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HAS THE PHILLIPS CURVE FLATTENED?

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We contribute to the recent debate on the instability of the slope of the Phillips curve by offering insights from a flexible time-varying instrumental variable (IV) approach robust to weak instruments. Our robust approach focuses directly on the Phillips curve and allows general forms of instability, in contrast to current approaches based either on structural models with time-varying parameters or IV estimates in ad-hoc sub-samples. We find evidence of a weakening of the slope of the Phillips curve starting around 1980. We also offer novel insights on the Phillips curve during the recent pandemic: The flattening has reverted and the Phillips curve is back.

1. INTRODUCTION

The Phillips curve (Phillips, 1958) hypothesizes that fluctuations in (wage) inflation can be caused by fluctuations in unemployment. The statistical relationship between inflation and unemployment, however, is prone to changes over time. In Phillips' (1958) words, the statistical "relation between unemployment and the rate of change of wage rates is (…) likely to be highly non-linear," and, in fact, was originally studied by Phillips (1958) in separate sub-samples, showing interesting shapes in "loops," whose pattern seems to change over time.

Indeed, inflation and unemployment seem to have become disconnected in the last decades. The correlation between inflation and real activity at business cycle frequencies has decreased in the 1990s (e.g., Atkeson and Ohanian, 2001; Stock and Watson, 2007, 2008, 2020), especially during the years of the expansion

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that followed the recent financial crisis of 2007–2009—the so-called missing deflation (see, among others, Hall (2011) and Ball and Mazumder (2011, 2021)). The decrease in the cyclical correlation between inflation and unemployment has been attributed by some to a flattening of the slope of the Phillips curve. The presence of parameter instability in several macroeconomic models (including the Phillips curve) has been emphasized by many researchers, including Boldea and Hall (2013) and Antoine and Boldea (2018). More recently, the dramatic increase in both inflation and inflation expectations has raised important policy questions: if the Phillips curve indeed flattened, it would imply that more extreme policy measures would be necessary to maintain inflation at its target value. Thus, the question of whether the Phillips curve flattened is of high empirical relevance.

One of the main challenges in the estimation of the Phillips curve is the presence of endogeneity, as inflation and unemployment are jointly determined in equilibrium. There are two main approaches to handling endogeneity: estimating the Phillips curve as part of structural macroeconomic models (either Structural VARs or DSGEs) or focusing on the Phillips curve relationship by relying on instrumental variable (IV) methods. Both have advantages and disadvantages. On the one hand, it is well-known how to estimate DSGEs and Structural VARs in the presence of instabilities; however, DSGEs and Structural VARs are full-information estimation procedures, and potential misspecification in any other part of the model might contaminate the estimates of the Phillips curve parameters. This could be a serious problem during the recent financial crisis, when structural models have to confront serious misspecification challenges (Canova, Ferroni, and Matthes, 2020; Kuo, Inoue, and Rossi, 2020; Den Haan and Drechsel, 2021). On the other hand, limited-information approaches, such as IVs (e.g., Galí and Gertler, 1999, or Galí, Gertler, and López-Salido, 2001, 2005), are less affected by potential misspecification but methodologies to address time variation in IV models in the presence of weak instruments are lacking in the literature.

The main contribution of this article is to directly estimate the time-varying structural Phillips curve. In particular, we are interested in empirically estimating the *slope* of the Phillips curve over time: the slope is defined and measured as the trade-off between inflation and unemployment in a typical New Keynesian Phillips curve (NKPC) structural model, where current inflation is determined by expected future inflation and marginal costs. The slope is theoretically a function of structural parameters, which we won't attempt to separately estimate; for example, the Phillips curve is flat if all the firms follow the Calvo pricing: the more firms follow the Calvo pricing, the flatter the Phillips curve will be (see Galí and Gertler, 1999, and Section 2.1 in Mavroeidis, Plagborg-Moller, and Stock, 2014, for theoretical derivations). The Phillips curve will be estimated via limited-information methods in an environment robust to instabilities as well as potentially weak instruments; the definition of robustness to weak instruments will be precisely clarified below. To our knowledge, this is the first article that estimates a general time-varying Phillips curve using IVs in the presence of potentially weak instruments. Our approach relies on the novel methodology proposed by Inoue,

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Rossi, and Wang (2024), building on Müller and Petalas (2010), to estimate local projections and IV models with time-varying parameters (TVP-IV). Importantly, their approach allows time variation in the parameters of the first-stage regression as well as in the main regression, thus permitting a time-varying relationship between the instruments and the endogenous variables. The presence of weak instruments, however, might invalidate IV estimation. We invert the weighted average risk (WAR) minimizing interval estimator (Müller and Petalas, 2010) for the strongly identified reduced-form parameters to obtain intervals for the structural parameters of the Phillips curve. Given the empirical importance of weak instrument issues in the estimation of the Phillips curve, the novel TVP-IV estimator is potentially robust to weak instruments.

Using the framework above, we shed light on whether and how the Phillips curve changed over time in a way consistent with the seminal, IV-based approach by Galí and Gertler (1999). We find that the slope of the Phillips curve weakened since the early 1980s: the slope decreased, in absolute value, by 68% in the last two decades. However, we also find that it started reverting back in the most recent COVID-19 pandemic period. We also find that the decrease in the correlation between unemployment and inflation cannot be attributed to monetary policy[1]; rather, to the decrease in the slope of the Phillips curve. Our results are complemented by a comparison with the commonly used benchmark IV model with constant parameters and the importance of instabilities is also demonstrated via structural break tests. We establish our main results in a specification that relies on a valid and strong set of instruments. However, we show that our results are robust to using the same set of instruments and specifications in the literature, carefully accounting for the presence of weak instruments. In addition, our findings do not rely on the rational expectations assumption, as our specification uses SPF survey measures of inflation expectations.

It is crucial to note that even small changes (in magnitude) in the slope of the Phillips curve could have important economic consequences. When shocks are big or their effect compounds, even a small flattening (in magnitude) could have important effects on the economy and on the conduct of monetary policy. For example, changes in the inflation target represent significant policy changes, and firms will likely respond by adjusting prices more frequently with higher trend inflation; thus, the slope of the Phillips curve would become steeper (see Ball, Mankiw, and Romer, 1988). In order to stimulate the economy in this environment, central banks would decrease interest rates more than under constant price flexibility, as pointed out by L'Huillier and Schoenle (2022).[2]

[1] According to our analysis, the empirical correlation between unemployment and inflation does not seem to be explained by changes in monetary policy.

[2] The flattening of the Phillips curve is not the only potential explanation for the disconnect between inflation and real variables during the Great Recession; other explanations which we entertain in our analysis include the possibility that inflation or economic slack are mis-measured and that monetary policy is better at stabilizing inflation, thus flattening aggregate demand.

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Relative to the existing literature, our more general approach allows us to confirm the change in the slope of the Phillips curve in the 1990s as a robust result that emerges directly in the classical and appealing IV approach—a result that was infeasible prior to our work due to lacking methodologies. The decrease in the slope of the Phillips curve, and its recent rise during the pandemic, come to the fore as the reason behind the decrease in the correlation between inflation and unemployment, without requiring auxiliary and restrictive assumptions on other parts of the model describing the economy, nor prior knowledge of the time and pattern of the instability. As we show, our results are very robust to using different measures of slack, different approaches to measure inflation expectations (including using survey's forecasts) and different model specifications.

In contrast to our work, most of the existing literature relies on one of the following approaches: reduced-form time-varying parameter models (Stock and Watson, 2009; Ball and Mazumder, 2019; Ashley and Verbrugge, 2019); semi-structural time-varying parameter approaches (Galí and Gambetti, 2019; Bergholt, Furlanetto, and Vaccaro-Grange, 2022); structural models' estimation in given sub-samples (e.g., Del Negro et al., 2020); estimation of full-information models, such as time-varying parameter VARs, Markov-switching models (Groen and Mumtaz, 2008) and non-linearities (Forbes, Gagnon, and Collins, 2021); and IV estimation in either given sub-samples (Barnichon and Mesters, 2020, 2021) or with discrete, estimated breakpoints (Hall, Han, and Boldea, 2012; Smith, Timmermann, and Wright, 2025; Antoine and Boldea, 2018). Only a handful of papers consider IV estimation in time-varying settings, such as Chen (2015) and Giraitis, Kapetanios, and Marcellino (2021). We summarize our relationship with each of these approaches below.

More thoroughly, a first strand of the literature uses time-varying parameter methods in reduced-form or semi-structural models. Reduced-form approaches study the correlation between inflation and unemployment without resolving the endogeneity problem. For example, Stock and Watson (2009) survey the literature on the evaluation of inflation forecasts in the United States and suggest that Phillips curve forecasts are better than competing multivariate forecasts, although their performance is episodic relative to a univariate benchmark, pointing to the presence of instabilities. Ball and Mazumder (2019) argue that expected inflation was backward-looking until the late 1990s, but then became strongly anchored at the central bank's target value, which would explain why inflation did not decrease in the high unemployment period around the Great Recession. Differently from Stock and Watson (2009) and Ball and Mazumder (2019), we study not only the correlation but also the structural Phillips curve. Ashley and Verbrugge (2019) show that the slope of the correlation varies across business cycle phases. Using a model where unemployment is decomposed into persistent and transitory components, they find that inflation responds differently depending on whether fluctuations in the unemployment gap are persistent or transitory. More broadly, a large literature has pointed to the existence of time variation in inflation (see, e.g., Mertens and Nason, 2020). Our findings of time variation in the slope of the

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Phillips correlation are in line with theirs; the difference between their approach and ours is that we take a model-free approach to time variation.

A second strand of the literature focuses on instabilities in semi-structural models. Galí and Gambetti (2019) adopt a semi-structural approach by estimating a time-varying parameter vector autoregression (VAR) model to identify economic shocks via sign and long-run restriction, then use such shocks to purge the Phillips curve variables and achieve identification of the Phillips curve parameters. In a similar spirit, Bergholt et al. (2022) estimate structural shocks using sign restrictions in constant-parameter VARs; then, they investigate changes in the Phillips curve over time using inflation and unemployment data purged by the relevant shocks in either sub-samples or rolling windows. Differently from Galí and Gambetti (2019) and Bergholt et al. (2022), we directly estimate the Phillips curve using IV methods that do not require identifying all the structural shocks in the economy.

A third strand of the literature relies on structural models. For example, Cogley and Sargent (2005) and Primiceri (2006) estimate time-varying parameter structural VAR models for the U.S. economy. Cogley and Sbordone (2008) include a time-varying trend component in inflation, modeled as a random walk; they estimate the time-varying parameter VAR under cross-equation restrictions following the approach in Sbordone (2002, 2006). In related work, Ascari, Bonomolo, and Haque (2022) use a piece-wise long-run trend in a time-varying VAR to describe a long-run Phillips curve. Del Negro et al. (2020) investigate whether the flattening of the Phillips curve explains the disconnect between inflation and unemployment by focusing on time-varying parameter structural VARs and DSGE models, accounting for the potential time variation in the relationship between inflation and real activity by separately estimating their models in two sub-samples, before and after 1989. The break date is determined by an a-priori choice, as a compromise between choosing a date where the economy became more stable (i.e., the Great Moderation, that started in 1984) and the stability of inflation itself, which seems to date back to the mid-1990s. Differently from their work, we rely directly on estimating the structural Phillips curve via limited-information methods, which are more robust to misspecification, and let the instability in inflation dynamics freely emerge within our time-varying IV estimator.

A fourth strand of the literature focuses on IVs or external information. McLeay and Tenreyro (2019) argue that the fact that inflation follows a seemingly exogenous statistical process, unrelated to the output gap, does not mean that the Phillips curve has disappeared. They show that, in a theoretical model, monetary policy can generate a negative correlation between inflation and the output gap by increasing inflation when output is below potential, thus blurring the identification of the Phillips curve. They find evidence against the disappearance of the Phillips curve using regional data. The identification problem pointed out by McLeay and Tenreyro (2019), however, can be addressed by using IVs, like we do. Barnichon and Mesters (2020, 2021) estimate the Phillips curve and the Phillips multiplier using narrative monetary policy shocks as instruments to address the endogeneity

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problem. To take into account time variation, they split the sample at a known break date. The more general framework by Hall, Han and Boldea (2012) allows for multiple discrete shifts at unknown breakpoints in the conditional mean parameters, while the variance is assumed to be constant; Antoine and Boldea (2018) additionally allow for time-varying information, which includes changes in instrument strength, in second moment of instruments, and in the variance of moment conditions. Overall, while these papers perform sub-sample analysis (e.g., Del Negro et al. (2020) use 1990 as the break point estimate), none of these papers allow for general patterns of time variation, which is instead the main contribution of our article.

Our article is closest to Chen (2015) and Giraitis et al. (2021). Chen (2015) proposes a time-varying coefficient model with time-varying endogeneity, and estimates it via a non-parametric procedure that involves a local linear two-stage least square estimation. Giraitis et al. (2021) develop a non-parametric IV estimation method where the coefficient can potentially vary over time. Both Chen (2015) and Giraitis et al. (2021) use their proposed methods to estimate a time-varying Phillips curve. Chen (2015) assumes a deterministic structural change while Giraitis et al. (2021) consider either deterministic or random coefficient models, and a general non-parametric, kernel-based estimation. We differ from their work because of several reasons: first, we consider local time variation whereas they consider global time variation; second, we estimate time-varying parameter models in the presence of potentially weak instruments; and third, our estimator and credible intervals minimize the WAR. The local time variation assumption is the crucial assumption that allows us to use a linear Gaussian pseudo model approximation.

The remainder of the article is organized as follows. The next section presents the methodology. Section 3 discusses the empirical evidence on the evolution of both the Phillips relation as well as the slope of the structural Phillips curve over time. Section 4 investigates whether the decrease in the correlation between unemployment and inflation is due to monetary policy or to a decrease in the slope of the Phillips curve, and Section 5 discusses the most recent evidence on the Phillips curve, including the recent pandemic. Section 6 concludes. The Appendix contains the data description, theoretical proofs and assumptions, as well as complementary empirical results. For Monte Carlo simulation to evaluate the performance of the proposed weak IV-robust TVP-IV estimators in finite samples and comparisons with alternative estimators, see the online supplementary material in Inoue, Rossi and Wang (2025).

2. THE PHILLIPS CURVE MODEL AND THE METHODOLOGY

Our benchmark Phillips curve is the classic version by Galí and Gertler (1999):

$$\pi_t = c + \gamma_f E_t(\pi_{t+1}) + \gamma_b \pi_{t-1} + \lambda x_t + u_t,$$

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where π_t denotes inflation, x_t denotes the measure of real marginal cost, $E_t(.)$ denotes conditional expectations at time t, and u_t is an unobserved shock. This specification is the same as Galí and Gertler (1999) and Galí et al. (2005): a hybrid NKPC with lagged inflation and the unemployment gap as the forcing variable. We will estimate the NKPC using IVs under the rational expectations assumption.

Our main focus is on estimating the slope of the Phillips curve, namely, λ, which is the object of a lively debate. On the one hand, several researchers found that the slope of the Phillips curve has flattened (i.e., that λ decreased in absolute value over time) or even that the Phillips curve "died," that is, its slope approached zero (see Coibion and Gorodnichenko (2015), Blanchard (2016), Ball and Mazumder (2019), and Stock and Watson (2020), among others). On the other hand, Barnichon and Mesters (2021) and Bergholt et al. (2022), among others, argue that it did not. For example, using a Phillips-multiplier approach, Barnichon and Mesters (2021) argue that the inflation–unemployment trade-off went from being very large before 1990 to being small, but still significant, after 1990, and that the decline in the trade-off is mostly due to the anchoring of inflation expectations. Tenreyro and Thwaites (2016) argue that the disconnect between inflation and real activity may not only be due to a flat Phillips curve but also to a flat aggregate demand, such as one where monetary policy strongly responds to inflation. For example, if the central bank achieves perfect inflation stability, the researcher would observe inflation to be uncorrelated with real activity even if the Phillips curve slope was not zero. Del Negro et al. (2020) find that the slope of the Phillips curve substantially weakened over time, and that is the main reason for the disconnect between inflation and unemployment. Bergholt et al. (2022) find that the Phillips curve is "dead" only unconditionally: once it is cleaned of supply shocks, the Phillips curve is alive and well, and may even have steepened since the financial crisis.[3]

As is well-known in the literature, there are several econometric challenges in estimating the structural Phillips curve. A first challenge is that the forcing variable x_t may be correlated with the structural error term u_t, thus resulting in an endogeneity problem. An additional challenge is that the expected inflation term $E_t(\pi_{t+1})$ is not only endogenous but also unobservable. To address these issues, we consider an IV approach to identification, as in Galí and Gertler (1999), Galí et al. (2005) and Barnichon and Mesters (2020), among others. Suppose Z_t is a vector of valid instruments such that

$$E[Z_t(\pi_t - \gamma_b \pi_{t-1} - \gamma_f \pi_{t+1} - \lambda x_t - c)] = 0.$$

Under the rational expectations assumption, we can include as instruments in Z_t any predetermined variables, such as lags of the endogenous variables.

