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ON THE ONTOLOGICAL COMMITMENT OF MEREOLOGY

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Abstract. In *Parts of Classes* (1991) and *Mathematics Is Megethology* (1993) David Lewis defends both the innocence of plural quantification and of mereology. However, he himself claims that the innocence of mereology is different from that of plural reference, where reference to some objects does not require the existence of a single entity picking them out as a whole. In the case of plural quantification "we have many things, in no way do we mention one thing that is the many taken together". Instead, in the mereological case: "we have many things, we do mention one thing that is the many taken together, but this one thing is nothing different from the many" (Lewis, 1991, p. 87). The aim of the paper is to argue that—for a certain use of mereology, weaker than Lewis' one—an innocence thesis similar to that of plural reference is defensible. To give a precise account of plural reference, we use the idea of *plural choice*. We then propose a *virtual theory of mereology* in which the role of individuals is played by plural choices of atoms.

§1. Introduction. In *Parts of Classes* (1991) David Lewis argues that, like logic, but unlike set theory, mereology is "ontologically innocent". *Prima facie*, Lewis' innocence thesis seems to be ambiguous. On the one hand, he seems to argue that, given certain objects Xs, referring to their sum is ontologically innocent because there is no *new* entity as the referent of the expression "the sum of the Xs". So, talking of the sum of the Xs would simply be a different way of talking of the Xs, *looking at them as a whole*. However, on the other hand, Lewis' innocence is not understood as a mere linguistic use, where sums are not reified. He himself claims that the innocence of mereology is different from that of plural reference, where reference to some objects does not require the existence of a single entity picking them out in a whole. In the case of plural quantification "we have many things, in no way do we mention one thing that is the many taken together". Instead, in the mereological case: "we have many things, we do mention one thing that is the many taken together, but this one thing is nothing different from the many" (Lewis, 1991, p. 87).

But, because Lewis explicitly uses sums as objects in their own right, we think that Lewis' innocence thesis must be understood in the sense that, even if the sum of the Xs is a well-determined object, distinct from the Xs, the existence of such an object must be accepted by anyone who has already accepted the existence of the Xs. In other words, committing oneself to the existence of the Xs would be an implicit commitment to some other entities—among them, the sum of the Xs. On the other hand, the existence of the set of the Xs would not be implicitly guaranteed by the existence of the Xs.

The aim of the paper is to argue that—for a certain use of mereology, weaker than Lewis'—an innocence thesis similar to that of plural reference is defensible. In order to give a precise account of plural reference, we use the idea of a *plural choice*. We then propose a *virtual theory of mereology*, where the role of individuals is played by plural choices of atoms. A choice is not an authentic object: its existence is merely potential

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and it consists in the act of performing it. Accordingly, in order to interpret a formal first-order mereological language, like Goodman's *calculus of individuals* (CG), we introduce a semantics of plural choices. We argue that our development of virtual mereology, grounded on the notion of plural choice, is ontologically innocent in a way completely analogous to that of plural reference: our claim is that mereological sums—unlike atoms—are not real objects. Referring to a sum of atoms is nothing but a way of referring to certain atoms. Our approach is adequate to interpret a first-order mereological language. It is inadequate for Lewis' mereology, because his plural quantification over *all* objects is incompatible with our notion of plural choice, where only atoms are capable of being chosen.

We shall consider some attempts present in the literature to defend the ontological innocence of mereology. Such attempts suggest the possibility of a fictional ontological commitment to mereological sums. Then, we shall develop a conception of a fictional ontological commitment that is adequate to a virtual (and so ontologically innocent) interpretation of CG.

§2. Arguments for a fictional ontological commitment. We shall consider some arguments that, however certain individuals Xs are given, aim to deny the additional existence of the sum of the Xs, by introducing a weak use of sum, according to which the latter is not an individual in its own right. The claim that the commitment to the existence of the fusion of the Xs is not a further commitment beyond the existence of the Xs is maintained in a fictional way by arguing in favor of the identification of the fusion of several individuals with the individual themselves. This identification seems to be suggested by Lewis' argument as follows:

(P1) Composition—a many–one relation—is like identity.
(P2) The commitment to sums is already presupposed in the acceptance of the objects that are summed.
(P3) Nothing could be considered more ontologically innocent than the request to accept something identical to things already accepted.
(C) Mereology is ontologically innocent.

