 10^6$.

We consider only the electrostatic modes in a dusty plasma in the low-frequency limit using perturbed hydrodynamic equations where only the interactions of plasma particles with dust and neutrals are taken into account. The dust will be treated in the collisionless limit, neglecting dust–dust collisions and dust–plasma particle collisions. This is valid provided the dust particles are much heavier than the plasma particles. Under these conditions the equations determining the perturbation in the plasma particle momentum and continuity are

$$(-i\omega + \tilde{\nu}_{e,i})\mathbf{u}_{e,i} = -v_{Te,i}^2 i\mathbf{k}\frac{\delta n_{e,i}}{n_{e,i}}(-+)\frac{e_{e,i}}{m_{e,i}}\frac{\mathbf{k}}{k}E, \tag{5}$$

$$(-i\omega + \bar{\nu}_{e,i})\frac{\delta n_{e,i}}{n_{e,i}} + i\mathbf{k}\cdot\mathbf{u}_{e,i} = -\bar{\nu}_{e,i}\frac{\delta n_d}{n_d} - \delta\bar{\nu}_{e,i}. \tag{6}$$

The subscripts e, i, d denote electron, ion and dust, respectively, $\mathbf{u}$ is the velocity perturbation (the velocity in the equilibrium state is zero, therefore the perturbation is the velocity itself) and $-i\omega$ describes the inertia, which in the low-frequency limit will be neglected. The first term in the right-hand side of (5) describes the pressure of electrons and ions, respectively, δn denotes the deviation of the density from its equilibrium value n, and the effective frequencies $\tilde{\nu}$ in the momentum equation and $\bar{\nu}$ in the continuity equation are different.

The generation of electrons and ions is taken into account in the continuity equation by a term which, from the balance in the equilibrium state, is given by $\bar{\nu}_{n_d}$. The linear perturbation of the recombination term in the continuity equation gives the terms on the right-hand side of (6). The first of them is due to variations of dust density, while the second is due to variations of dust charges. The frequencies $\tilde{\nu}$ and $\bar{\nu}$ can be found from the Limiting Orbit Theory approach. The $\tilde{\nu}$ also contains collisions of plasma particles with neutrals, $\nu_n^{e,i}$. The Limiting Orbit Theory approach also provides expressions for the deviations of the charging collision frequencies about their equilibrium values, $\delta\bar{\nu}_{e,i}$. We find

$$\tilde{\nu}_e = N_n n_e \sigma_n^{(e)} v_{Te} + \nu_{ch}\frac{P(\tau+z)}{z(1+\tau+z)}\left(4 + z + \frac{2z^2}{3}e^z \ln\frac{d}{a}\right), \tag{7}$$

$$\tilde{\nu}_i = N_n n_e \sigma_n^{(i)} v_{Ti}(1+P) + \nu_{ch}\frac{P}{(1+P)z(1+\tau+z)}\left(z + \frac{4}{3}\tau + \frac{2z^2}{3\tau}\ln\frac{d}{a}\right), \tag{8}$$

$$\bar{\nu}_e = \nu_{ch}\frac{P}{z}\frac{(\tau+z)}{(1+\tau+z)}; \quad \bar{\nu}_i = \frac{\bar{\nu}_e}{1+P}, \tag{9}$$

$$\delta\bar{\nu}_e = -z\frac{\delta Z_d}{Z_d}\bar{\nu}_e; \quad \delta\bar{\nu}_i = \frac{z}{\tau+z}\bar{\nu}_i\frac{\delta Z_d}{Z_d}. \tag{10}$$

The first term in (7) and (8) describes the collisions with neutrals; the second term in (7) and (8) contains both inelastic (recombination) and elastic collisions of electrons and ions with dust particles (the terms in the brackets not containing $\ln d/a$).

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To find the variation of charges on dust particles we use the limiting orbit approach for the dust charging equations in the limit $\omega \ll v_{ch}$:

$$\frac{\delta Z_d}{Z_d} = \frac{(\tau + z)}{z(1 + \tau + z)} \left(\frac{\delta n_e}{n_e} - \frac{\delta n_i}{n_i} \right). \tag{11}$$