[3] Bergholt et al. (2022) estimate the Phillips curve using OLS after cleaning the variables for the effects of supply shocks.

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2.1. The Time-Varying IV (TVP-IV) Path Estimator

Our contribution is to estimate the Phillips curve slope allowing the parameters to be time-varying using the time-varying parameter IV (TVP-IV) approach by Inoue et al. (2024). We consider the following time-varying parameter Phillips curve:

$$\pi_t = c_t + \gamma_{f,t}\pi_{t+1} + \gamma_{b,t}\pi_{t-1} + \lambda_t x_t + u_t,$$

where the variance of the error term is also allowed to be time-varying. The parameters of primary interest are thus $c_t, \gamma_{f,t}, \gamma_{b,t}, \lambda_t$ and the shock volatility—in particular, the slope of the Phillips curve, λ_t.

We estimate the parameters of interest using an IV approach. The TVP-IV regression is as follows[4]:

$$\begin{bmatrix} x_t \\ \pi_{t+1} \\ \pi_t \end{bmatrix} = \begin{bmatrix} \beta'_{x,z,t} & c_{x,t} & \beta_{x,\pi_b,t} \\ \beta'_{\pi_f,z,t} & c_{\pi_f,t} & \beta_{\pi_f,\pi_b,t} \\ \lambda_t\beta'_{x,z,t} + \gamma_{f,t}\beta'_{\pi_f,z,t} & \lambda_t c_{x,t} + \gamma_{f,t}c_{\pi_f,t} + c_t & \beta_{\pi,\pi_b,t} \end{bmatrix} \begin{bmatrix} z_{2,t} \\ 1 \\ \pi_{t-1} \end{bmatrix} + \begin{bmatrix} v_{x,t} \\ v_{\pi_f,t} \\ v_{\pi,t} \end{bmatrix}, \tag{1}$$

where $(x_t, \pi_{t+1})'$ are endogenous variables, $z_{2,t}$ are the excluded instruments, and $\beta_{\pi,\pi_b,t} = \lambda_t\beta_{x,\pi_b,t} + \gamma_{f,t}\beta_{\pi_f,\pi_b,t} + \gamma_{b,t}$. Note that the estimation directly constrains the parameter $\beta_{\pi,\pi_b,t}$ to be a function of the actual parameters of interest. The parameter path is estimated according to a minimum WAR criterion, as in Müller and Petalas (2010) and Inoue et al. (2024).[5]

Let $\theta_t = [\beta'_{x,z,t}, c_{x,t}, \beta_{x,\pi_b,t}, \beta'_{\pi_f,z,t}, c_{\pi_f,t}, \beta_{\pi_f,\pi_b,t}, \lambda_t, c_t, \gamma_{f,t}, \gamma_{b,t}, vech(\Sigma_{v,t})']'$, where $\Sigma_{v,t}$ is the covariance matrix of $[v_{x,t}, v_{\pi_f,t}, v_{\pi,t}]'$. Let θ_t evolve slowly over time according to a deterministic function of time, where $\theta_t = \theta + \delta_t$ and δ_t describes small amounts of time variation (of magnitude $T^{-1/2}$). The risk of estimating the wrong parameter path depends on the true parameter path θ_t and no estimator achieves uniformly low risk over all such paths. As explained below, we estimate the parameter path based on Müller and Petalas (2010) and Inoue et al. (2024), by minimizing the WAR, where the weighting is over alternative true parameter paths. The optimal parameter path estimator will assume a weighting function for δ_t such that θ_t is a multivariate Gaussian random walk with a small variance:

$$\theta_t = \theta_{t-1} + \epsilon_t,$$

[4]The first two equations below are the first-stage equations, and the third equation is obtained by substituting the first-stage equations into the Phillips curve.

[5]The notion of risk is associated with a given (bounded) loss function. For instance, as in our case, when estimating the parameter path under a (truncated) mean squared error loss, risk becomes mean squared error averaged over t. The weighting is over alternative true parameter paths.

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where ϵ_t is an i.i.d. disturbance with variance $(1/T)\sigma^2_{\epsilon,t}$, and T is the total sample size.

Modeling the time variation in the parameters as a random walk has a long tradition in the literature (see, e.g., Canova (1983), Cogley and Sargent (2005) and Primiceri (2006), among others).

Let us highlight that we not only allow the coefficients of the Phillips curve to be time-varying in the main regression but also in the first-stage regression, thus allowing a time-varying relationship between the instruments and the endogenous variables.

2.2. The TVP-IV Estimator Robust to Weak Instruments

We estimate the reduced-form parameters in eq. (1) by rewriting it as eq. (2):

$$\begin{bmatrix} x_t \\ \pi_{t+1} \\ \pi_t \end{bmatrix} = \begin{bmatrix} \beta'_{x,z,t} & c_{x,t} & \beta_{x,\pi_b,t} \\ \beta'_{\pi_f,z,t} & c_{\pi_f,t} & \beta_{\pi_f,\pi_b,t} \\ \beta'_{\pi,z,t} & c_{\pi,t} & \beta_{\pi,\pi_b,t} \end{bmatrix} \begin{bmatrix} z_{2,t} \\ 1 \\ \pi_{t-1} \end{bmatrix} + \begin{bmatrix} v_{x,t} \\ v_{\pi_f,t} \\ v_{\pi,t} \end{bmatrix}, \tag{2}$$

where $\beta'_{\pi,z,t} = \lambda_t\beta'_{x,z,t} + \gamma_{f,t}\beta'_{\pi_f,z,t}$, $c_{\pi,t} = \lambda_t c_{x,t} + \gamma_{f,t}c_{\pi_f,t} + c_t$, and $\beta_{\pi,\pi_b,t} = \lambda_t\beta_{x,\pi_b,t} + \gamma_{f,t}\beta_{\pi_f,\pi_b,t} + \gamma_{b,t}$. In particular, the reduced-form parameters are $\beta'_{\pi,z,t}$, $c_{\pi,t}$ and $\beta_{\pi,\pi_b,t}$, rather than λ_t, γ_f, and γ_b. We let θ^{rf}_t denote the vector of reduced-form (''rf'') parameters: $\theta^{rf}_t = [\beta'_{x,z,t}, c_{x,t}, \beta_{x,\pi_b,t}, \beta'_{\pi_f,z,t}, c_{\pi_f,t}, \beta_{\pi_f,\pi_b,t}, \beta'_{\pi,z,t}, c_{\pi,t}, \beta_{\pi,\pi_b,t}, vech(\Sigma_{v,t})']'$, where $\Sigma_{v,t}$ is the covariance matrix of $[v_{x,t}, v_{\pi_f,t}, v_{\pi,t}]'$. Note that, unlike the previous section, in this section, we estimate directly the vector of reduced-form parameters, rather than the structural parameters underlying them. The parameters of primary interest $(\widehat{\lambda}_t, \widehat{\gamma}_f, \widehat{\gamma}_b)$ can then be recovered from the reduced-from parameter estimate $\widehat{\theta}^{rf}_t$.

As Kleibergen and Mavroeidis (2014) point out, the posterior piles up in non-identified parameter regions. Rather than using posterior draws of the structural parameters, we invert a WAR-minimizing set of the reduced-form parameters to obtain a set of structural parameters. Here below, we provide the detailed algorithm we used to estimate the time-varying parameters, adapted from Müller and Petalas (2010), as discussed in Inoue et al. (2024).

2.3. Description of the Estimation Algorithm

Let θ_t include all the slope and covariance matrix parameters in eq. (2), which can be written as $\{\theta_t\}_{t=1}^T = \{\theta + \delta_t\}_{t=1}^T$, with $\sum_{t=1}^T \delta_t = 0$. Consider the corresponding stable system of eq. (2), where all the parameters are time-invariant, denoted as θ. Let $y_t = (x_t, \pi_{t+1}, \pi_t)'$, $v_t = (v_{x,t}, v_{\pi_f,t}, v_{\pi,t})'$, and $f(y_t|z_t, \pi_{t-1}, \theta)$ denote a family of conditional density functions for y_t under the assumption that $\{v_t\}$ has zero mean and covariance matrix Σ_v. In particular, let $\sum_{t=1}^T \ell_t(\theta), \ell_t(\theta) = \ln f(y_t|z_t, \pi_{t-1}, \theta)$ denote the (potentially mis-specified) likelihood of eq. (2), let $s_t(\theta) = \partial\ell_t(\theta)/\partial\theta, t = 1, \ldots, T$, denote the sequence of $(q \times 1)$ score vectors,

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let $h_t(\theta) = -\partial s_t(\theta)/\partial\theta', t = 1, \ldots, T$, denote the sequence of $(q \times q)$ Hessians. Then, $\widehat{\theta} = argmax_{\theta\in\Theta} \sum_{t=1}^{T} \ell_t(\theta)$ satisfies $\sqrt{T}(\widehat{\theta} - \theta_0) \Rightarrow \mathcal{N}(0, S)$, where Θ is the parameter space, the sandwich matrix S is typically estimated as $\widehat{S} = \widehat{H}^{-1}\widehat{V}\widehat{H}^{-1}$ and $\widehat{H} = \frac{1}{T}\sum_{t=1}^{T} h_t(\widehat{\theta})$. If the score vectors are i.i.d., then $\widehat{V} = \frac{1}{T}\sum_{t=1}^{T} s_t(\widehat{\theta})s_t(\widehat{\theta})'$; however, in this context, the residuals in eq. (2) might be serially correlated, thus HAC estimators, such as Newey and West (1987), may be used to account for the serial correlation.

The sample information about θ and $\{\delta_t\}_{t=1}^{T}$ is approximately independent and described by the pseudo model (see Müller and Petalas, 2010):

$$\begin{aligned} \widehat{\theta} &= \theta + T^{-1/2}\widehat{S}\nu_0, \\ \widehat{H}\widehat{V}^{-1}s_t(\widehat{\theta}) &= \widehat{S}^{-1}\delta_t + \nu_t, \quad t = 1, \ldots, T, \end{aligned} \tag{3}$$

with $\nu_t \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \widehat{H})$. Recall that $\widehat{H}$, $\widehat{V}$, and $\widehat{S}$ are consistent estimators of the counterparts defined below. $s_t(\widehat{\theta})$ is the score vector defined above evaluated at the constant parameter estimator $\widehat{\theta}$, ignoring parameter instability. The first equation in (3) follows from the asymptotic normality of the maximum likelihood estimator ignoring local parameter instability subject to $\sum_{t=1}^{T}\delta_t = 0$. The second equation in (3) follows from the second-order Taylor series expansion of the log-likelihood around $\widehat{\theta}$ and approximating $h_t(\widetilde{\theta}_t)$ by $\widehat{H} = \frac{1}{T}\sum_{t=1}^{T} h_t(\widehat{\theta})$, again under the local parameter instability assumption, where $\widetilde{\theta}_t$ is a point between $\widehat{\theta}$ and $\theta + \delta_t$.[6]

Assuming an approximately stationary model and a weighting function for $\{\delta_t\}_{t=1}^{T}$ that is a demeaned multivariate Gaussian random walk, as in Müller and Petalas (2010), the asymptotically WAR minimizing path estimators $\{\widehat{\theta}_t\}_{t=1}^{T}$ under truncated quadratic loss can be obtained as follows:

1. For $t = 1, \ldots, T$, let $\widetilde{x}_t$ and $\widetilde{y}_t$ be all the elements of $\widehat{H}^{-1}s_t(\widehat{\theta})$ and $\widehat{H}\widehat{V}^{-1}s_t(\widehat{\theta})$, respectively.
2. For $c_i \in C = \{0, c_1, c_2, \ldots, c_{n_G}\}$,[7] $i = 0, 1, \ldots, n_G$, compute
   (a) $r_i = 1 - \frac{c_i}{T}$, $\xi_{i,1} = \widetilde{x}_1$, and $\xi_{i,t} = r_i\xi_{i,t-1} + \widetilde{x}_t - \widetilde{x}_{t-1}, t = 2, \ldots, T$;
   (b) the residuals $\{\widetilde{\xi}_{i,t}\}_{t=1}^{T}$ of a linear regression of $\{\xi_{i,t}\}_{t=1}^{T}$ on $\{r_i^{t-1}I_q\}_{t=1}^{T}$;
   (c) $\overline{\xi}_{i,T} = \widetilde{\xi}_{i,T}$, and $\overline{\xi}_{i,t} = r_i\overline{\xi}_{i,t+1} + \widetilde{\xi}_{i,t} - \widetilde{\xi}_{i,t+1}, t = 1, \ldots, T-1$;
   (d) $\{\widehat{\theta}_{i,t}\}_{t=1}^{T} = \{\widehat{\theta} + \widetilde{x}_t - r_i\overline{\xi}_{i,t}\}_{t=1}^{T}$;
   (e) $qLL(c_i) = \sum_{t=1}^{T}(r_i\overline{\xi}_{i,t} - \widetilde{x}_t)'\widetilde{y}_t$, where $\widetilde{w}_0 = 1$ and
   $$\widetilde{w}_i = \sqrt{T(1-r_i^2)r_i^{T-1}/((1-r_i^{2T}))}\,exp[-\tfrac{1}{2}qLL(c_i)].$$
3. Compute $w_i = \widetilde{w}_i / \sum_{j=0}^{n_G}\widetilde{w}_j$.
4. The parameter path estimator is given by $\{\widehat{\theta}_t\}_{t=1}^{T} = \{\sum_{i=0}^{n_G} w_i\widehat{\theta}_{i,t}\}_{t=1}^{T}$.
5. With the weighting functions for $\{\delta_t\}_{t=1}^{T}$ and θ interpreted as priors from a Bayesian perspective, the approximate posterior for θ_t is a mixture of

[6] See Müller and Petalas (2010, p. 1511)) for more detailed discussions.

[7] For the factor of proportionality $\frac{c^2}{T^2}$, Müller and Petalas (2010) suggest a default choice of minimizing WAR relative to an equal-probability mixture of $c \in \{0, 5, 10, \ldots, 50\}$, which represents the standard deviation of the endpoint of the random walk weighting function and covers a wide range of magnitudes for the time variation.

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multivariate normals $\mathcal{N}\left(\widehat{\theta}_{i,t}, T^{-1}\widehat{S}\kappa_t(c_i)\right), i = 0, \ldots, n_G$ with mixing probabilities w_i, where $\widehat{S} = \widehat{H}^{-1}\widehat{V}\widehat{H}^{-1}$, $\kappa_t(c) = \frac{c(1+e^{2c}+e^{2ct/T}+e^{2c(1-t/T)})}{2e^{2c}-2}$, and $\kappa_t(0) = 1$.

In the weak instrument case, we need an extra step to recover the structural parameter estimates $(\widehat{\lambda}_t, \widehat{\gamma}_{f,t}, \widehat{\gamma}_{b,t})$ from the reduced-form parameters θ_t^{rf}. Therefore, in the weak instrument case, we implement the procedure described above in points 1–5, with θ_t^{rf} instead of θ_t, followed by the additional step below:

6. Given the point estimates of the reduced-form parameters $\widehat{\theta}_t^{rf}$ (from Step 4) and their approximate posterior distribution (from Step 5), we recover the structural parameter estimates $\widehat{\lambda}_t, \widehat{\gamma}_{f,t}, \widehat{\gamma}_{b,t}$ and the associated credible sets. We randomly draw $\mathcal{M}$ times from the approximate posterior distribution of the reduced-form parameters and repeatedly recover $\widehat{\lambda}_t^i, \widehat{\gamma}_{f,t}^i, \widehat{\gamma}_{b,t}^i, i = 1, 2, \ldots, \mathcal{M}$. The medians of these recovered values are the structural parameter estimates, and the $(1-a)$ credible set is the set of structural parameter estimates corresponding to the $(1-a)$ WAR minimizing interval of the reduced-from parameters.

The intuition behind the algorithm is as follows. The algorithm is designed to calculate the posterior mean of each normal component and its weight for the mixture of normals, using a version of the Kalman smoother for the Gaussian local level model.

The estimation procedure described above differs from that in Inoue et al. (2024) in Step 6; in fact, our estimation procedure is robust to weak instruments since that will not affect the estimation of the reduced form parameters θ_t. The detailed formulas used for the calculations are provided in the Appendix.

2.4. Theoretical Justification of the Methodology

The theoretical justification for the robustness of the proposed methodology to weak instruments is discussed in the next propositions, while the theoretical assumptions are presented in Section D of the Appendix. While Inoue et al. (2024) focus on a local projections approach when the data generating process follows a time-varying parameter vector moving process, this article considers IV estimation of time-series models, such as the Phillips curve, allowing parameters to vary over time.