Lewis' argument rests on the thesis (P1) of *composition as identity*. However, Lewis criticizes the following strong version of (P1):

(StrongCom) The predicate "are" used for the composition relation is literally the plural for the "is" of identity.

Formally:

$$\forall X\, \forall y((y \text{ is the sum of the } X) \rightarrow y = X)$$

One of Lewis' argument against (StrongCom) concerns the indiscernibility of identicals (InId) that is:

$$\text{(InId)} \quad \forall x\, \forall y(x = y \rightarrow \forall F(Fx \leftrightarrow Fy))$$

where the third universal quantification is of second order and "F" is a predicate variable. "Even though—Lewis argues—the many and the one are the same portion of Reality, and the character of that portion is given once and for all whether we take it as many or take it as one, still we do not really have a generalized principle of indiscernibility of identicals ... What is true of the many is not exactly what is true of the one. After all they are many while it is one" (Lewis, 1991, p. 87).

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Consider the following example. Suppose that the number of the Xs is n, where $n > 1$. Then, the plural predicate "... are exactly n" should apply—given (InId)—to y too, but the number of y is one.

This argument shows how in Lewis' conception the sum of several individuals is a single object in its own right, but the presence of such a single object would not undermine the ontological innocence of mereology.

Lewis argues for the innocence of mereology grounding it on a weak reading of (P1):

(WeakCom) The predicate "are" used for the composition relation is *analogous* to the plural form of the "is" of identity.

The problem is that Lewis' notion of analogy is rather vague and smoky (for a criticism see Carrara & Martino, 2008). We think that the alleged innocence of mereology is better defended within a fictional conception. An example of that might be, at least for mereological questions, the formulation offered by Baxter (1988), who argues for a way to maintain (InId) and (StrongCom). He introduces two kinds of identity, a *strict* and a *popular* identity. Baxter gives the following exemplification of the above distinction: "Suppose a man owned some land which he divides into six parcels. Overcome with enthusiasm for [the denial of composition as identity] he might try to perpetrate the following scam. He sells off the six parcels while retaining ownership of the whole. That way he gets some cash while hanging on to his land. Suppose the six buyers of the parcels argue that they jointly own the whole and the original owner now owns nothing. Their argument seems right. But it suggests that the whole was not a seventh thing" (Baxter, 1988, p. 579), as quoted in Lewis 1991, p. 83.

A justification of (StrongCom) is to argue that to count the many strictly is to count the one loosely:

(BT) The whole is the many parts counted as one thing (Baxter, 1988, p. 579).

Even if Baxter argues that (BT) does not deny the existence of the whole, but just the *additional* existence of the whole, it seems to us that this popular mood does not reify the whole. Baxter's example demonstrates a *weak* use of the sum, not involving its existence as an entity. It seems to be a use of sums similar to that of sets in a sentence as:

(1) The set of the Germans trekking on the Plose has cardinality 600.

This sentence can be reformulated without involving the notion of set:

(2) The number of the Germans trekking on the Plose is 600.

Likewise, the sentence:

(3) I have seen a flock of seven bee-eaters

can be rewritten in the form

(4) I have seen seven bee-eaters.

(4) does not involve any entity denoted by the word "flock".

Obviously, such examples are inadequate to defend Lewis' realistic conception of mereology. They suggest, however, the possibility of a fictional conception of mereological sums.

In what follows we shall develop such a conception, which will be called *virtual*, grounded on the notion of *arbitrary plural choice.*

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§3. Arbitrary plural choices for a second-order logic.[1] We start by making explicit a certain notion of arbitrary reference which is implicitly presupposed both by first- and second-order logic.

First-order logic implicitly presupposes the possibility of singular reference to any individual of the universe of discourse. That can be shown by analyzing the quantification rules of the system of natural deduction for first- and second-order logic.