The system of (5 and 6) then becomes

$$(1 + k_e^2) \frac{\delta n_e}{n_e} = \mathscr{E}_e - \frac{\delta n_d}{n_d} + \frac{(\tau + z)}{1 + \tau + z)} \left(\frac{\delta n_e}{n_e} - \frac{\delta n_i}{n_i} \right), \tag{12}$$

$$(1 + k_i^2) \frac{\delta n_i}{n_i} = -\mathscr{E}_i - \frac{\delta n_d}{n_d} - \frac{1}{1 + \tau + z)} \left(\frac{\delta n_e}{n_e} - \frac{\delta n_i}{n_i} \right), \tag{13}$$

where

$$k_e^2 = \frac{k^2 v_{Te}^2}{\bar{\nu}_e \tilde{\nu}_e}; \quad k_i^2 = \frac{k^2 v_{Ti}^2}{\bar{\nu}_i \tilde{\nu}_i}, \tag{14}$$

and

$$\mathscr{E}_e = \frac{iekE}{m_e \bar{\nu}_e \tilde{\nu}_e}; \quad \mathscr{E}_i = \frac{iekE}{m_i \bar{\nu}_i \tilde{\nu}_i}. \tag{15}$$

We also introduce the dimentionless electron and ion plasma frequencies:

$$\Omega_e^2 = \frac{\omega_{pe}^2}{\bar{\nu}_e \tilde{\nu}_e}; \quad \Omega_i^2 = \frac{\omega_{pi}^2}{\bar{\nu}_i \tilde{\nu}_i}. \tag{16}$$

As the dust is considered collisionless, we can use a simple expression for changes in dust density. For $\omega \gg k v_{Td}$, where v_{Td} is the dust thermal velocity, we have

$$\frac{\delta n_d}{n_d} = -\frac{ieZ_d}{\omega^2 m_d} kE \to \left(\epsilon_{\omega k}^d - 1 \right) ik\delta E. \tag{17}$$

The dispersion relation has the following form:

$$\epsilon_{\omega,k} = A(k^2) - \frac{\omega_{pd}^2}{\omega^2} B(k^2) = 0, \tag{18}$$

where

$$A(k^2) = 1 + \Omega_e^2 \frac{\left[k_i^2 + \dfrac{\tau + z}{1 + \tau + z} + \dfrac{1 + P}{1 + \tau + z} + \dfrac{P(\tau + z)}{z(1 + \tau + z)} (k_i^2 + 1) \right]}{\left[\dfrac{k_e^2(\tau + z)}{1 + \tau + z} + \dfrac{k_i^2}{1 + \tau + z} + k_i^2 k_e^2 (k_i^2 + 1) \right]}$$

$$+ \Omega_i^2 \frac{\left[k_e^2 + \dfrac{1}{1 + \tau + z} + \dfrac{\tau + z}{(1 + \tau + z)(1 + P)} + \dfrac{P(\tau + z)}{(1 + P)(1 + \tau + z)} k_e^2 \right]}{\left[\dfrac{k_e^2(\tau + z)}{1 + \tau + z} + \dfrac{k_i^2}{1 + \tau + z} + k_i^2 k_e^2 \right]} \tag{19}$$

and

$$B(k^2) = 1 + \frac{k_e^2(1 + P) - k_i^2}{P \left[\dfrac{k_e^2(\tau + z)}{1 + \tau + z} + \dfrac{k_i^2}{1 + \tau + z} + k_i^2 k_e^2 \right]}. \tag{20}$$

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3. Negative mass instability

Let us denote the ratio of the square of the effective mean free path of ions to the square of the mean free path of electrons divided by $1 + P$ by

$$F = \frac{\lambda_i^2}{(1+P)\lambda_e^2}. \tag{21}$$

The term effective mean free path is determined by $\lambda_{e,i} = \sqrt{\lambda_{ch}^{e,i}\lambda_m^{e,i}}$, where $\lambda_{ch}^{e,i} = v_{T_{e,i}}/\bar{\nu}_{e,i}$ is the mean free path for the charging process for electrons and ions respectively, and $\lambda_m^{e,i} = v_{T_{e,i}}/\tilde{\nu}_{e,i}$ is the mean free path for momentum losses of electrons and ions respectively. Using previous definitions, F is also given by

$$F(\tau, P, N) = \frac{k_i^2}{k_e^2(1+P)}. \tag{22}$$

The condition for development of a negative mass instability is to have the value of B as negative. We may write B in the following form:

$$-B = \frac{k_{cr}^2 - k_i^2}{k_0^2 + k_i^2}, \tag{23}$$

where k_{cr} is the critical value of the wavenumber. The instability can develop only for $k < k_{cr}$ with

$$k_{cr}^2 = F\frac{(1+P)(1+\tau+z)+P\tau}{zP(1+\tau+z)} - \frac{1+P}{P} - \frac{\tau+z}{1+\tau+z}\frac{z+1}{z} \tag{24}$$

and

$$k_0^2 = \frac{F(1+P)+\tau+z}{1+\tau+z}. \tag{25}$$

A necessary condition for development of instability is that the coefficient of F in the numerator is to be positive

$$P < P_{cr} = 1 + \tau + z(P_{cr}, \tau). \tag{26}$$

The solution of this equation is different for different temperatures. For $P < P_{cr}$ the instability will develop if F exceeds some critical value $F > F_{cr}$

$$F_{cr} = \frac{1+\tau+z}{1+\tau+z-P}\left[1 + \frac{Pk_i^2}{1+P} + \frac{P(\tau+z)}{(1+P)(1+\tau+z)}\right]. \tag{27}$$