The first proposition shows that the time-varying estimates are optimal, in the sense that the expected loss of estimating the parameter path in the misspecified TVP-IV model and the best inference in a corresponding correctly specified model is identical (asymptotically). The second proposition instead states that inference is robust even in the presence of weak instruments.

PROPOSITION 1. *Let Δ denote a mixture of n_G multivariate normal distributions $N(e\widehat{\theta} + \Sigma_i\widehat{s}, \Sigma_i)$, $i = 1, 2, \ldots, n_G$, with mixing probabilities proportional to $\widetilde{w}_i$ and Δ^r a mixture of n_G multivariate normal distributions $N(e\widehat{\theta}^r + \Sigma_i^r\widehat{s}^r, \Sigma_i^r)$ with mixing probabilities proportional to $\widetilde{w}_i^r$, where $e = [I_k \ \cdot \ I_k]'$, $\widehat{s} =$*

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$[s_1(\widehat{\theta})', s_2(\widehat{\theta})', \ldots, s_T(\widehat{\theta})']'$, $\underline{\widehat{s}}^r$ *is the vector of stacked scores* $\widehat{\underline{s}}_t^r$, *where* $\widehat{\underline{s}}_t^r = \widehat{\Xi}^{-1}\widehat{V}^{-1}s_t(\theta_0)$.[8]

Let θ_T^* and θ_T^{r*} be the time-varying parameter paths under the correctly specified model and a misspecified model, respectively. Specifically, θ_T^* minimizes the expected risk relative to Δ in the correctly specified model in the sense of Müller and Petalas (2010, Condition 1),[9] Similarly, let θ_T^{r*} be the time-varying parameter path that minimizes the expected risk relative to Δ^r in a misspecified model satisfying the assumptions in the Appendix.

Assume that the researcher minimizes a loss function $L_T(\cdot)$ defined over θ, δ, and action a^*, which is the parameter path estimate) such that $L_T(\theta_0, [\delta_0\left(\frac{1}{T}\right), \ldots, \delta_0\left(\frac{T}{T}\right)], a^*(\Delta_{1,T})) - L_T\left(\theta_0, \left[\delta_0\left(\frac{1}{T}\right), \ldots, \delta_0\left(\frac{T}{T}\right)\right], a^*(\Delta_{2,T})\right) \rightarrow 0$ whenever the total variation between the two mixtures $\Delta_{1,T}$ and $\Delta_{2,T}$ converges to zero, where Δ_{1T} and Δ_{2T} are two mixtures of n_G normal distributions.

Then, the difference between $L_T\left(\theta_0, \left[\delta_0\left(\frac{1}{T}\right), \ldots, \delta_0\left(\frac{T}{T}\right)\right], \theta_T^*\right)$ in the correctly specified model and $L_T\left(\theta_0, \left[\delta_0\left(\frac{1}{T}\right), \ldots, \delta_0\left(\frac{T}{T}\right)\right], \theta_T^{r*}\right)$ in the misspecified TVP-LP model converges to zero in the Prohorov metric.

PROPOSITION 2. *Let* $C_{\theta_t^{rf}, 1-a}$ *be a* $(1-a)$ *WAR minimizing interval estimator for reduced-form parameters* θ_t^{rf} *that has coverage probability* $(1-a)$ *with respect to the quasi-posterior probability. Define a* $(1-a)$ *WAR minimizing interval estimator for the structural parameters* θ_t *by*

$$C_{\theta_t, 1-a} = \{\theta_t \in \Theta : \textit{There is } \theta_t^{rf} \in C_{\theta_t^{rf}, 1-a} \textit{ such that } \theta_t^{rf} = g(\theta_t)\}. \quad \textbf{(4)}$$

Here, we focus on the just-identified case so that $C_{\theta_t, 1-a}$ *is not empty. By construction, it has coverage probability at least* $1-a$ *with respect to the quasi-posterior probability.*[10]

---

[8] Suppose that the distribution of δ, Q_T, is the distribution of the mixture of $\{T^{-\frac{1}{2}}G_i(t/T)\}$ with mixing probabilities p_i and denote the covariance matrix of $T^{-\frac{1}{2}}\left(G_i\left(\frac{1}{T}\right)', G_i\left(\frac{2}{T}\right)', \ldots, G_i(1)'\right)'$ by $\Sigma_{\delta(i)} = E(\delta(i)\delta(i)')$ for $i = 1, \ldots, n_G$. Then, as in equations (19) and (21) in Müller and Petalas (2010), the mixing probabilities for Δ are defined as

$$\widetilde{w}_i = p_i \left|D_{\widetilde{h}}\Sigma_{\delta(i)} + I_{Tk}\right|^{-\frac{1}{2}} \left|e'D_{\widetilde{h}}e - e'D_{\widetilde{h}}K_iD_{\widetilde{h}}e\right|^{-\frac{1}{2}} \exp\left(\frac{1}{2}\widehat{s}'\Sigma_i\widehat{s}\right),$$

$$D_{\widetilde{h}} = I_T \otimes \widetilde{h},$$

$$K_i = \Sigma_{\delta(i)}(D_{\widetilde{h}}\Sigma_{\delta(i)} + I_{Tk})^{-1},$$

$$\Sigma_i = K_i + (I_{Tk} - KD_{\widetilde{h}})e(e'D_{\widetilde{h}}e - e'D_{\widetilde{h}}K_iD_{\widetilde{h}}e)^{-1}e'(I_{Tk} - D_{\widetilde{h}}K_i),$$

for i=1,.., n_G. $\widetilde{w}_i^r$ is the same as $\widetilde{w}_i$ with $\widehat{s}$ and $\widetilde{h}$ in the definition of w_i, K_i, and Σ_i replaced by $\widehat{s}^r$ and $\widetilde{h}^r = \widehat{S}^{-1} = \widehat{H}\widehat{V}^{-1}\widehat{H}$, respectively.

[9] Although their assumptions are too long to present here, they are standard high-level assumptions for correctly specified models, including the assumptions of uniquely and globally identified parameters, scores forming martingale difference arrays, and a version of the information matrix equality.

[10] The fact that the time variation in the parameters is related to the score (see eq. (3)) is reminiscent of score-driven models, such as Creal et al. (2013) and Harvey (2013). However, the latter consider a predictive framework, and do not consider optimality properties of their estimator, IV approaches, or weak instruments.

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Throughout the article, we will compare our results to the benchmark model with constant parameters. In particular, we report results based on instability tests and comment on the differences between our time-varying approach and the constant parameter IV model typically considered in the literature.

3. HAS THE PHILLIPS CURVE FLATTENED OVER TIME?

In this section, we discuss our main empirical evidence on the three main concepts surrounding the estimation of the Phillips curve in the literature: the Phillips relation, the slope of the Phillips curve, and the Phillips multiplier.

3.1. The Phillips Relation

We start by empirically investigating the reduced-form relationship between inflation and the labor share over time, following Stock and Watson (2020). We focus on the estimated slope ($\beta_{1,t}$) in the following Phillips relation:

$$E_t \Delta_4 \pi_t^4 = \beta_{0,t} + \beta_{1,t} x_t^4, \tag{5}$$

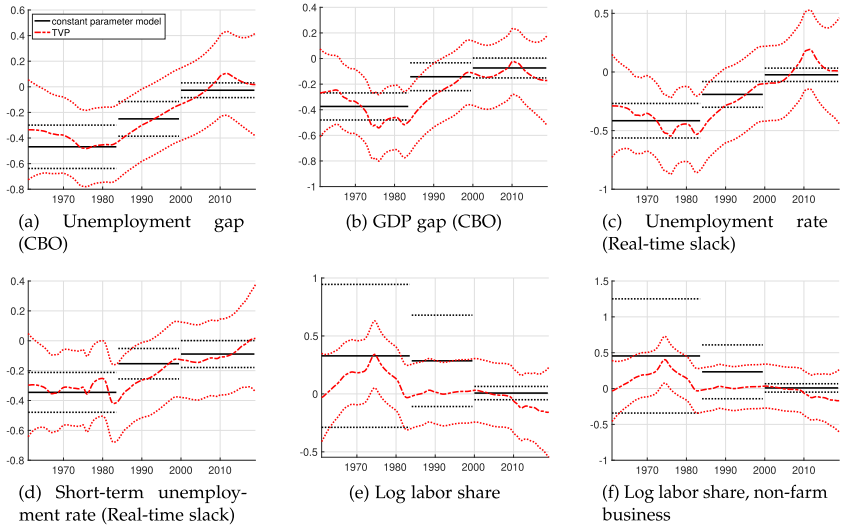
where x_t^4 is the change in the average value of variable "x" between times t and $t-3$ and $\Delta_4 = (1-L^4)$, L denotes the lag operator such that $Lx_t = x_{t-1}$. There are several candidate choices for both inflation and real marginal cost measures (see, e.g., the literature review in Mavroeidis et al. (2014)). In our analysis, inflation (π_t) is measured by personal consumption expenditure price index (PCE excluding food and energy, PCExFE) and x_t is a measure of slack.

Figure 1 reports the time-varying estimate of $\beta_{1,t}$ for various measures of slack for the United States. The estimate based on our path estimator is reported by the dashed line (in red). Each panel in the figure corresponds to a different measure of slack, inspired by Stock and Watson (2020, Table 1). We consider: the unemployment gap, as measured by the Congressional Budget Office (CBO) in panel (a); the GDP gap, also from the CBO, in panel (b); the unemployment rate measured as a real-time slack in panel (c); the short-term unemployment rate in panel (d); the log labor share in panel (e), and the log labor share (non-farm business) in panel (f). In addition, Figure A1 in Appendix B reports the estimates using additional measures of slack.[11]

The instability in the correlation between inflation and the real marginal cost measure is confirmed by standard parameter instability tests, no matter which measure we use. For example, Elliott and Müller's (2006) qLL test on $\beta_{1,t}$ ranges from -12.50 to -3.1671 across the various specifications that we consider. The null hypothesis of no time variation is rejected at the 5% or 10% significance level

[11] All slack measures are standardized, and they have the same mean and standard deviation as the unemployment gap from the CBO. They have also been transformed in order to be positively correlated with the CBO output gap. Appendix A provides more details on the data.

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(a) Unemployment gap (CBO) (b) GDP gap (CBO) (c) Unemployment rate (Real-time slack)

(d) Short-term unemployment rate (Real-time slack) (e) Log labor share (f) Log labor share, non-farm business

FIGURE 1. The time-varying Phillips relation. The figure shows the estimated slope (β_1) in the Phillips relation: $E_t\Delta_4\pi_t^4 = \beta_0 + \beta_1 x_t^4$, where x_t^4 is the change in the average value of variable "x" between times t and $t-3$ and $\Delta_4 = (1-L^4)$. Inflation is measured by PCE-xFE and x_t are the various measures of slack for the United States (see Stock and Watson, 2020, Table 1). The period is 1961–2019.

for all slack measures except the CBO output gap, the short-term unemployment rate measured in real time, and log labor share measures.

We compare our results with Stock and Watson's (2020), who estimate the Phillips relation in three sub-samples. Their estimate is depicted by the solid (black) line, together with 90% confidence bands (black dotted lines).

As Figure 1 shows, the slope of the Phillips relation substantially flattened over time, and this emerges clearly in the data no matter whether we estimate the relation in sub-samples or using our time-varying estimator. Relative to Stock and Watson's (2020) estimates, we find a more gradual decline in the slope magnitude in the 1970s, as well as a flatter curve in the most recent period. Thus, the Phillips relation (i.e., correlation) has disappeared in the data in the most recent period.

But does it mean that the structural Phillips curve has disappeared? Not necessarily, as the Phillips relation measures the (reduced-form) correlation between inflation and unemployment, while the Phillips curve measures the trade-off between inflation and unemployment due to supply shocks. The latter will be considered in the next section.

3.2. The Slope of the Phillips Curve

In what follows, we will directly estimate the structural Phillips curve using the TVP-IV-based approach that flexibly allows the parameters to change over time

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while, at the same time, avoiding the endogeneity problem. In fact, the estimator has the advantage of letting the parameters change over time in a flexible way, including the variance of the error term, as well as the advantage of being robust to endogeneity, as it relies on IVs.[12]

Understanding whether the structural Phillips curve flattened is important, as it implies that more extreme policy measures become necessary to maintain inflation at its target value. It is also an important issue for the design of optimal monetary policy and the desired inflation target. Even small changes (in magnitude) in the slope of the Phillips curve could have important consequences. For example, changes in the inflation target represent significant policy changes, and firms will likely respond by adjusting prices more frequently with higher trend inflation; thus, the slope of the Phillips curve would become steeper (see Ball et al., 1988). In order to stimulate the economy in this environment, central banks should decrease interest rates more than under constant price flexibility, as pointed out by L'Huillier and Schoenle (2022). As a consequence, when shocks are big or their effects compound, as per the mechanisms described in the aforementioned papers, even a small flattening in the slope of the Phillips curve may have important effects on the macroeconomy and on the conduct of monetary policy.

Figure 2 plots the time-varying estimates of the slope of the Phillips curve λ_t, the forward-looking parameter $\gamma_{f,t}$, and the backward-looking parameter $\gamma_{b,t}$; dotted lines report the 90% WAR minimizing interval estimates robust to weak instruments. The main sample ranges from 1974Q1 to 2008Q1. We focus on a model specification where x_t is the output gap, as considered in, e.g., Barnichon and Mesters (2022), and expected inflation is the two-quarter-ahead forecast of the mean GDP deflator inflation from the Survey of Professional Forecasters. The set of instruments includes one lag of the forcing variable and the second lag of inflation.[13]

Our results reveal a flattening of the slope of the Phillips curve (λ_t) in the last two decades. Specifically, the slope has decreased slightly from approximately 0.06 in the early 1970s. Notably, starting from 1975, this slope exhibited a downward trend, ultimately becoming effectively statistically indistinguishable from zero. Interestingly, however, both the forward-looking parameter, $\gamma_{f,t}$, and the backward-looking parameter, $\gamma_{b,t}$, are relatively more stable over time (see panels b and c in Figure 2). In particular, the coefficient on lagged inflation, $\gamma_{b,t}$, affects inflation persistence and, hence, is very relevant for monetary policy. In fact, while over the medium-run inflation is mostly a monetary phenomenon, in the short-run economic shocks create a wedge between actual inflation and the central bank's objective. Inflation persistence provides a measure of the speed of inflation to such shocks and hence it is important to central banks whose policy aims at price

[12] Instabilities are empirically relevant. In fact, Elliott and Müller's (2006) qLL test statistic on the reduced-form regression, which is included as eq. (2) below, is −904.7377. Thus, the null hypothesis of no time variation is rejected at a 1% significance level, reflecting instabilities in the system.

[13] The first lag of inflation is already included as a regressor.

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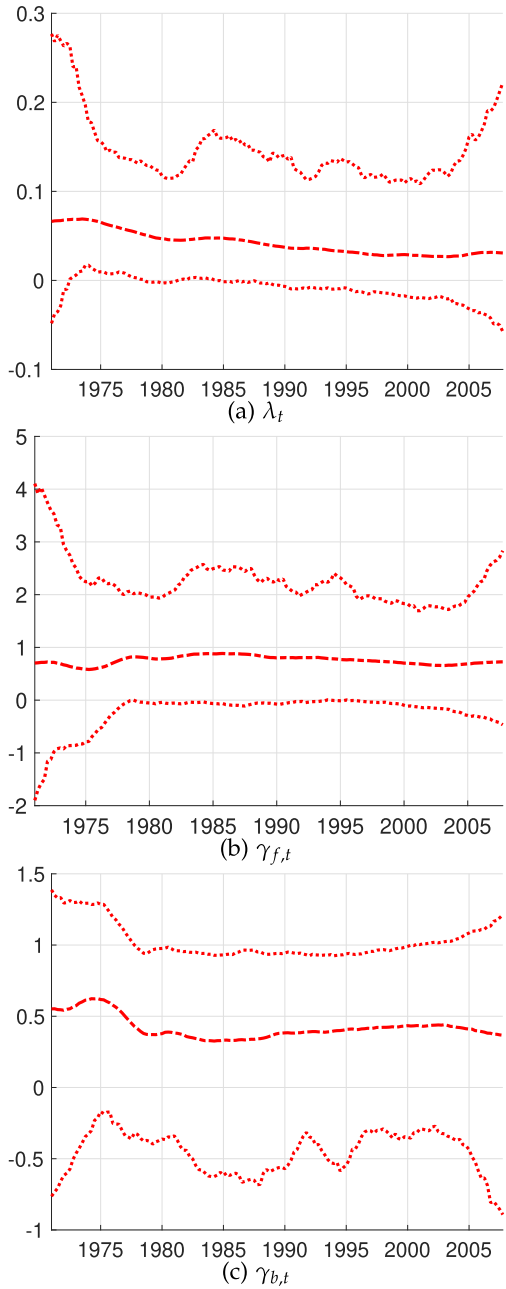


Figure 2. The time-varying Phillips curve using output gap (CBO) as the forcing variable. The figure shows the estimated weak-IV-robust TVP-IV coefficients together with 90% WAR minimizing interval estimates. The sample period is 1971Q1:2007Q4. We estimate the hybrid NKPC with one lag of inflation and the output gap as forcing variable. The set of instruments includes one lag of the forcing variable and the second lag of inflation. HAC-robust variance estimates are implemented with Lazarus et al.'s (2018) recommendation, which implies to 16 lags. The bands are smoothed using a seven quarter centered moving average.