The *introduction rule* of the universal quantifier (I∀) allows the inference of $\forall x\,Ax$ from the premise Ab in the usual way.

$$\begin{array}{ll} . & \\ . & \\ . & \\ (n) & Ab \\ (n+1) & \forall x\,Ax \end{array}$$

Where 'b' is an arbitrary name (or a free variable) not occurring in any assumption on which Ab depends. The soundness of the rule is grounded on the consideration that if one has proved that b enjoys the property $\lambda x\,Ax$, without any specific piece of information about b, then *any* individual enjoys the property in question. The above remark clearly presupposes that b can denote *any* individual. Similarly for the elimination rule of the existential quantifier (E∃).

So, the logical use of quantification presupposes the ideal possibility of singularly referring to any individual.

The problem arises: how can one refer to an *arbitrary* individual? Perhaps, one might think, by means of some characterizing property. Unfortunately, that involves a problematic universe of properties, suitable for characterizing any individual. Since this option involves the general notion of a property of individuals, it seems to be inappropriate to first-order logic. Besides, it faces the problem of how to refer to an arbitrary property. Therefore, it seems that the notion of reference to an arbitrary individual, which is presupposed by first-order logic, is more primitive than any linguistic notion of reference. We think that the most appropriate idealization for justifying arbitrary reference should be grounded on the ideal possibility of a *direct* access to any individual. We shall invoke an *ideal agent* who is supposed to be able, by means of an arbitrary act of choice, to isolate any individual. In such a conceptual frame the *introduction rule* of the universal quantifier (I∀) is justified in the following way.

Let us imagine an *ideal chooser* who arbitrarily chooses an individual b, about which we have no information at all. If we are able, just by reasoning about b, to conclude that it enjoys $\lambda x\,Ax$, since, so far as we know, *each* individual could be the chosen one, we can conclude that any individual has the property in question and, thus, infer $\forall x\,Ax$.

As to second-order logic, this presupposes the possibility of referring to any set of individuals. In fact, the semantics of second-order logic quantifies over all subsets of the domain of individuals. The problem is *how* to refer to an arbitrary set of individuals.

Since all we know about an arbitrary set of individuals is that it is specified by its elements, the reference to a set is not realizable except by simultaneously referring to its elements. The comprehension principle of second-order logic:

$$\text{(CP)} \qquad \exists X \forall x (Xx \leftrightarrow A(x)),$$

[1] In this paragraph we summarize some ideas developed in Martino (2001, 2004).

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in the usual set-theoretical interpretation, where the second-order variables X range over all subsets of the individuals, rests on the assumption that the individuals satisfying the formula $A(x)$ form a set. That is justified by the general set-theoretical principle:

(CP*) *However certain individuals are given, they form a set.*

The problem arises: how can arbitrary individuals be given? One might think: by means of some property they share. But this option is unsatisfactory, in the light of the well-known criticism of impredicative definitions. Indeed, the formula $A(x)$ could contain some second-order quantification. In that case, the property $\lambda x\, Ax$ expressed by the formula $A(x)$ is defined in terms of the totality of sets of individuals. If every set presupposes the existence of a property that is able to characterize its elements, the property $\lambda x\, Ax$ is circularly defined in terms of the totality of properties of individuals. The circularity could be avoided by means of some *reducibility axiom à la* Russell: one could assume that, for every property definable in second-order logic, there is a coextensive elementary property (i.e. one expressible without second-order quantification). In this way, quantifying over all sets would implicitly presuppose only quantification over such elementary properties. But, in general, these are not expressible in the given logical language and their nature seems to be highly problematic. (Consider, for example, the well-known criticism of the *Reducibility Axiom*.[2])

For these reasons we propose an alternative approach: the idea is that the members of a set are not isolated by means of a property, but by means of *an act of an arbitrary plural choice*. To this end, let us extend the idealization of singular choices to that of plural choices. Imagine that there is an agent for every individual. For the sake of simplicity, we identify the individuals with the agents themselves. A plural choice consists of an arbitrary simultaneous choice by each agent between the numbers 0, 1. The agents whose choice is 1 are the individuals *designated* by the choice in question. Within this framework (CP*) can be made explicit as follows:

(SET) *The individuals designated by any plural choice are the members of a suitable set.*

(SET) supplies evidence (in a noncircular way) for the comprehension principle (CP): since, in an act of plural choice, each agent can arbitrarily choose 0 or 1, it is certainly possible that a plural choice be performed, where the agents choosing 1 are precisely those satisfying the formula $A(x)$; so these agents, in virtue of (SET), determine a set.