We find that the negative mass instability can develop when the effective mean free path for ions is larger than the mean free path for electrons. The physical interpretation of this result is that electrons under these conditions will not be able to reach the dust as quickly as the ions and the dust will be discharged. This is the source driving the instability. More explicitly, the instability will develop if

$$\frac{\lambda_i^2}{\lambda_e^2} > (1+P)F_{cr}. \tag{28}$$

For laboratory plasmas $P \geqslant 1$ and $\tau \ll 1$. From (3), (26) and (27) it is readily seen that

$$F_{cr} \approx \frac{P_{cr}}{P_{cr}-P}\left[1 + \frac{P(P_{cr}-1)}{(1+P)P_{cr}} + \frac{Pk_i^2}{1+P}\right] \geqslant 1. \tag{29}$$

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For space plasmas, $P \ll 1$ and $\tau \geqslant 1$, and using the same argument as above results in

$$F_{cr} \simeq \frac{P_{cr}}{P_{cr} - P} \geqslant 1. \tag{30}$$

Expressing (28) in terms of collision frequencies, the negative mass instability will develop in laboratory or space plasmas if

$$\frac{\tilde{\nu}_e}{\tilde{\nu}_i} \geqslant \left(\frac{T_e}{T_i}\right)\left(\frac{m_i}{m_e}\right). \tag{31}$$

For a laboratory Hydrogen plasma $T_e/T_i \geqslant 1$ and $m_i/m_e \simeq 1840$ so that $\tilde{\nu}_e/\tilde{\nu}_i \geqslant 1.84 * 10^3$. Keeping only the neutral component in the collision frequency and employing elementary kinetic theory results for the neutral collisions (Ginzburg 1970), we get

$$\frac{\tilde{\nu}_e}{\tilde{\nu}_i} = \frac{1.7 * 10^{11}}{4 * 10^9}\sqrt{\frac{T_e}{T_i}},$$

which is less than 1840. The charging frequency ratio contribution is much smaller, of the order of 1, as is readily verified. Hence, no negative mass instability develops in laboratory plasmas under etching conditions. Only in the much lower frequency domain, when the dust–dust attraction forces are taken into account, the other type of instability related to these forces can develop (Tsytovich et al. 1996). In space plasmas with no neutral contribution to the collision frequency and no charging collision frequency, the contribution is also of the order of 1. For instability to develop, one must have $T_e/T_i \simeq m_i/m_e$. Note that $T_e/T_i \simeq 1840$ for a Hydrogen plasma. However, these conclusions depend on the limiting orbit theory results for the collision frequencies.

4. The new dust charging mode and the new electron dust sound wave mode

The condition where the mean free path of electrons is much larger than the mean free path of ions is often met in laboratory plasmas and etching experiments. Under these conditions the negative mass instability will not develop. For wavelengths much less then the effective mean free paths of either electrons or ions, the dispersion properties of a dusty plasma will change drastically and will be quite different than those expected if the account of charging and other effective collisions of plasma particles with dust particles are not considered. First we check that for wavelengths much less than the effective mean free paths we have introduced above, the usual results known for dust sound waves are recovered.

In the limit $k_i^2 \gg 1$ and $k_e^2 \gg 1$ the coefficient B approaches the value 1 and A approaches a value that corresponds to the Debye screening. Then the dust waves will be the usual dust sound waves with

$$\omega^2 = \omega_{pd}^2 \frac{1}{1 + \dfrac{\Omega_e^2}{k_e^2} + \dfrac{\Omega_i^2}{k_i^2}}. \tag{32}$$

There always exists a domain of wavenumbers where the usual dust sound waves exist because the k values for which the relation $k = k_i$ is less than the inverse Debye

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length by a factor a/d_i where d_i, the ion Debye radius, can always be satisfied. Nevertheless the latter ratio is not very small in existing experiments and it is of interest to consider the case where the inequality is not satisfied.