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stability. Our results indicate that inflation persistence has decreased slightly but not substantially over time.

In addition, we report estimates based on a uniform L_2 norm measure, calculated as $||f'|| = (\int_a^b f'(r)^2 dr)^{1/2}$. The L_2 norm measure helps assess the magnitude of the changes over time. The integral measure is also calculated separately over signed derivatives to help assess the direction of the changes over time.[14] Regarding the slope of the Phillips curve (λ_t), we calculate the L_2 norm measure as: $(T-1)^{-1}\sum_{t=2}^{T}(\lambda_t - \lambda_{t-1})^2$.[15] On the one hand, the changes in the slope of the Phillips curve are small in magnitude: over the full sample, the estimate is 0.1923, 39% of which is associated with positive changes and 61% is associated with negative changes. Thus, over most of the sample, the decrease of the slope explains most of the variation over time. On the other hand, the changes in $\gamma_{f,t}$ and $\gamma_{b,t}$ over time are bigger in magnitude than those in λ_t.[16] For $\gamma_{f,t}$, the estimate is 0.6036, of which 52% is due to positive changes and 48% is due to negative changes. For $\gamma_{b,t}$, the estimate is 0.4491, of which 43% is due to positive changes and 57% is due to negative changes. Therefore, while most of the changes in the forward-looking parameter are due to increases, most of the changes in the backward looking parameter are associated with decreases.

Our estimates of the Phillips curve parameters are based on a time-varying path estimator robust to weak instruments, motivated by the extensive evidence of two empirical features of the data: the presence of instabilities and weak instruments. Here, we discuss complementary evidence on both features. Regarding the first issue, namely, the presence of instability, standard tests for parameter instability, such as Elliott and Müller's (2006) qLL test, provide statistical evidence of instability in the parameters of the Phillips curve. Elliott and Müller's (2006) qLL test statistic for structural breaks on the reduced-form regression is −8888.39. The qLL test rejects the null hypothesis of no time variation at the 1% significance level (−103.8677), reflecting instabilities in the system. The qLL test, however, is not robust to weak instruments. Magnusson and Mavroeidis (2014) provide a parameter stability and overidentifying restriction test that is robust to weak identification. The magnitude of the parameter estimates is consistent with that obtained based on full-sample estimates robust to instability and weak identification, such as Magnusson and Mavroeidis (2014) (see Appendix B).[17]

A second, important issue in the estimation of the structural Phillips curve via IVs is the presence of a weak instrument problem (Kleibergen and Mavroeidis, 2009; Mavroeidis et al., 2014). Standard tests find mixed evidence on the existence of a weak instrument problem. For example, Lewis and Mertens' (2022) weak

[14] We thank the editor for suggesting to report the L_2 norm measure.

[15] Given that the magnitude is small, we report estimates multiplied by 100,000.

[16] In fact, for $\gamma_{f,t}$ and $\gamma_{b,t}$, we report the magnitudes multiplied by 1,000.

[17] Figure A7 in Appendix B plots the Magnusson and Mavroeidis' (2014) qLL-AR confidence set for the structural parameters λ, γ_f, and γ_b over the full sample. The qLL-AR results show that, in the full sample, the range of values of the estimated parameters are included in the WAR minimizing interval estimators of the time-varying parameter paths reported in Figure 2.

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IV test statistic equals 6.507, and it is smaller than the 90% critical value (26.235512), indicating that the instruments are weak. The Ganics et al. (2021) weak-instrument robust confidence interval for the strength of identification points to strong instruments (the minimum eigenvalue is 7.57808, with a confidence interval equal to (6.439, 8.126)). The Lewis and Mertens (2022) test and the Ganics et al. (2021) confidence intervals assume parameter stability, however.[18] The mixed results from various statistics measuring instrument strength underscore substantial uncertainty regarding the strength of the instruments. Robust methods are essential when instrument strength is uncertain, as they yield reliable estimates regardless of instrument strength. This is the reason why this article provides WAR minimizing interval estimators that address both the instability and the weak instruments problems.

Figure A4 in Appendix B plots the weak-IV-robust path estimates and credible sets for λ_t together with the TVP-IV estimates for strong instruments (from Steps 1–5 of the algorithm). The TVP-IV estimates also indicate a flattening of the Phillips curve slope from the mid-1970s through the 2000s. The results show that the weak-IV robust point estimate closely aligns with the strong-IV estimate, while the weak-IV credible set is wider, reflecting the trade-off when instrument strength is uncertain.

To provide a comparison with the existing method, Figure A3 in Appendix B plots our weak-IV-robust path estimates of λ_t together with Giraitis et al.'s (2021) nonparametric TVP-IV estimates. Both estimates indicate a flattening of the slope of the Phillips curve.

The main difference between the two estimates is that Giraitis et al.'s (2021) is not robust to weak instruments while ours is, and that is reflected in the larger bands of our method.

Previous papers, notably Galí and Gambetti (2019), have estimated the Phillips curve allowing the parameters to be time-varying. In order to address the endogeneity problem in the estimation of the Phillips curve, their approach relies on purging the OLS estimates using shocks identified via time-varying parameter Structural VARs—hence, it is a semi-structural approach. In particular, their Structural VAR identifies several macroeconomic shocks, using both sign and long-run restrictions. Bergholt et al. (2022) also rely on a semi-structural approach. While the semi-structural approach addresses the endogeneity problem, it requires researchers to separately identify several shocks underlying the economy. Our approach instead does not require researchers to address the challenging task of separately identifying the shocks, as it relies on IVs, in a way that directly parallels the pioneering work of Galí and Gertler (1999).

[18] We also report estimates and confidence intervals for the strength of identification over time in Figure A2 in Appendix B. They are based on Ganics et al.'s (2021) weak-instrument robust estimates and confidence intervals. The confidence interval excludes zeros, thus implying that this set of instruments is not weak in this specification when considering unstable environments.

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3.3. The Phillips Multiplier

In a recent paper, Barnichon and Mesters (2021) propose the "Phillips multiplier" as a measure of the inflation–unemployment trade-off faced by policymakers, different from the slope of the Phillips curve. Relative to Barnichon and Mesters (2021), our approach in the previous section can directly and flexibly estimate the time-varying trade-off between inflation and unemployment in the classical specification of the Phillips curve.[19] Furthermore, we can use the same instruments and specification as in the seminal contribution by Galí and Gertler (1999), which makes our approach more directly comparable to theirs.

However, the Phillips multiplier is an alternative measure of the trade-off between inflation and unemployment that we can estimate using our time-varying method. By being estimated using IV methods, the Phillips multiplier also avoids the typical endogeneity problems afflicting the estimation of the Phillips curve. We focus on the same model specification as in Barnichon and Mesters (2021) with time-varying parameters:

$$\sum_{j=0}^{h}\pi_{t+j} = \mathcal{P}_{h,t}\sum_{j=0}^{h}x_{t+j} + \gamma'_{h,t}W_t + e_{t+h}, \tag{6}$$

where x_t is unemployment, W_t denote control variables, including four lags of inflation and unemployment following Barnichon and Mesters (2021) and monetary policy shocks (η_t) are used as instruments for $\sum_{j=0}^{h}x_{t+j}$, as monetary policy shocks are uncorrelated with supply shocks. The parameter $\mathcal{P}_{h,t}$ denotes the Phillips multiplier in unstable environments and can be estimated based on the TVP-LP-IV estimator in Inoue et al. (2024). Therefore, the Phillips multiplier will estimate the effect of an increase in unemployment on inflation conditional on the presence of monetary policy shocks. Additionally, time-varying parameter impulse responses of inflation and unemployment to monetary policy shocks can be obtained by estimating the following TVP-LP:

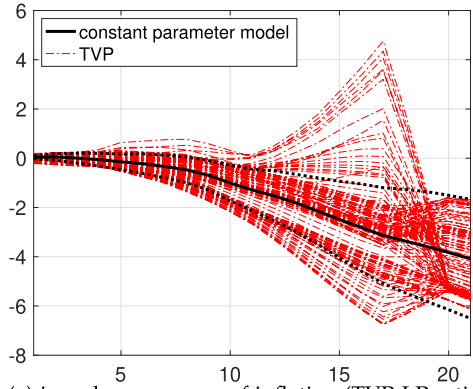
$$\begin{aligned}\pi_{t+h} &= \beta_{\pi,h,t+h}\eta_t + \gamma'_{\pi,h,t}W_t + e_{\pi,t+h},\\ x_{t+h} &= \beta_{x,h,t+h}\eta_t + \gamma'_{u,h,t}W_t + e_{x,t+h},\end{aligned} \tag{7}$$

where $\beta_{\pi,h,t+h}$ and $\beta_{u,h,t+h}$ denote the impulse responses of inflation and unemployment to monetary policy shocks in unstable environments and can be estimated based on the TVP-LP estimator in Inoue et al. (2024).

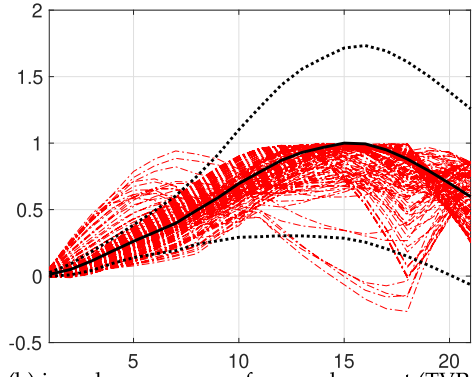
The top two panels in Figure 3 report the impulse responses of inflation and unemployment to a monetary policy shock, whereas the bottom panel depicts the Phillips multiplier. The dashed (red) lines depict our time-varying estimates (each line corresponds to an impulse response estimated at a given point in time), while the continuous (black) line reports the full-sample estimate. The top

[19] Under some assumptions, in particular, a constant forward-looking behavior of inflation, Barnichon and Mesters (2021) relate the Phillips multiplier to the slope of the Phillips curve. In our approach, we instead let the forward-looking inflation coefficient be freely time-varying.

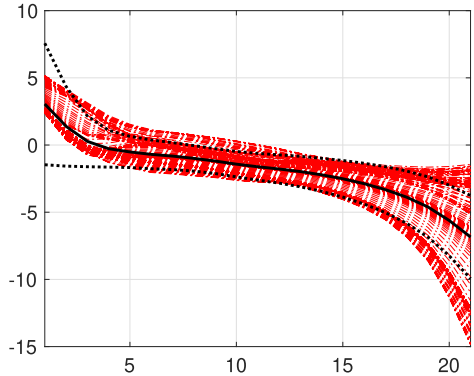
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(a) impulse responses of inflation (TVP-LP estimation)



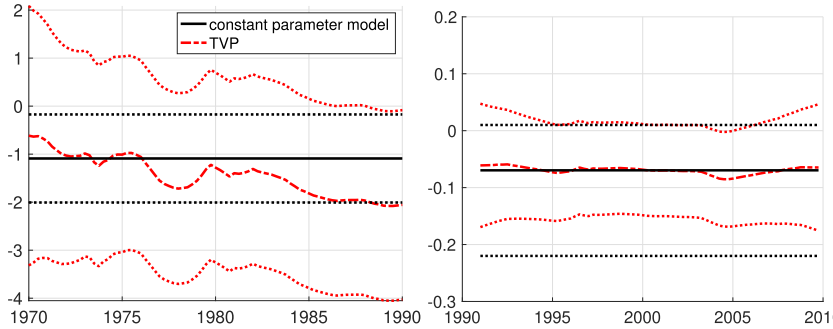
(b) impulse responses of unemployment (TVP-LP estimation)



(c) Phillips multiplier (one-step TVP-LP-IV estimation)

FIGURE 3. The time-varying Phillips multiplier. The figure shows the U.S. Phillips multiplier in the sample period 1969Q1–2007Q4. The black continuous line is Barnichon and Mesters' (2021, Figure 1) full-sample multiplier and the red dashed lines are the TVP-LP-IV multipliers.

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(a) $h = 12$, pre-1990, RR shocks (b) $h = 12$, post-1990, HFI shocks

FIGURE 4. The time-varying Phillips multiplier. The figure shows the TVP-LP estimated U.S. Phillips multiplier over time for selected horizons (dashed red line), using Romer and Romer's shocks as instruments for pre-1990 sub-samples and HFI shocks as instruments for post-1990 sub-samples as to compare with Barnichon and Mesters' (2021) constant multiplier estimates, reported by the black solid line.

two panels in the picture show that, conditionally on a contractionary monetary policy shock, inflation decreases and unemployment increases. The time-varying estimates show that the quantitative extent to which inflation and unemployment respond to a monetary policy shock changes significantly over time. As a result, the estimated Phillips multiplier also varies over time, as shown in the bottom panel of Figure 3.

In order to shed more light on the nature of the time variation in the Phillips multiplier, Figure 4 depicts the Phillips multiplier over time for a selected horizon ($h = 12$). Figure 4(a) depicts the Phillips multiplier over time before 1990, using Romer and Romer's (2004) monetary policy shocks as instruments. Figure 4(b) depicts the Phillips multiplier over time after 1990, using high-frequency-identified (HFI) monetary surprises as instruments. For each sub-figure, the TVP-IV estimates are reported by dash-dot red lines, while the continuous black lines report the sub-sample estimates of the Phillips multiplier in Barnichon and Mesters (2021). The figure shows that the Phillips multiplier also decreased substantially over time, and the decrease dates back to the 1970s. After 1990, the TVP-IV multiplier estimates are close to 0.

Comparing our results in Figure 2 to Figures 3 and 4, the flattening of the Phillips curve is a robust result in our data, no matter whether we consider the Phillips (cor)relation,[20] the Phillips multiplier or the slope of the structural Phillips curve.

[20]We use the term ''Phillips (cor)relation" to refer to the correlation between inflation and real activity, which has been referred to by Stock and Watson (2020) as the ''Phillips curve relation."

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4. WHY HAS THE CYCLICAL CORRELATION BETWEEN INFLATION AND UNEMPLOYMENT DECREASED?

Several researchers have made compelling arguments that the reason for the decrease in the cyclical correlation between inflation and unemployment is related to monetary policy actions. According to such researchers, a more responsive monetary policy to inflation and economic conditions would tighten monetary policy more when it perceives inflation to be increasing, in order to keep the latter under control: this causes unemployment to rise, resulting in a positive correlation between inflation and unemployment that biases the slope coefficient of the Phillips curve toward zero. See Haldane and Quah (1999); Roberts (2006); Williams (2006); Mishkin (2007); Carlstrom, Fuerst and Paustian (2009); and, more recently, McLeay and Tenreyro (2018).

As is well-known, the correlation between inflation and unemployment is the same as the slope of the Phillips curve only in the presence of no endogeneity bias and no measurement error. Thus, the endogeneity problem can be solved using valid and relevant instruments.[21] In the presence of an endogeneity bias due to monetary policy actions, IV estimates will still be consistent provided the instruments satisfy the required statistical conditions—that is, the chosen instruments should be both valid and relevant.

The IV approach we discussed in the previous section, that uses lagged macroeconomic variables as instruments, suggests that the slope of the Phillips curve decreased over time in a manner similar to the decrease in the correlation between inflation and unemployment. Our results therefore suggest that the decrease in the correlation between inflation and unemployment is due to a decrease in the slope of the Phillips curve and not to other factors, among which monetary policy.

As mentioned, our analysis in the previous section is based on an IV method where instruments are both valid and relevant. However, the theoretical validity of the instruments also requires that the residuals of the Phillips curve are not serially correlated; otherwise, the IV procedure may not correctly estimate the slope of the Phillips curve. Lurking inside the residual of the Phillips curve there are both cost push shocks as well as measurement error, and both might be correlated with the lagged instruments via a correlation with their own lags. In our analysis, the residuals of the Phillips curve show some evidence of serial correlation.

To be robust to this potential problem, we consider aggregate demand shocks as instruments, as in Barnichon and Mesters (2020). In particular, we consider monetary policy shocks, which are potentially valid instruments, as both the measurement error and unobserved supply shocks (such as labor supply, price, or technology shock) would be uncorrelated with such shocks. Although we use

[21] Some researchers have attempted to solve the endogeneity problem by using regional data; however, there are several issues in using cross-sectional data for this purpose (see Canova, 2024). Here, we will instead maintain the same framework as in the classical literature on the Phillips curve debate, which focuses on macroeconomic data; however, we shed light on the issue by using both valid and strong instruments, as well as weak identification robust WAR minimizing interval estimators.