Boolos (1984, 1985) has argued for the ontological innocence of second-order monadic logic by proposing an interpretation grounded on the notion of plural quantification. In Boolos' interpretation second-order variables do not range over sets of individuals, but over individuals plurally. By contrast, first-order variables range over individuals singularly.

Boolos' basic idea consists in interpreting the atomic formulas of the form 'Xy' as "y is one of the Xs", and the existential formulas of form '$\exists X\ldots$' as "There are some individuals Xs such that …". The universal quantifier $\forall X$ is expressible in terms of the existential one in the usual way.

Boolos' proposal has been criticized in various ways: one wonders if speaking of pluralities of individuals is just a rough manner of speaking of sets.

The idea of plural choices can clarify Boolos' proposal. For, the project of reinterpreting second-order logic without involving second-order entities, such as sets or properties, can

[2] For criticism of the Reducibility Axiom see Copi (1971). On the other hand, for a defense see Myhill (1979).

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be carried out by reasoning in terms of acts of choice. Instead of assuming (SET), that is the existence of an entity collecting together the individuals designated by any arbitrary plural choice, we can use, as it were, a *virtual* theory of sets, in which the set-theoretical language is paraphrasable in terms of plural choices.

To be more precise, we introduce the following semantics of plural choices.

Let ϕ be a second-order monadic formula whose free first-order variables are among $x_1, \ldots, x_m$ and free second-order variables among $X_1, \ldots, X_n$. Consider, for each variable x_i, a singular choice $x_i^*(i = 1, \ldots, m)$ and, for each variable X_j, a plural choice $X_j^*(j = 1, \ldots, n)$. We inductively define the truth value of ϕ relative to the choices $x_1^*, \ldots, x_m^*; X_1^*, \ldots, X_n^*$. We state only the clauses for atomic formulas and for second-order quantifiers, the others being as usual:

1. if $\phi \equiv x_j x_i$, it is true if the individual designated by choice x_i^* is designated by choice X_j^*;
2. if $\phi \equiv \forall Y \psi$, it is true if, however a plural choice Y^* is performed, ψ is true relative to choices $x_1^*, \ldots, x_m^*, X_1^*, \ldots, X_n^*, Y^*$;
3. if $\phi \equiv \exists Y \psi$, is true if it is possible to perform a plural choice Y^* in such a way that ψ turns out to be true relative to choices $x_1^*, \ldots, x_m^*, X_1^*, \ldots, X_n^*, Y^*$.

The above semantics is ontologically innocent because the acts of choice are not objects. An act of choice satisfying certain conditions, such as the three conditions specified above, is not an entity that exists or not in itself, but an action which might or might not be performed. Accordingly, the interpretation of quantifiers is merely potential. That does not undermine the validity of classical logic, since the obtaining or not of a plural choice satisfying clause (3) is completely determined by which individuals are available. Everything is perfectly determined once the domain of individuals is fixed.

An important aspect of the difference between acts of choice and outright sets is that sets, as individuals, are capable of being chosen in turn. In contrast, it would be meaningless to perform a simultaneous choice of infinitely many acts of choice, because the latter are never simultaneously available. For this reason, the semantics of plural choices cannot be extended to a logic of order higher than the second. Still less might it be used for interpreting general set theory as *ZF* set theory. The above limitations contribute to show that the semantics of arbitrary choices is not an expedient for introducing sets surreptitiously.

In the next section we shall show that the semantics of arbitrary choices, suitably modified, can be used for developing a virtual mereology.