As in many cases the condition $k_i \approx 1$ correspondes to values of k much larger than $k_e \approx 1$, we can consider the case $k_i \ll 1$ but $k_e \gg 1$. Then we find the following approximate expressions for A and B:

$$B \approx 1 + \frac{(1+P)(1+\tau+z)}{P(\tau+z)}, \tag{33}$$

$$A \approx 1 + \Omega_i^2 \frac{(1+\tau+z)}{(\tau+z)} + \frac{\Omega_e^2}{k_e^2}\left[1 + \frac{1+P}{\tau+z}\right], \tag{34}$$

which leads to a new dust mode

$$\omega^2 = \omega_{pd}^2 \frac{1 + \dfrac{(1+P)(1+\tau+z)}{P(\tau+z)}}{1 + \Omega_i^2 \dfrac{(\tau+z+1)}{(\tau+z)} + \dfrac{\Omega_e^2}{k_e^2}\left[1 + \dfrac{1+P}{\tau+z}\right]}. \tag{35}$$

For $k_e \gg \Omega_e/\Omega_i$ this leads to a new dust oscillation mode related to the charging process. For $\Omega_i \gg 1$

$$\omega \approx \frac{\omega_{pd}}{\Omega_i}\sqrt{\frac{\tau+z+\frac{(1+P)}{P}(1+\tau+z)}{1+\tau+z}}. \tag{36}$$

As $\Omega_i \gg 1$, this frequency is much less than the dust plasma frequency and as Ω_i is inversely proportional to $\bar{\nu}_i\tilde{\nu}_i$, the frequency of the new dust mode will be proportional to $\bar{\nu}_i\tilde{\nu}_i$. From the latter, it can be seen that its presence is determined by the charging process.

For $k_e \ll \Omega_e/\Omega_i$ we find a new dust sound mode, the velocity of which will be determined by the electron temperature (but not the ion temperature as is the case for a usual dust acoustic mode of $T_i \ll T_e$). We shall call this new acoustic mode an electron dust sound wave. The dispersion relation for this is

$$\omega^2 \approx k^2 v_{eds}^2, \tag{37}$$

where

$$v_{eds}^2 = \frac{T_e P Z_d}{m_d} \frac{z+\tau+\dfrac{(1+P)(1+\tau+z)}{P}}{\tau+z+1+P}. \tag{38}$$

Thus, we find that the charging process can substantially change the modes of a dusty plasma for wavelengths less than the effective mean free path of the plasma particle due to the charging process.

5. Limitations of results obtained

We first discuss the limitations arising from the process of electrostatic dust long-range repulsion in a plasma. The long-range electrostatic repulsion potential is not screened by plasma particles and creates long-range interactions corresponding to the low values of wavenumbers (Tsytovich et al. 1996). Thus, the results presented here are not only limited by the upper values of wavenumbers but also by the

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lower values. Fot $\tau \ll 1$, the new charging dust modes appear for $k \ll a/d_i\sqrt{\tau}$ when collisions with neutrals are negligible, while the conditions for the dust long-range repulsion to be negligible contains, in the right-hand side, an additional factor of the order of P^2/τ, which implies that for $P \approx 1$ the new dust mode exists, but for $P \ll 1$ the domain of existence is limited by the relation $P > \sqrt{\tau}$, the ratio of the minimum wave vector of this mode to the maximum wave vector is $P/\sqrt{\tau}$. Comparing the larger value of the electron dust sound wave with the effects of dust unscreened repulsion we get a condition where $k \gg a/d_e^2$, the latter can be neglected while the electron dust sound wave exists for $k \ll Pa/d_e d_i \tau$ (for $\tau \ll 1$). This shows that there always exists a range of wave vectors for the electron dust sound mode if $\tau \ll 1$ and $P \ll \tau^2$. These conditions are always fulfilled in etching experiments. All these constraints are softened when neutrals determine the mean free path because they do not appreciably influence the dust long-range interaction. For $\tau \gg 1$ the electron sound wave and the negative mass instability for long wavelengths exists for $P > \tau$.

6. Conclusions

The results obtained in the present paper can have applications in both space research and laboratory dust-plasma experiments, including plasma etching. The new modes can be excited by using a resonance with external radio frequency (RF) field by matching frequencies. On the other hand, the strong dust sound waves will obviously turn to shock waves and their consideration is an important issue in supernova shocks and cometary physics. Thus, the results of the present paper can be considered as a starting point for research on a dust shock wave with applications to many astrophysical problems. Our main conclusions are the following: (i) The new modes can be used as a laboratory dust plasma diagnostic, and other applications on plasma etching; (ii) the negative mass instability develops for sufficiently large τ; (iii) the absence of a negative mass instability does not imply the absence of other dust instabilities, including the electrostatic dust attraction instability, which in most cases can be complementary to the negative mass instability (see Tsytovich et al. 1996).

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