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the same model specification and the same identification strategy as Barnichon and Mesters (2020), we will use it in our framework to analyze whether the culprit behind the decrease in the correlation between inflation and output is monetary policy, while their approach does not allow for time-varying parameters.

We estimate the following Phillips curve specification[22]:

$$\pi_t = c_t + \lambda_t x_t + \gamma_{f,t}\pi_{t+1} + \gamma_{b,t}\pi_{t-1} + u_t,$$

where the set of instruments is the Almond parameterization of 20 lags of the Romer and Romer (2004) monetary policy shocks and the forcing variable x_t is the unemployment gap.

We estimate the equation in the sample starting in 1974:Q1 and ending in 2007:Q4, due to data limitations in the availability of the monetary policy shock data.[23]

Ganics et al.'s (2021) weak instrument-robust procedure implies weak instruments, as the minimum eigenvalue is zero, with a confidence interval equal to $(0, 0.0004)$. Lewis and Mertens' (2022) weak IV test statistic equals 2.618 and it is smaller than the 90% critical value (21.547624), also indicating that the instruments are weak.[24]

Figure 5 shows the time-varying estimate of the slope of the Phillips curve; dotted lines report the 90% WAR minimizing interval estimates robust to weak instruments. The figure confirms our result that the slope of the Phillips curve substantially flattened in the 1980s and 1990s.[25] The slope has decreased, in absolute value, by 70% approximately.[26]

In conclusion, using a variety of different specifications and weak instrument-robust techniques, we find convincing and robust evidence of a decrease in the correlation between inflation and unemployment in the structural Phillips curve. Our result, which is robust to endogeneity, measurement error, and correlation, highlights that the decrease in the correlation is due to a flattening of the Phillips curve, rather than to monetary policy.[27]

[22] This is the same model specification and sample as in Barnichon and Mesters (2020).

[23] The monetary policy shock series that we use ends in 2007. During the zero lower bound period, there are fewer monetary policy shocks anyway, which might invalidate the strength of the instrument.

[24] Newey and West's (1994) HAC-robust variance estimates are implemented following Barnichon and Mesters' (2020) choice, which is 5 lags.

[25] Barnichon and Mesters (2020) also study the specification considering the output gap as the forcing variable. We report our result of this specification in Figure A5 in Appendix B, which also confirms that the slope of the Phillips curve substantially flattened in the 1980s and 1990s.

[26] Elliott and Müller's (2006) qLL test statistic of the reduced-form regression in eq. (2) is -908.5 for this specification. Thus, the null hypothesis of no time variation is rejected at the 1% significance level, thus pointing to time variation in the system. While the qLL test provides complementary evidence, it is important to keep in mind that the test is not robust to weak identification.

[27] Antoine and Renault (2023) and Kleibergen (2023) propose alternative methods to deal with potential misspecification as well as weak identification.

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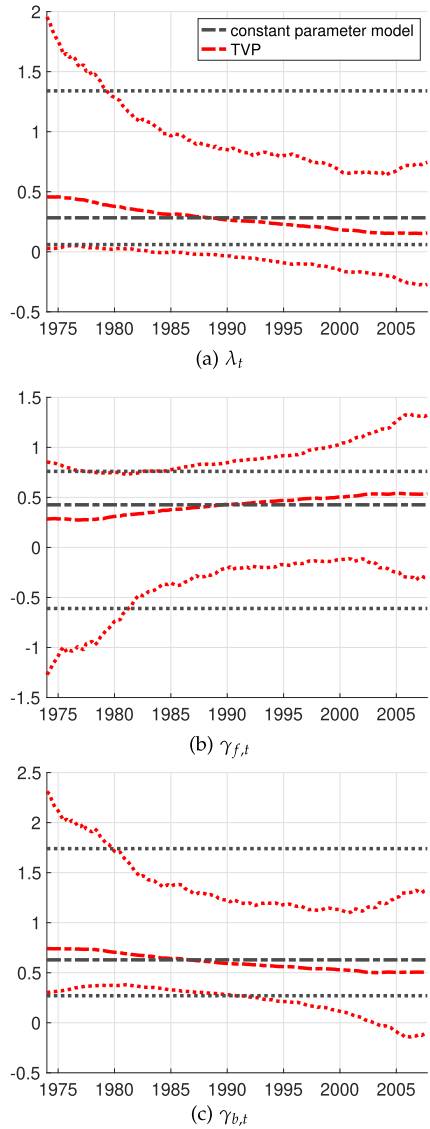


FIGURE 5. The time-varying Phillips curve using monetary policy shocks as instruments and the output gap as the forcing variable. The figure shows the TVP-IV estimated coefficients and the full-sample estimates, together with the 90% WAR minimizing interval estimates robust to weak instruments. The sample period is 1974Q1:2007Q4. The specification uses as instruments the Almond parameterization of 20 lags of the Romer and Romer monetary policy shocks, and the output gap obtained via the Hodrick–Prescott filter as the forcing variable. HAC-robust variance estimates are implemented with five lags, following Barnichon and Mesters (2020). The bands are smoothed using a 7-quarter moving average.

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5. WHAT'S UP WITH THE PHILLIPS CURVE IN THE RECENT PANDEMIC?

Finally, we turn to the Phillips curve during the recent financial crisis and, especially, the recent pandemic. Both have contributed to a substantially unstable macroeconomic environment. Elliott and Müller's (2006) qLL test statistic of the reduced-form regression in eq. (2) is −12331.5 for this specification. Thus, the null hypothesis of no time variation is rejected at a 1% significance level (−103.8), reflecting instabilities in the reduced form system.

In this analysis, Lewis and Mertens' (2022) weak IV test statistic equals 8.87, and it is smaller than the 90% critical value (23.234407), indicating that the instruments are weak. The Ganics et al.'s (2021) weak-instrument robust confidence interval for the strength of identification also points to weak instruments (the minimum eigenvalue is 1.444, with a 95% confidence interval equal to (0.034, 1.715).). We also report estimates and confidence intervals for the strength of identification over time in Figure A6 in Appendix B. The confidence interval includes zeros, thus implying that this set of instruments is weak in this specification when considering unstable environments.

Figure 6 plots the time-varying estimate of the slope of the Phillips curve λ_t, the forward-looking parameter $\gamma_{f,t}$, and the backward-looking parameter $\gamma_{b,t}$, using data up to 2024Q1 focusing on the same specification as employed in Figure 2. The dotted lines report the 90% interval estimates robust to weak instruments. The top panel in the picture shows the slope of the Phillips curve (λ_t). It is clear from the figure that, after hovering toward zero (in absolute value) from 2010 to 2020, the slope has started to increase again since the beginning of the 2020s. This upward trend has brought the parameter to 0.009 in 2024. Thus, relative to the literature that attributes the missing disinflation during the recent financial crisis to the weakening of the Phillips curve, we find evidence that the Phillips curve is again alive and well.

What else happened during the financial and pandemic crises? Panel (b) of Figure 6 indicates that the coefficient on the forward-looking component of inflation was roughly constant between the start of the great moderation period (1985) and 2010, after which it began to rise, trending toward 1 in the early 2020s. In contrast, panel (c) illustrates a stable degree of backward-looking behavior in inflation between 1985 until 2010, followed by a slight decrease in magnitude. This downward trend has reduced the parameter to approximately 0.3 in 2024.

Overall, our findings suggest that, in setting prices, agents pay more attention to the future and less to the past. The fact that past inflation has lost importance in the agents' price-setting behavior may explain the decrease in the overall serial correlation in inflation and its lack of predictability over time (Stock and Watson, 2007). Our results on the degree of forward- and backward-looking indexation are also consistent with those in Cogley and Sbordone (2008), who, using a very different time-varying parameter VAR-based methodology, similarly find that estimates of the backward-looking indexation parameter are close to zero and that the forward-looking component is instead prominent.

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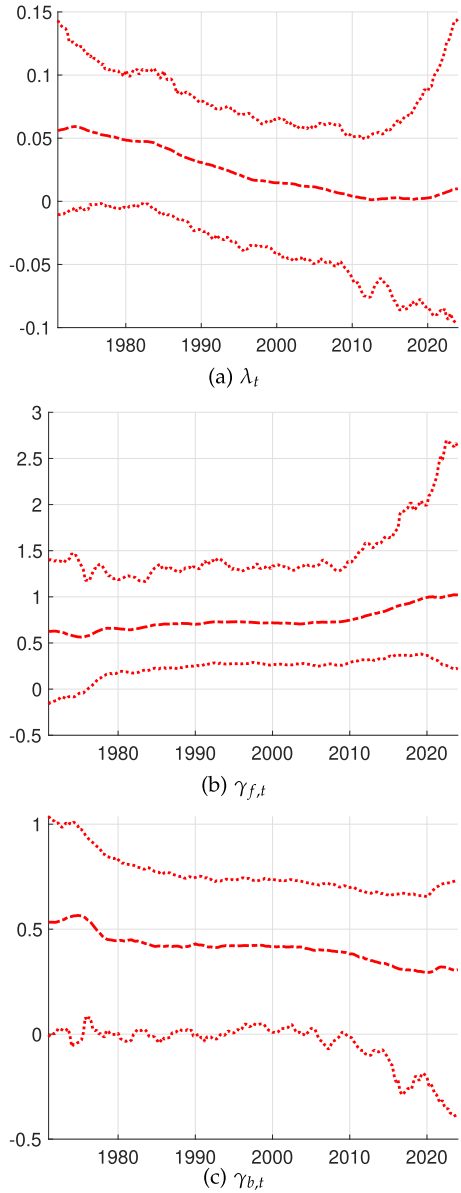


FIGURE 6. The time-varying Phillips curve during the pandemic. The figure shows the estimated weak-IV-robust TVP-IV coefficients together with 90% WAR minimizing interval estimates. The sample period is 1974Q1:2024Q1. We estimate the hybrid NKPC with one lag of inflation and the labor share as forcing variable. The set of instruments includes two lags of the forcing variable. HAC-robust variance estimates are implemented with Lazarus et al.'s (2018) recommendation, which implies to 19 lags. The bands are smoothed using a seven quarter centered moving average.

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6. CONCLUSION

We contribute to the debate surrounding the instability of the relation between unemployment and inflation over time by offering insights from a flexible time-varying IV approach.

We find that the weakening of the cyclical correlation between inflation and unemployment is due to a flattening in the slope of the Phillips curve, rather than to monetary policy. The slope of the structural Phillips curve has decreased over time since the 1980s.

In the most recent period, since the Great Recession and during the recent pandemic, the slope of the Phillips curve has increased again.

Our results are based on an approach that has the advantage of avoiding endogeneity while, at the same time, being robust to changes in the economic environment. In addition, by virtue of the approach taken in this article, our conclusions do not require making auxiliary assumptions on the rest of the economy nor estimating a fully specified model, and hence are more robust to misspecification than existing, full-information approaches. We demonstrate the robustness of our results to various specifications that feature both strong instruments as well as weak-identification robust ones.

Appendix

There are six sections in this Appendix. Section A contains a detailed description of the data. Section B reports additional empirical results. Section C outlines formulas for the scores and Hessians in the multivariate system. Section D presents the high-level assumptions, and Section F contains the proofs of the theoretical results. Section DF contains a table summarizing the empirical specifications used throughout the article.

A. DATA DESCRIPTION

The data used in this article are quarterly and their span is determined by data availability.

Inflation is measured by the "Implicit GDP deflator" (mnemonics "GDPDEF"); the raw data is transformed as follows: 100 times the log difference of the GDP deflator. For the labor share, we use the "Business Sector: Labor Share for All Employed Persons" (mnemonics "PRS84006173"); the raw data is transformed as: 100*ln(PRS84006173/100). For labor share (non-farm business), we use the "Nonfarm Business Sector: Labor Share for All Employed Persons" (mnemonics "PRS85006173"). The data is transformed as: 100*ln(PRS85006173/100). We also consider other analogs of the labor share, following Mavroeidis et al. (2014). For the labor share, estimated via a cointegrating relation (Nason and Smith, 2008), the data is transformed as follows: 100*(ln(ULCBS/100)-0.8952*ln(GDPDEF/100)). For labor share estimated via a cointegrating relation (Nason and Smith, 2008) based on non-farm business data, the data is transformed as follows: 100*(ln(ULCNFB/100)-0.9802*ln(IPDNBS/100)). For the cointegrating relation between the real wage and productivity (Sbordone, 2005), the data is transformed as follows: 100*(ln(RCPHBS/100)+0.9558*ln(PRS84006173/RCPHBS)). For the cointegrating

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relation between the real wage and productivity (Sbordone, 2005) based on nonfarm business data, the data is transformed as follows: 100*(ln(COMPRNFB/100) + 0.9558*ln(PRS85006173/COMPRNFB)).

For wage inflation, we use the "Business Sector: Hourly Compensation for All Employed Persons" (mnemonics "HCOMPBS"); the data is transformed as follows: 100 times the log difference of the HCOMPBS. The output gap is an economic measure of the difference between the actual output of an economy and its potential output. To calculate output, we use "Real Gross Domestic Product" (mnemonics "GDPC96") and the "Population Level" (mnemonics "CNP16OV"); the data is transformed as follows: 100*ln(GDPC96/CNP16OV). To calculate potential output, we use "Real Potential Gross Domestic Product" (mnemonics "GDPPOT"); the data is transformed as follows: 100*ln(GDPPOT/CNP16OV). All data are available from the Federal Reserve Bank of St. Louis's FRED database. "GDPDEF" is available from 1947Q1 to 2024Q1. "PRS84006173" is available from 1947Q1 to 2024Q1. "HCOMPBS" is available from 1947Q1 to 2024Q1. "GDPC96" is available from 1947Q1 to 2017Q2. (This series has been discontinued. It was a duplicate of "GDPC1," which will continue to be updated.) "CNP16OV" is available from 1948Q1 to 2024Q1. "GDPPOT" is available from 1949Q1 to 2024Q1.

The monetary policy shock is from Romer and Romer (2004). Their original series is available from 1969Q1 to 2007Q4; the updated data we use is available from Wieland and Yang (2020) at: https://www.openicpsr.org/openicpsr/project/135741/version/V1/view.[28] The high-frequency-identified (HFI) monetary surprises we use are from Barnichon and Mesters (2021), available from 1990Q1 to 2007Q4.

B. ADDITIONAL EMPIRICAL RESULTS

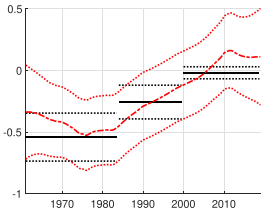
B.1. The Time-Varying Phillips Relation: Additional Results

The figure shows the estimated slope of the Phillips relation for several combinations of inflation and slack measures considered in Stock and Watson (2020). While we focus on real-time data and one-sided filters in the main text, here we report results for two-sided filtered data for completeness.

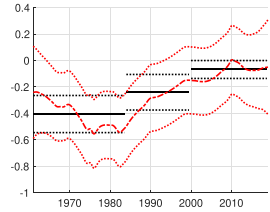
[28]The Romer and Romer (2004) monetary policy shock series is available from 1969Q1 to 2007Q4.

The monetary policy shock series is available at the monthly frequency, and we aggregate it at the quarterly frequency by summing the monthly values.

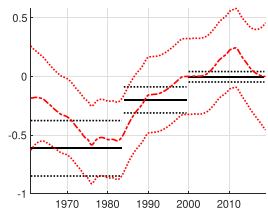
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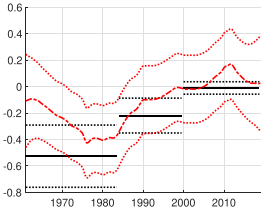
(a) Unemployment gap (two-sided filtered)



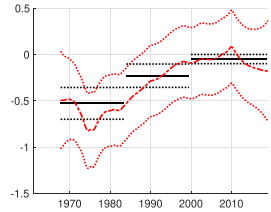
(b) Short-term unemployment gap (two-sided filtered)



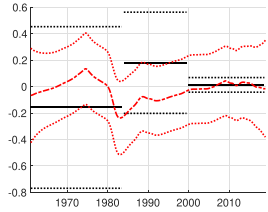
(c) Employment-population ratio (two-sided filtered)



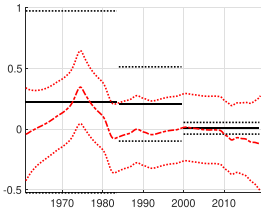
(d) Employment-population ratio ages 25-54 (two-sided filtered)



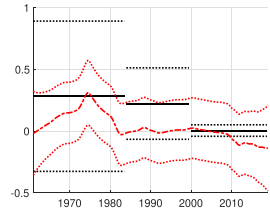
(e) Capacity utilization rate (two-sided filtered)



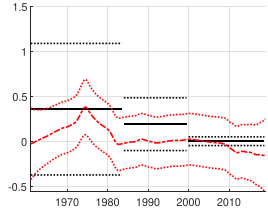
(f) Log labor share, estimated cointegrating relation (Nason and Smith, 2008)



(g) Log labor share, estimated cointegrating relation (Nason and Smith, 2008), non-farm business



(h) Log labor share, cointegrating relation btw real wage and productivity (Sbordone, 2005)



(i) Log labor share, cointegrating relation btw real wage and productivity (Sbordone, 2005), non-farm business

FIGURE A1. The time-varying Phillips relation: Additional results. The figure shows the estimated slope (β_1) in the Phillips relation: $E_t\Delta_4\pi_t^4 = \beta_0 + \beta_1 x_t^4$, where x_t^4 is the change in the average value of variable "x" between times t and $t-3$ and $\Delta_4 = (1-L^4)$. Inflation is measured by PCE-xFE and x_t are the various measures of slack for the United States (see Stock and Watson, 2020, Table 1). The sample period is 1961–2019. The data are from Stock and Watson (2020) and an extended version of Mavroeidis et al. (2014).

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B.2. The Time-Varying Ganics et al.'s (2021) Weak-Instrument Robust Confidence Interval of the Strength of Identification

This figure shows results for the Ganics et al.'s (2021) weak-instrument robust confidence interval of the strength of identification, estimated in rolling windows.

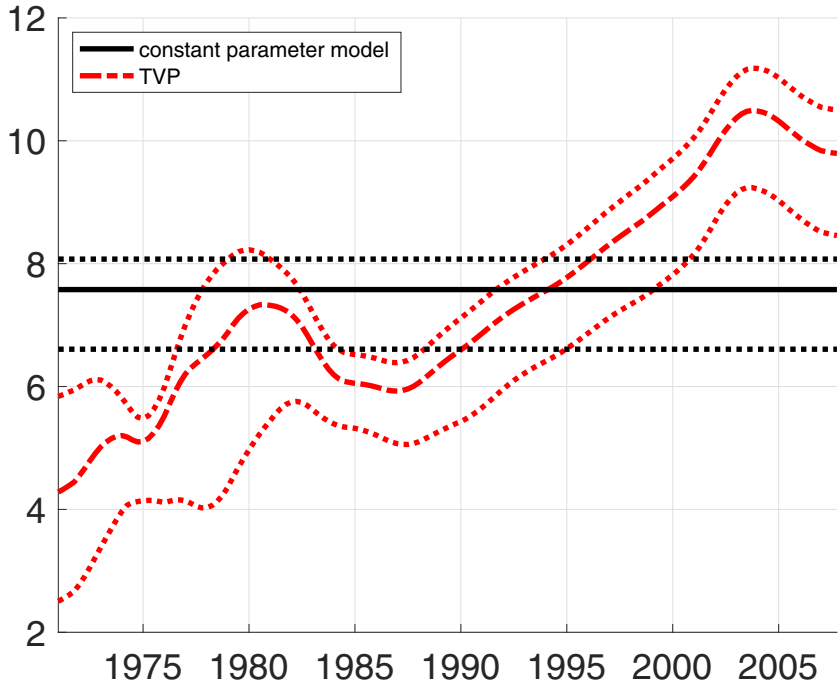


FIGURE A2. The time-varying Ganics et al.'s (2021) weak-instrument robust confidence interval of the strength of identification. The figure shows the Ganics et al.'s (2021) weak-instrument robust estimate of the strength of identification. The sample period is 1971Q1:2007Q4. The picture compares results for the constant parameter model (the black solid line) and the TVP-IV model (the red dashed line). The dotted lines are the corresponding 90% confidence intervals.

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B.3. The Time-Varying Phillips Curve: A Comparison With Giraitis et al. (2021)

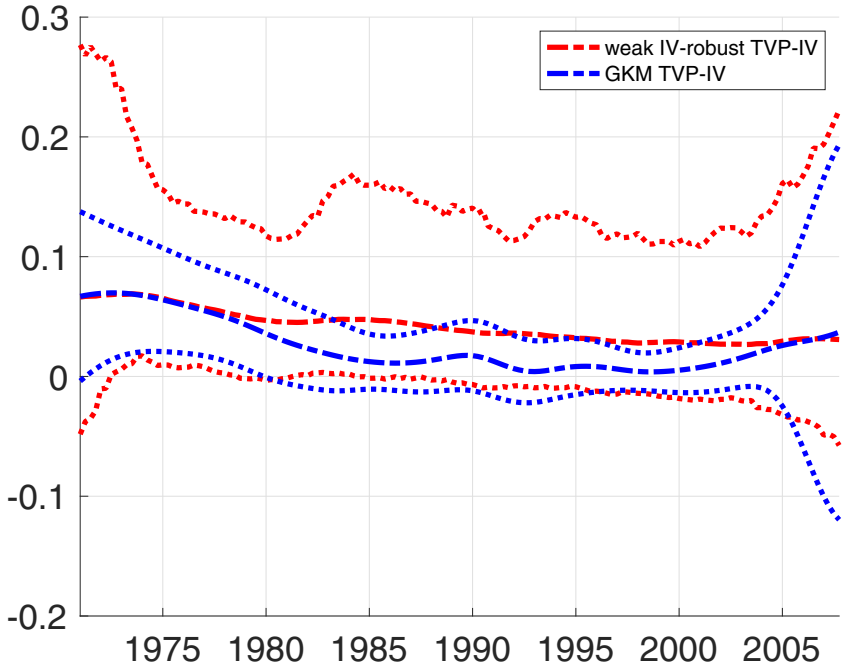


FIGURE A3. The time-varying Phillips curve using output gap (CBO) as the forcing variable, compared with GKM. The figure shows the estimated weak-IV-robust TVP-IV coefficients as well as Giraitis et al.'s (2021) nonparametric TVP-IV coefficients together with their respective 90% confidence bands. The sample period is 1971Q1:2007Q4. We estimate the hybrid NKPC with one lag of inflation and the output gap as forcing variable. The set of instruments includes one lag of the forcing variable and the second lag of inflation. HAC-robust variance estimates are implemented with Lazarus et al.'s (2018) recommendation, which implies 16 lags. The bands are smoothed using a seven quarter centered moving average.

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B.4. The Time-Varying Phillips Curve: A Comparison With TVP-IV (Procedure for Strong Instruments)

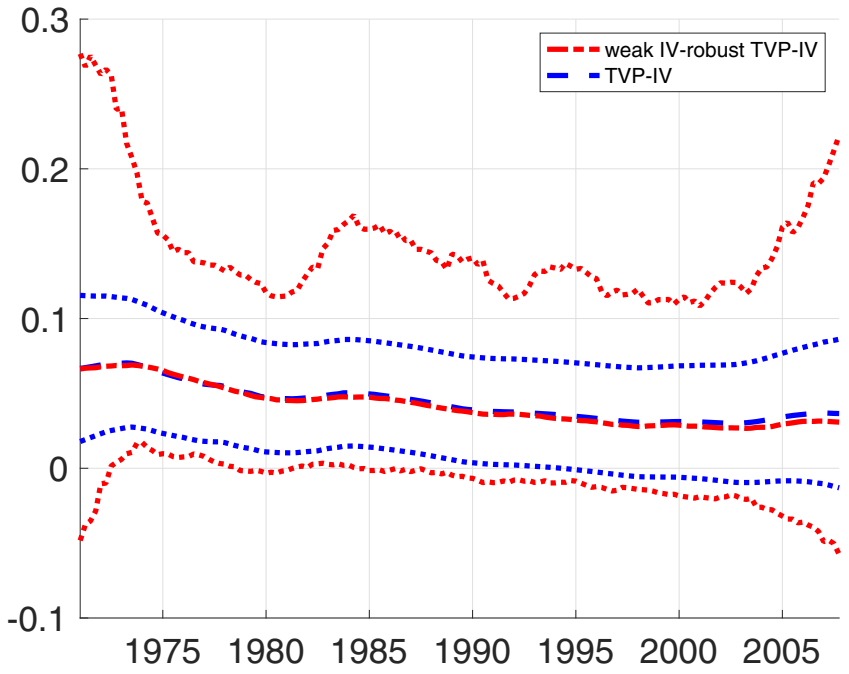


FIGURE A4. The time-varying Phillips curve using output gap (CBO) as the forcing variable, compared with estimate and set obtained using TVP-IV algorithm. The figure shows the estimated weak-IV-robust TVP-IV coefficients as well as TVP-IV coefficients together with their respective 90% credible sets. The sample period is 1971Q1:2007Q4. We estimate the hybrid NKPC with one lag of inflation and the output gap as forcing variable. The set of instruments includes one lag of the forcing variable and the second lag of inflation. HAC-robust variance estimates are implemented with Lazarus et al.'s (2018) recommendation, which implies 16 lags. The bands are smoothed using a seven quarter centered moving average.

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B.5. The Time-Varying Phillips Curve Using Monetary Policy Shocks as Instruments

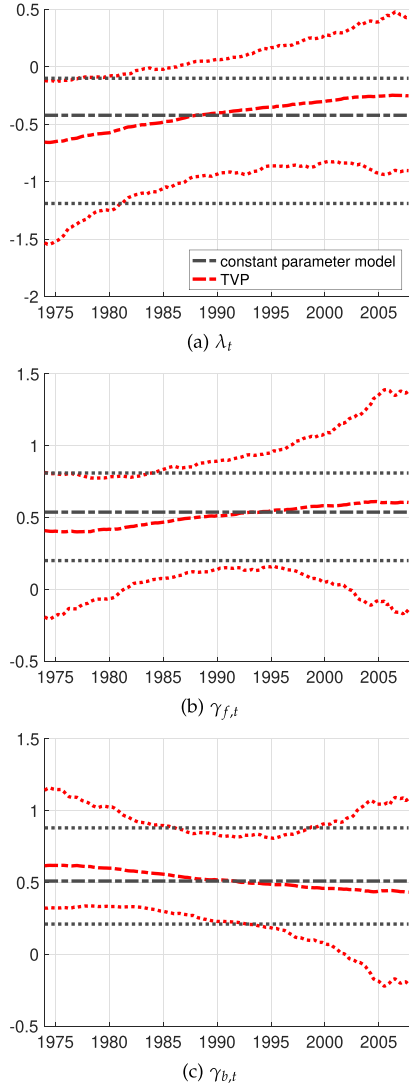


FIGURE A5. The time-varying Phillips curve using monetary policy shocks as instruments. The figure reports the estimated TVP-IV coefficients as well as their full-sample counterparts, together with 90% weak-instrument robust WAR minimizing interval estimates. The sample period is 1974Q1:2007Q4. The specification is as follows: we use the Almond parameterization of the 20 lags of Romer and Romer monetary policy shocks, and the unemployment gap obtained via the Hodrick–Prescott filter. The choice of the number of lags considered in the Newey and West's (1987) estimator, set equal to five, closely follows Barnichon and Mesters (2020). The bands are smoothed using a seven quarter centered moving average.

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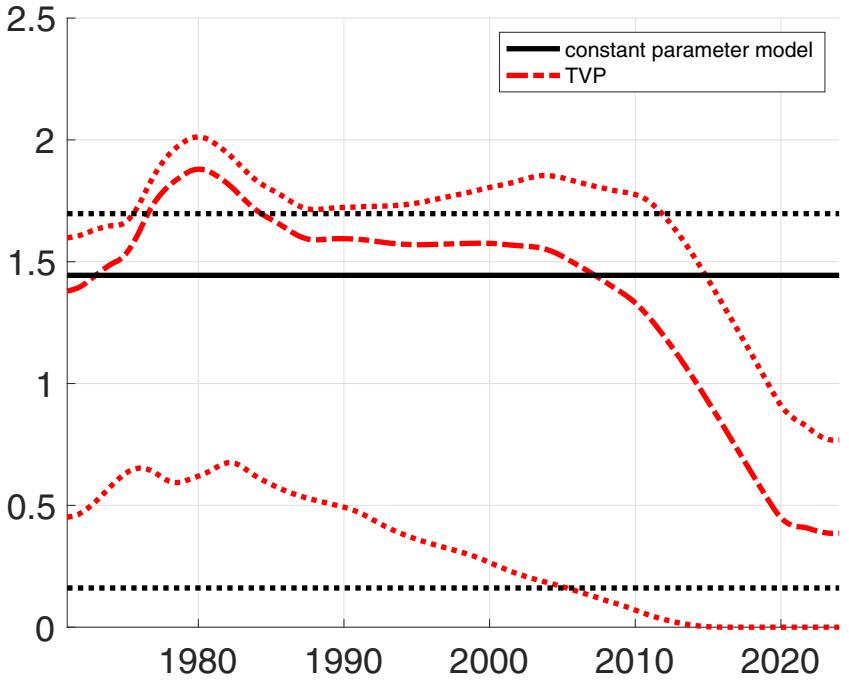


Figure A6. The time-varying Ganics et al.'s (2021) weak-instrument robust confidence interval of the strength of identification, 1971–2024. The figure shows the Ganics et al.'s (2021) weak-instrument robust estimate for the strength of identification, considering constant parameter model (the black solid line) and considering TVP-IV model (the red dashed line). The dotted lines are the corresponding 90% confidence intervals. The sample period is 1971Q1:2024Q1.

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B.6. Magnusson and Mavroeidis' (2014) qLL-AR Confidence Set for the Phillips Curve Parameters

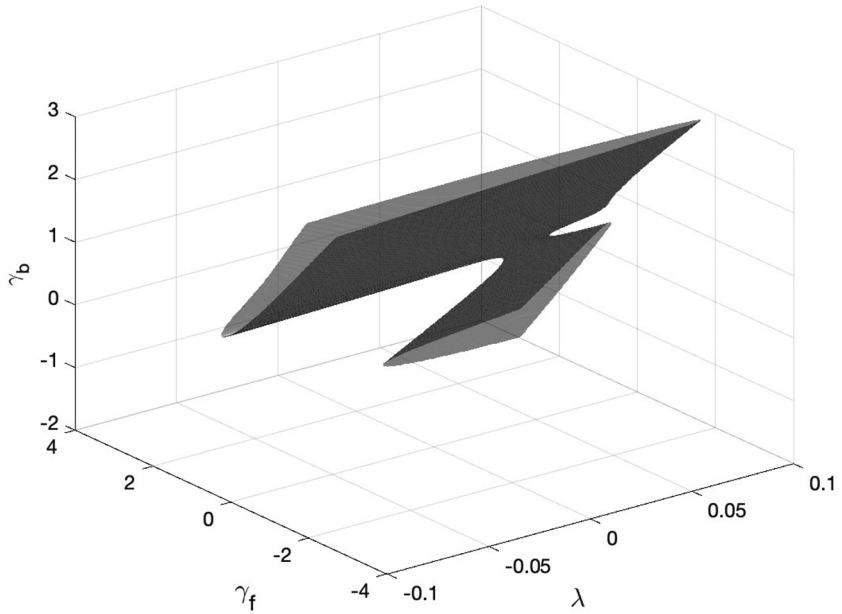


FIGURE A7. Magnusson and Mavroeidis' (2014) qLL-AR confidence set for the structural parameters λ, γ_f, and γ_b. The figure shows the 90% qLL-AR confidence sets for the structural parameters λ, γ_f, and γ_b. The sample period is 1974Q1:2007Q4. We estimate the hybrid NKPC using the labor share as the forcing variable. The set of instruments includes one lag of the forcing variable and the second lag of inflation. HAC-robust variance estimates are implemented with Lazarus et al.'s (2018) recommendation, which implies 16 lags.

C. LIKELIHOOD, SCORES, AND HESSIANS IN THE MULTIVARIATE SYSTEM

For notational simplicity, we omit the subscript t in all the parameters in what follows.

C.1. General Framework

Suppose we are interested in the following hybrid Phillips curve:

$$\pi_t = \lambda x_t + \gamma_f \pi_{t+1} + c + \gamma_b \pi_{t-1} + u_t, \tag{C.1}$$

where instruments are needed for x_t and π_{t+1}. Let Z_t be the vector of instruments $[z_{2,t}', z_{1,t}']'$, where $z_{1,t} = (1, \pi_{t-1})'$ includes constant and exogenous control variables, and $z_{2,t}$ denotes

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the external IVs (the excluded exogenous variables). The reduced-form equations for x_t and π_{t+1} are:

$$\begin{aligned} x_t &= \beta'_{x,z} z_{2,t} + c_x + \beta_{x,\pi_b}\pi_{t-1} + u_{x,t}, \\ \pi_{t+1} &= \beta'_{\pi_f,z} z_{2,t} + c_{\pi_f} + \beta_{\pi_f,\pi_b}\pi_{t-1} + u_{\pi_f,t}. \end{aligned} \tag{C.2}$$

Combining eqs. (C.1) and (C.2), we have

$$\begin{aligned} \pi_t &= c + \lambda\left(\beta'_{x,z} z_{2,t} + c_x + \beta_{x,\pi_b}\pi_{t-1} + u_{x,t}\right) \\ &\quad + \gamma_f\left(\beta'_{\pi_f,z} z_{2,t} + c_{\pi_f} + \beta_{\pi_f,\pi_b}\pi_{t-1} + u_{\pi_f,t}\right) + \gamma_b\pi_{t-1} + u_t \\ &= \left(c + \lambda c_x + \gamma_f c_{\pi_f}\right) + \left(\lambda\beta'_{x,z} + \gamma_f\beta'_{\pi_f,z}\right) z_{2,t} + \left(\lambda\beta_{x,\pi_b} + \gamma_f\beta_{\pi_f,\pi_b} + \gamma_b\right)\pi_{t-1} \\ &\quad + \left(\lambda u_{x,t} + \gamma_f u_{\pi_f,t} + u_t\right), \end{aligned}$$

and, equivalently, we have the following reduced-form equations:

$$\begin{bmatrix} x_t \\ \pi_{t+1} \\ \pi_t \end{bmatrix} = \begin{bmatrix} \beta'_{x,z} & c_x & \beta_{x,\pi_b} \\ \beta'_{\pi_f,z} & c_{\pi_f} & \beta_{\pi_f,\pi_b} \\ \lambda\beta'_{x,z} + \gamma_f\beta'_{\pi_f,z} & \lambda c_x + \gamma_f c_{\pi_f} + c & \lambda\beta_{x,\pi_b} + \gamma_f\beta_{\pi_f,\pi_b} + \gamma_b \end{bmatrix} \begin{bmatrix} z_{2,t} \\ 1 \\ \pi_{t-1} \end{bmatrix} + \begin{bmatrix} v_{x,t} \\ v_{\pi_f,t} \\ v_{\pi,t} \end{bmatrix}, \tag{C.4}$$

where $(x_t, \pi_{t+1})'$ are (2×1) endogenous variables, $v_{x,t} = u_{x,t}$, $v_{\pi_f,t} = u_{\pi_f,t}$, and $v_{\pi,t} = \lambda u_{x,t} + \gamma_f u_{\pi_f,t} + u_t$.

We now rewrite the Phillips curve model above using a more general notation. Let $y_{1,t}$ denote the vector of endogenous variables $(x_t, \pi_{t+1})'$, and $y_{2,t}$ denote the LHS variable of interest in eq. (C.1), that is, π_t.

In addition, let $m = (\lambda, \gamma_f)$ and $\mu = (c, \gamma_b)$. Then, the *structural model* in eq. (C.1), whose parameters are the goal of our estimation, can be written as:

$$y_{2,t} = m y_{1,t} + \mu z_{1,t} + u_t. \tag{C.5}$$

Furthermore, let $\alpha_2 = (\beta_{x,z}, \beta_{\pi_f,z})'$ and $\alpha_1 = \begin{pmatrix} c_x & \beta_{x,\pi_b} \\ c_{\pi_f} & \beta_{\pi_f,\pi_b} \end{pmatrix}$; also, let $v_{1,t} = (v'_x, v_{\pi_f})'$ and $v_{2,t} = v_\pi$. Then, equation (C.4) above can be rewritten as:

$$\begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} = \begin{pmatrix} \alpha_2 & \alpha_1 \\ m\alpha_2 & m\alpha_1 + \mu \end{pmatrix} \begin{pmatrix} z_{2,t} \\ z_{1,t} \end{pmatrix} + \begin{pmatrix} v_{1,t} \\ v_{2,t} \end{pmatrix}, \tag{C.6}$$

or, equivalently:

$$\begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} = \begin{pmatrix} \alpha \\ m\alpha + [0 \;\; \mu] \end{pmatrix} \begin{pmatrix} z_{2,t} \\ z_{1,t} \end{pmatrix} + \begin{pmatrix} v_{1,t} \\ v_{2,t} \end{pmatrix}. \tag{C.7}$$

More generally, let equations (C.5) and (C.7) hold, where $y_{1,t}$ is an $(n_1 \times 1)$ vector and $y_{2,t}$ is an $(n_2 \times 1)$ vector. Let Z_t be the $(k \times 1)$ vector of instruments $[z'_{2,t}, z'_{1,t}]'$, where $z_{1,t}$ is $(k_1 \times 1)$, $z_{2,t}$ is $(k_2 \times 1)$, such that $k = k_1 + k_2$. Note that $z_{1,t}$ includes constant

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and exogenous control variables, and $z_{2,t}$ includes excluded exogenous variables. Also, let $\alpha = [\alpha_2\ \alpha_1]$ is $(n_1 \times k)$, m is $(n_2 \times n_1)$, μ is $(n_2 \times k_1)$.

Let the $\bar{q} \times 1$ vector $\bar{B}$ contain the *structural parameters*:

$$\bar{B} = \left(\alpha_{1,:} \cdots \alpha_{n_1,:}, m_{1,:} \cdots m_{n_2,:}, \mu_{1,:} \cdots \mu_{n_2,:}\right)', \tag{C.8}$$

where subscripts refer to the elements (e.g., $\alpha_{1,:}$ refers to the first row elements in α) and $\bar{q} = n_1 k + n_2(n_1 + k_1)$.[29]

C.2. Log-Likelihood

Write (C.7) as

$$y_t = \begin{bmatrix} \alpha \\ \widetilde{\alpha} \end{bmatrix} \begin{bmatrix} z_{2,t} \\ z_{1,t} \end{bmatrix} + \begin{bmatrix} v_{1,t} \\ v_{2,t} \end{bmatrix} = X_t' B + v_t, \tag{C.9}$$

where $\widetilde{\alpha} = m\alpha + \begin{bmatrix} 0 & \mu \end{bmatrix}$, $X_t' = (I_n \otimes \begin{bmatrix} z_{2,t}' & z_{1,t}' \end{bmatrix})$is $(n \times nk)$, v_t has zero mean and covariance matrix Σ, B is the $(q \times 1)$ vector of *reduced-form parameters*, where $q = nk$, obtained by stacking all the parameters in the following order:

$$B = \left(\alpha_{1,:} \cdots \alpha_{n_1,:}, \widetilde{\alpha}_{1,:} \cdots \widetilde{\alpha}_{n_2,:}\right)'. \tag{C.10}$$

Once we knowingly incorrectly assume that v_t is an i.i.d. normal random variable, we can write the quasi-log-likelihood based on eq. (C.9):

$$\log L = \sum_t \ell_t = -\frac{T(n_1 + n_2)}{2} \log 2\pi - \frac{T}{2} \sum_t \log |\Sigma| - \frac{1}{2} \sum_t \left(y_t - X_t' B\right)' \Sigma^{-1} \left(y_t - X_t' B\right). \tag{C.11}$$

C.3. Scores and Hessians

We estimate eq. (C.9) with θ^{rf} defined as $\theta^{rf} = (B', vech(\Sigma)')'$. The corresponding scores are $s_t = \begin{bmatrix} \frac{\partial l_t}{\partial B} \\ \frac{\partial l_t}{\partial vech(\Sigma)} \end{bmatrix}$, with

$$\frac{\partial l_t}{\partial B} = X_t \Sigma^{-1} (y_t - X_t' B), \tag{C.12}$$

$$\frac{\partial l_t}{\partial \Sigma} = -\frac{1}{2} \Sigma^{-1} + \frac{1}{2} \Sigma^{-1} (y_t - X_t' B)(y_t - X_t' B)' \Sigma^{-1}, \tag{C.13}$$

$$\frac{\partial l_t}{\partial vech(\Sigma)} = D' vec\left(\frac{\partial l_t}{\partial \Sigma}\right), \qquad \frac{\partial l_t}{\partial vec(\Sigma)} = DD' vec\left(\frac{\partial l_t}{\partial \Sigma}\right), \tag{C.14}$$

where D denote the duplication matrix such that $D \cdot vech(V) = vec(V)$. Besides, denote $D^+ = (D'D)^{-1} D'$ such that $D^+ \cdot vec(V) = vech(V)$.

[29] This is just a vectorized version of the parameters in equations (C.5) and (C.7). For example, in our Phillips curve model, the parameters of interest are m, the $(n_1 \times k) + 1 : (n_1 \times k) + n_1 n_2$-th elements in $\bar{B}$, and γ_b, the $(n_1 \times k) + n_1 n_2 + n_2 k_1$-th element in $\bar{B}$.

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The corresponding Hessians are $h_t = -\begin{bmatrix} \frac{\partial^2 l_t}{\partial B \partial B'} & \frac{\partial^2 l_t}{\partial B \partial vech(\Sigma)'} \\ \frac{\partial^2 l_t}{\partial vech(\Sigma) \partial B'} & \frac{\partial^2 l_t}{\partial vech(\Sigma) \partial vech(\Sigma)'} \end{bmatrix}$, with

$$\begin{aligned} \frac{\partial^2 l_t}{\partial B \partial B'} &= -X_t \Sigma^{-1} X_t', \\ \frac{\partial^2 l_t}{\partial B \partial vech(\Sigma)'} &= \left[-D' \left(\Sigma^{-1} (y_t - X_t' B) \otimes \Sigma^{-1} \right) X_t' \right]', \\ \frac{\partial^2 l_t}{\partial B \partial vec(\Sigma)'} &= \left[-DD' \left(\Sigma^{-1} (y_t - X_t' B) \otimes \Sigma^{-1} \right) X_t' \right]' \end{aligned} \tag{C.15}$$

$$\begin{aligned} \frac{\partial^2 l_t}{\partial vech(\Sigma) \partial vech(\Sigma)'} &= D' \left(\Sigma^{-1} \otimes \left(\frac{1}{2} \Sigma^{-1} - \Sigma^{-1} (y_t - X_t' B)(y_t - X_t' B)' \Sigma^{-1} \right) \right) D, \\ \frac{\partial^2 l_t}{\partial vec(\Sigma) \partial vec(\Sigma)'} &= DD' \left(\Sigma^{-1} \otimes \left(\frac{1}{2} \Sigma^{-1} - \Sigma^{-1} (y_t - X_t' B)(y_t - X_t' B)' \Sigma^{-1} \right) \right) DD'. \end{aligned} \tag{C.16}$$

Strongly Identified Case:

If we have strong instruments and want to estimate $\bar{B}$ directly. Then, we estimate eq. (C.7) with θ defined as $\theta = (\bar{B}', vech(\Sigma)')'$.

To obtain the derivatives w.r.t. $\theta = (\bar{B}', vech(\Sigma)')'$, we only need to obtain the derivative of B w.r.t. $\bar{B}$ so that we can apply the chain rule: $\frac{\partial B}{\partial \bar{B}} \frac{\partial l_t}{\partial B} = \frac{\partial l_t}{\partial \bar{B}}$, $\frac{\partial l_t}{\partial B'} \left(\frac{\partial B}{\partial \bar{B}} \right)' = \frac{\partial l_t}{\partial \bar{B}'}$, and

$$\underbrace{\begin{bmatrix} \frac{\partial B_1}{\partial \bar{B}_1} & \cdots & \frac{\partial B_1}{\partial \bar{B}_{\bar{q}}} \\ \vdots & \ddots & \vdots \\ \frac{\partial B_q}{\partial \bar{B}_1} & \cdots & \frac{\partial B_q}{\partial \bar{B}_{\bar{q}}} \end{bmatrix}}_{\left(\frac{\partial B}{\partial \bar{B}} \right)'} = \begin{bmatrix} I_{(n_1 k \times n_1 k)} & O_{(n_1 k \times n_1 n_2)} & O_{(n_1 k \times n_2 k_1)} \\ m \otimes I_{k(n_2 k \times n_1 k)} & I_{n_2} \otimes \alpha'_{(n_2 k \times n_1 n_2)} & \begin{bmatrix} O_{(n_2 k_2 \times (n_2 k_1))} \\ I_{n_2 k_1} \end{bmatrix} \end{bmatrix}_{(q \times \bar{q})}. \tag{C.17}$$

We also need the second-order derivatives $\frac{\partial^2 l_t}{\partial \bar{B}_t \partial \bar{B}_t'}$, which is $(\bar{q} \times \bar{q})$:

$$\begin{aligned} \frac{\partial^2 l_t}{\partial \bar{B} \partial \bar{B}'} &= \frac{\partial \frac{\partial l_t}{\partial \bar{B}}}{\partial \bar{B}'} = \frac{\partial \frac{\partial B}{\partial \bar{B}} \frac{\partial l_t}{\partial B_t}}{\partial \bar{B}_t'} \\ &= \left[\frac{\partial \frac{\partial B}{\partial \bar{B}} \frac{\partial l_t}{\partial B}}{\partial \bar{B}_1} \cdots \frac{\partial \frac{\partial B}{\partial \bar{B}} \frac{\partial l_t}{\partial B}}{\partial \bar{B}_{\bar{q}}} \right] = \left[\frac{\partial B}{\partial \bar{B}} \cdot \frac{\partial \frac{\partial l_t}{\partial B}}{\partial \bar{B}_1} + \frac{\partial \frac{\partial B}{\partial \bar{B}}}{\partial \bar{B}_1} \cdot \frac{\partial l_t}{\partial B} \cdots \frac{\partial B}{\partial \bar{B}} \cdot \frac{\partial \frac{\partial l_t}{\partial B}}{\partial \bar{B}_{\bar{q}}} + \frac{\partial \frac{\partial B}{\partial \bar{B}}}{\partial \bar{B}_{\bar{q}}} \cdot \frac{\partial l_t}{\partial B} \right] \\ &= \frac{\partial B}{\partial \bar{B}} \cdot \frac{\partial \frac{\partial l_t}{\partial B}}{\partial \bar{B}'} + \left[\frac{\partial \frac{\partial B}{\partial \bar{B}}}{\partial \bar{B}_1} \cdot \frac{\partial l_t}{\partial B} \cdots \frac{\partial \frac{\partial B}{\partial \bar{B}}}{\partial \bar{B}_{\bar{q}}} \cdot \frac{\partial l_t}{\partial B} \right] \\ &= \frac{\partial B}{\partial \bar{B}} \cdot \frac{\partial^2 l_t}{\partial B \partial B'} \left(\frac{\partial B}{\partial \bar{B}} \right)' + \underbrace{\left[\frac{\partial \frac{\partial B}{\partial \bar{B}}}{\partial \bar{B}_1} \cdots \frac{\partial \frac{\partial B}{\partial \bar{B}}}{\partial \bar{B}_{\bar{q}}} \right]}_{\Delta_{(\bar{q} \times q \bar{q})}} \left(I_{\bar{q}} \otimes \frac{\partial l_t}{\partial B} \right). \end{aligned}$$

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We derive each submatrix in Δ in turn. For the first $n_1 k$ elements in Δ, we have

$$\left(\frac{\partial \frac{\partial B}{\partial \bar{B}}}{\partial \bar{B}_j}\right)' = \begin{bmatrix} O_{(n_1k \times n_1k)} & O_{(n_1k \times n_1n_2)} & O_{(n_1k \times n_2k_1)} \\ O_{(n_2k \times n_1k)} & I_{n_2} \otimes (e'_{\alpha,j})_{(n_2k \times n_1n_2)} & O_{(n_2k \times n_2k_1)} \end{bmatrix}_{(q \times \bar{q})},$$

where $e'_{\alpha,j}$ is a $(k \times n_1)$ matrix with a certain entry being 1 and the remaining entries being 0. For example, $e'_{\alpha,1} = \begin{bmatrix} 1 & 0 \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}$, $e'_{\alpha,2} = \begin{bmatrix} 0 & 0 \cdots & 0 \\ 1 & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}$, in line with the corresponding element in α'.

For the next $n_1 n_2$ elements, i.e., from the $(n_1 k+1)$-th to the $(n_1 k+n_1 n_2)$-th elements in Δ, we have

$$\left(\frac{\partial \frac{\partial B}{\partial \bar{B}}}{\partial \bar{B}_{n_1k_2+j}}\right)' = \begin{bmatrix} O_{(n_1k \times n_1k)} & O_{(n_1k \times n_1n_2)} & O_{(n_1k \times n_2k_1)} \\ e_{m,j} \otimes I_{k_{(n_2k \times n_1k)}} & O_{(n_2k \times n_1n_2)} & O_{(n_2k \times n_2k_1)} \end{bmatrix}_{(q \times \bar{q})},$$

where $e_{m,j}$ is an $(n_2 \times n_1)$ matrix whose $(1,j)$ entry is one and all other entries are zeros.

For the remaining elements, the derivatives are 0.

Below, we summarize the expressions for scores and Hessians for the strongly identified case.

The score is $s_t = \begin{bmatrix} \frac{\partial l_t}{\partial \bar{B}} \\ \frac{\partial l_t}{\partial vech(\Sigma)} \end{bmatrix} = \begin{bmatrix} \frac{\partial B}{\partial \bar{B}} \frac{\partial l_t}{\partial B} \\ \frac{\partial l_t}{\partial vech(\Sigma)} \end{bmatrix}$, where each component is defined earlier.

The Hessian is

$$\begin{aligned} h_t &= -\begin{bmatrix} \frac{\partial^2 l_t}{\partial \bar{B} \partial \bar{B}'} & \frac{\partial^2 l_t}{\partial \bar{B} \partial vech(\Sigma)'} \\ \frac{\partial^2 l_t}{\partial vech(\Sigma) \partial \bar{B}'} & \frac{\partial^2 l_t}{\partial vech(\Sigma) \partial vech(\Sigma)'} \end{bmatrix} \\ &= -\begin{bmatrix} \frac{\partial B}{\partial \bar{B}} \frac{\partial^2 l_t}{\partial B \partial B'} \left(\frac{\partial B}{\partial \bar{B}}\right)' + \Delta \left(I_{\bar{q}} \otimes \frac{\partial l_t}{\partial B}\right) & \frac{\partial B}{\partial \bar{B}} \frac{\partial^2 l_t}{\partial B \partial vech(\Sigma)'} \\ \frac{\partial^2 l_t}{\partial vech(\Sigma) \partial B'} \left(\frac{\partial B}{\partial \bar{B}}\right)' & \frac{\partial^2 l_t}{\partial vech(\Sigma) \partial vech(\Sigma)'} \end{bmatrix}, \end{aligned} \tag{C.18}$$

where each component is defined earlier.

D. ASSUMPTIONS

To use Theorem 5 of Müller and Petalas (2010), we impose the following high-level assumptions.

Assumption 1.

(a) y_t, $Z_t = [z'_{2,t}\ z'_{1,t}]'$ and $X'_t = (I_n \otimes [z'_{2,t} \quad z'_{1,t}])$ satisfy the reduced-form equation (C.9) and have finite fourth moments uniformly in t. $E(Z_t Z'_t)$ is nonsingular.

(b) $E(Z_t v_t) = 0$.

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(c) For the identification robust case, let $\theta_t = [B_t' \text{ vech}(\Sigma_t)']'$, $\theta_0 = [B_0' \text{ vech}(\Sigma_0)']'$, as defined by the notation θ^{rf} in Section 2.2; for the strongly identified case, let $\theta_t = [\bar{B}_t' \text{ vech}(\Sigma_t)']'$ and $\theta_0 = [\bar{B}_0' \text{ vech}(\Sigma_0)']'$, as defined by the notation θ in Section 2.1. They satisfy $\theta_t = \theta_0 + \delta_t = \theta_0 + T^{-\frac{1}{2}}\delta\left(\frac{t}{T}\right)$, where $\delta(\cdot)$ is piecewise continuous with at most a finite number of discontinuities and left and right limits everywhere.

(d) $T^{-\frac{1}{2}}\sum_{t=1}^{[\cdot T]} s_t(\theta_t) \Rightarrow V^{\frac{1}{2}}\mathcal{W}(\cdot)$, where long-run variance matrix V is positive definite, and $\mathcal{W}(\cdot)$ denotes the standard Brownian motion vector.

(e) $\sup_{\lambda\in[0,1]}\left|T^{-1}\sum_{t=1}^{[\lambda T]} h_t(\theta_0) - \lambda\Xi\right| = o_p(1)$.

Assumption 1(a) specifies the reduced-form relationship between endogenous variables y_t and exogenous variables Z_t. Assumption 1(b) imposes that Z_t is exogenous. According to Assumption 1(c), we estimate the structural parameters when they are known to be strongly identified. Otherwise, we estimate the reduced-form parameters. As in Müller and Petalas (2010), we focus on time variations of order $O(T^{-\frac{1}{2}})$. Assumption 1(d) is satisfied if $\text{vec}(X_t\Sigma^{-1}(y_t - X_t'B_t))$ and $\text{vech}\left(\Sigma^{-1}(y_t - X_t'B)(y_t - X_t'B_t)'\Sigma^{-1} - \Sigma^{-1}\right)$ satisfy assumptions for the invariance principle. Assumption 1(e) is a high-level assumption that imposes that the sample Hessian converges to a negative-definite matrix.

E. PROOFS

First, we show that Condition 3 in Müller and Petalas (2010) is satisfied for the weak instruments case. To be self-contained, we present the results as a lemma.

LEMMA. *Suppose that Assumption 1 holds. Then,*

(i) $T^{-\frac{1}{2}}\sum_{t=1}^{[\cdot T]}\widehat{s}_t^r \Rightarrow J^r(\cdot) - \iota(\cdot)J^r(1)$, *where* $\widehat{s}_t^r = \widehat{\Xi}\widehat{V}^{-1}\widehat{s}_t - \frac{1}{T}\sum_{j=1}^{T}\widehat{\Xi}\widehat{V}^{-1}\widehat{s}_j = \widehat{\Xi}\widehat{V}^{-1}\widehat{s}_t$, $\widehat{s}_t = s_t(\widehat{\theta})$, $J^r(\cdot) = \Gamma^{\frac{1}{2}}\mathcal{W}(\cdot) + \Gamma\int_0^{\iota(\cdot)}\delta(r)dr$, $\Gamma = \Xi V^{-1}\Xi$, $\Xi = plim_{T\to\infty}(1/T)\sum_{t=1}^{T} h_t(\theta_0)$, *and V is the long-run variance–covariance matrix of* $s_t(\theta_0)$*;*

(ii) $T^{\frac{1}{2}}(\widehat{\theta}^r - \theta_0) \Rightarrow \Gamma^{-\frac{1}{2}}\mathcal{W}(1) + \int_0^1\delta_0(r)dr$, *where* $\widehat{\theta}^r = \widehat{\theta} + (\sum_{t=1}^{T}\widehat{\Xi}\widehat{V}^{-1}\widehat{h}_t)^{-1}\sum_{t=1}^{T}\widehat{\Xi}\widehat{V}^{-1}\widehat{s}_t = \widehat{\theta}$ *is the robustified estimator,* $\widehat{\Xi} = \frac{1}{T}\sum_{t=1}^{T} h_t(\theta_0)$, $\widehat{h}_t = h_t(\widehat{\theta})$, *and* $\widehat{V}$ *is a consistent estimator of* V[30]*;*

(iii) *there exist matrices* $\widetilde{h}_t^r$ *such that* $\sup_{s\in[0,1]}\left|\frac{1}{T}\sum_{t=1}^{[sT]}\widetilde{h}_t^r - s\Gamma\right| = o_p(1)$.

Proof of (ii): Consider a compact set, $\overline{\Theta}_0$, that contains θ_0. It follows from the Chebyshev inequality that

$$\frac{1}{T}\sum_{t=1}^{T} h_t(\theta) \xrightarrow{p} \lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T} E(h_t(\theta)), \tag{E.1}$$

[30] In our setup, $\widehat{\Xi}_t$ and $\widehat{V}_t$ in Condition 3 of Müller and Petalas (2010) are $\widehat{\Xi}$ and $\widehat{V}$, respectively.

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for all $\theta \in \overline{\Theta}_0$. Because $\frac{1}{T}\sum_{t=1}^{T} h_t(\theta)$ satisfies a Lipschitz condition, given Assumption 1(a) and equations (C.15) and (C.16) in Appendix C.3,

$$\sup_{\theta\in\overline{\Theta}_0}\left|\frac{1}{T}\sum_{t=1}^{T}h_t(\theta)-\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}E(h_t(\theta))\right| = o_p(1), \tag{E.2}$$

(Andrews, 1992). Thus, for any sequence $\overline{\theta}_t \to \theta_0$,

$$\left|\frac{1}{T}\sum_{t=1}^{T}h_t(\overline{\theta}_t)-\frac{1}{T}\sum_{t=1}^{T}E(h_t(\overline{\theta}_t))\right| = \left|\frac{1}{T}\sum_{t=1}^{T}h_t(\overline{\theta}_t)-\Xi\right|+o_p(1) = o_p(1). \tag{E.3}$$

Applying the mean value theorem to the first-order condition for the MLE of θ based on the likelihood defined in eq. (C.11),

$$\begin{aligned} T^{\frac{1}{2}}(\widehat{\theta}-\theta_0) &= \left[\frac{1}{T}\sum_{t=1}^{T}h_t(\overline{\theta})\right]^{-1} T^{-\frac{1}{2}}\sum_{t=1}^{T}s_t(\theta_0) \\ &= \left[\frac{1}{T}\sum_{t=1}^{T}h_t(\overline{\theta})\right]^{-1} T^{-\frac{1}{2}}\sum_{t=1}^{T}\left[s_t(\theta_t)+h_t(\overline{\theta}_t)T^{-\frac{1}{2}}\delta\left(\frac{t}{T}\right)\right] \\ &= \left[\frac{1}{T}\sum_{t=1}^{T}h_t(\overline{\theta})\right]^{-1} T^{-\frac{1}{2}}\sum_{t=1}^{T}s_t(\theta_t)+\left[\frac{1}{T}\sum_{t=1}^{T}h_t(\overline{\theta})\right]^{-1}\frac{1}{T}\sum_{t=1}^{T}h_t(\overline{\theta}_t)\delta\left(\frac{t}{T}\right), \end{aligned} \tag{E.4}$$

where $\overline{\theta}$ and $\overline{\theta}_t$ are points between $\widehat{\theta}$ and θ_0 and between θ_t and θ_0, respectively. Using Assumption 1(c) and the Chebyshev inequality,

$$\begin{aligned} \frac{1}{T}\sum_{t=1}^{T}h_t(\overline{\theta}_t)\delta\left(\frac{t}{T}\right) &= \frac{1}{T}\sum_{t=1}^{T}(h_t(\theta_0)-\Xi)\delta\left(\frac{t}{T}\right)+\Xi\frac{1}{T}\sum_{t=1}^{T}\delta\left(\frac{t}{T}\right) \\ &\quad + O_p\left(\frac{1}{T}\sum_{t=1}^{T}D\mathrm{vec}(h_t(\overline{\overline{\theta}}_t))(\overline{\theta}_t-\theta_0)\right) \\ &\xrightarrow{p} \Xi\int_0^1\delta(r)dr, \end{aligned} \tag{E.5}$$

where $D\mathrm{vec}(h_t(\overline{\overline{\theta}}_t))$ is the Jacobian matrix of $vec(h_t(\overline{\overline{\theta}}_t))$ with respect to θ evaluated at $\overline{\overline{\theta}}_t$, and $\overline{\overline{\theta}}_t$ is an intermediate point between $\overline{\theta}_t$ and θ_0. It follows from (E.4), (E.5), and Assumption 1(c) that

$$T^{\frac{1}{2}}(\widehat{\theta}-\theta_0) \xrightarrow{d} \Xi^{-1}V^{\frac{1}{2}}\mathcal{W}(1)+\int_0^1\delta(r)dr. \tag{E.6}$$

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Since

$$\widehat{\theta}^r \equiv \widehat{\theta} + \widehat{\Xi}^{-1} \frac{1}{T} \sum_{t=1}^{T} s_t(\widehat{\theta}) + o_p(1) = \widehat{\theta} + o_p(1), \tag{E.7}$$

it follows that

$$\begin{aligned} T^{\frac{1}{2}}(\widehat{\theta}^r - \theta_0) &= T^{\frac{1}{2}}(\widehat{\theta} - \theta_0) + o_p(1) \\ &\xrightarrow{d} \Gamma^{-\frac{1}{2}} \mathcal{W}(1) + \int_0^1 \delta(r) dr. \end{aligned} \tag{E.8}$$

Because the RHS is $\Gamma^{-1} J^r(1)$ in our setup, condition (ii) is satisfied.

Proof of (i): Next, we show that condition (i) in the lemma is satisfied.

It follows from Assumptions 1(c)–1(e) and equation (31) that:

$$\begin{aligned} T^{-\frac{1}{2}} \sum_{t=1}^{[\cdot T]} \widehat{s}_t &= T^{-\frac{1}{2}} \sum_{t=1}^{[\cdot T]} s_t(\theta_t) - \frac{1}{T} \sum_{t=1}^{[\cdot T]} h_t(\overline{\theta}_t) T^{\frac{1}{2}}(\widehat{\theta} - \theta_t) \\ &= T^{-\frac{1}{2}} \sum_{t=1}^{[\cdot T]} s_t(\theta_t) - \frac{1}{T} \sum_{t=1}^{[\cdot T]} h_t(\overline{\theta}_t) T^{\frac{1}{2}}(\widehat{\theta} - \theta_0) + \frac{1}{T} \sum_{t=1}^{[\cdot T]} h_t(\overline{\theta}_t) T^{\frac{1}{2}}(\theta_t - \theta_0) \\ &= T^{-\frac{1}{2}} \sum_{t=1}^{[\cdot T]} s_t(\theta_t) - \frac{1}{T} \sum_{t=1}^{[\cdot T]} h_t(\overline{\theta}_t) T^{\frac{1}{2}}(\widehat{\theta} - \theta_0) + \frac{1}{T} \sum_{t=1}^{[\cdot T]} h_t(\overline{\theta}_t) \delta\left(\frac{t}{T}\right) \\ &\Rightarrow V^{\frac{1}{2}} \mathcal{W}(\cdot) + \Xi \int_0^{\iota(\cdot)} \delta(r) dr - \iota(\cdot) \left(V^{\frac{1}{2}} \mathcal{W}(1) + \Xi \int_0^1 \delta(r) dr \right), \end{aligned} \tag{E.9}$$

where $\overline{\theta}_t$ is a point between θ_t and θ_0 and $\iota(x) = x$. Thus,

$$\begin{aligned} T^{-\frac{1}{2}} \sum_{t=1}^{[\cdot T]} \widetilde{s}_t^r &= \widehat{\Xi} \widehat{V}^{-1} T^{-\frac{1}{2}} \sum_{t=1}^{[\cdot T]} s_t(\widehat{\theta}) \\ &\Rightarrow \Xi V^{-\frac{1}{2}} \mathcal{W}(\cdot) \\ &\quad + \Xi V^{-1} \Xi \int_0^{\iota(\cdot)} \delta(r) dr - \iota(\cdot) \left(\Xi V^{-\frac{1}{2}} \mathcal{W}(1) + \Xi V^{-1} \Xi \int_0^1 \delta(r) dr \right) \\ &\equiv J^r(\cdot) - \iota(\cdot) J^r(1). \end{aligned} \tag{E.10}$$

Thus, condition (i) is satisfied.[31]

Proof of (iii): Finally, it follows that

$$\sup_{\lambda \in [0,1]} \left| \frac{1}{T} \sum_{t=1}^{[\lambda T]} \widetilde{h}_t^r - \lambda \Xi V^{-1} \Xi \right| = o_p(1), \tag{E.11}$$

where $\widetilde{h}_t^r = \widehat{\Xi} \widehat{V}^{-1} \widehat{\Xi}$, satisfying the third condition.

[31] Note that the corresponding Condition 3 in Müller and Petalas (2010) is stated for $\widehat{\Xi}_t$ and $\widehat{V}_t^{-1}$ instead of $\widehat{\Xi}$ and $\widehat{V}^{-1}$; we let $\widehat{\Xi}_t = \widehat{\Xi}$ and $\widehat{V}_t^{-1} = \widehat{V}^{-1}$ to ensure that the estimate is positive definite in any given sample.

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Second, after having proved the lemma, we can proceed with the proof of the propositions.

Because Condition 3 is satisfied, WAR minimizing interval estimators, C_θ, for the reduced-form parameters, B and Σ, satisfy the conclusion of Theorem 5 of Müller and Petalas (2010). Recall that $m = (\lambda, \gamma_f)$, as defined in Section C1 above. Based on C_θ, a conservative WAR minimizing interval estimator for m can be constructed as

$$\{m : \exists \theta \text{ such that } \theta \in C_\theta\}, \tag{E.12}$$

for example.

F. ADDITIONAL TABLES

Table A1 provides a summary of the specifications investigated in the main text.

Table A1. Summary of specifications

Concept	Figure	Left variable	Right variable/ Forcing variable	Instruments
Phillips relation	Figure 1	Inflation (PCE-xFE)	Various measures of slack	-
Phillips curve	Figure 2	Inflation (GDP deflator)	Output gap (CBO)	One lag of the forcing variable and the second lag of inflation
Phillips multiplier	Figure 3	Cumulative inflation (PCE-xFE)	Cumulative unemployment	Monetary policy shocks (RR)
Phillips multiplier	Figure 4	Cumulative inflation (PCE-xFE)	Cumulative unemployment	Monetary policy shocks (RR, HFI)
Phillips curve	Figure 5	Inflation (PCE-xFE)	Output gap (HP filter)	Twenty lags of monetary policy shocks (RR)
Phillips curve	Figure 6	Inflation (GDP deflator)	Output gap (CBO)	One lag of the forcing variable and the second lag of inflation

SUPPLEMENTARY MATERIAL

The supplementary material for this article can be found at https://doi.org/10.1017/S0266466625100169.

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