§4. Virtual mereology (VM). An informal exposition. Imagine an infinite domain Σ of agents, which we shall also call mereological atoms. A plural choice consists in the simultaneous choice of an atom by each agent. The chosen atoms are said to be *designated* by the plural choice. Two plural choices c_1 and c_2 are *equivalent* if they designate the same atoms. As already observed, acts of choice are not individuals. Nevertheless, we can make them play the role of mereological individuals: we call individuals the acts of choice and say that equivalent choices are the same individual. We treat the equivalence relation between choices as the identity relation between individuals. Talk about individuals is to be understood as extensional talk about plural choices, that is talk identifying equivalent choices. If a choice designates a unique atom it is identified with the atom itself. If every atom designated by choice c_1 is also designated by choice c_2, we say that individual c_1 is part of individual c_2; in symbols $c_1 < c_2$.

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In this way, we obtain the fundamental relation of mereology. A mereological property P is a law that, with every plural choice, associates one of the values 0,1 in an extensional way (i.e. in such a way that the same value is associated with equivalent choices). Similarly, a binary relation is an extensional law that, with every ordered pair of plural choices, associates one of the values 0,1. We say that object c enjoys property P if the latter associates value 1 to choice c. Similarly, we say that objects c_1, c_2 are related by relation R if R associates 1 to the pair of choices c_1, c_2.

We define the sum of c_1 and c_2, (in symbols $c_1 + c_2$) as the individual whose atoms are all those of c_1 and all those of c_2. This sum exists since there is certainly a possible choice whose designated atoms are precisely those designated by c_1 and those designated by c_2. Similarly, if c_1 and c_2 share at least one atom, then there exists their product $c_1 \cdot c_2$ whose atoms are those common to c_1 and c_2.

More generally, if P is a property enjoyed by some objects, then there exists the sum of all objects enjoying P, that is the object built up from all atoms, such that each of them is part of at least one object enjoying P. Indeed, for any atom a it is a well-determined fact whether or not a plural choice c is possible such that a is designated by c and a enjoys P. There is therefore certainly a possible choice, that will be indicated by $\sigma x \mathrm{P}x$, whose designated atoms are precisely those satisfying the above condition.

Similarly, if at least one object satisfies P, and at least one atom is part of every object satisfying P, then there exists the product of all objects enjoying P, $\pi x \mathrm{P}x$, that is the object built up from all atoms common to all objects satisfying P.

As already observed for the virtual set theory, because the acts of choice are conceived of as performable in time, we can speak of individuals of virtual mereology only in a purely potential sense. To say that there exists at least one object satisfying property P amounts to saying that a choice c satisfying P is performable. To say that every object enjoys property P amounts to saying that, however a plural choice c is performed, it will enjoy property P. Again, we can observe that such a potential interpretation does not undermine the validity of classical logic. For, the possibility or not of performing a plural choice satisfying certain conditions is well determined by the objects available in the domain.

One might wonder whether, and, if so, in what sense, one can speak of the cardinality of the universe of virtual objects, and compare it with that of the universe of atoms. There is a merely potential sense in which one might claim that there are more individuals than atoms. It is possible, by means of a single plural choice to determine infinitely many objects. Precisely, one can produce, by a single plural choice, a family of objects indexed by atoms. If c is a plural choice of ordered pairs of atoms, and i is the first component of a pair designated by c, we indicate by c_i the object whose atoms are the second components of the pairs designated by c whose first component is i. Thus, we get the family $\{c_i\}_{i \in I}$ of objects c_i indexed by the first components Is of the pairs designated by c.

Suppose, for the sake of simplicity, that there are denumerably many atoms and indicate them by the natural numbers. We can obtain, by means of a single plural choice, a family of objects $c_i \in \mathrm{N}$. Using the well-known method of diagonalization, one can prove the existence of an object different from all objects of the family. Precisely, let P be the property that is enjoyed by an atom if and only if it is an atom of c_i. It is certainly possible to perform a choice c whose designated atoms are exactly those enjoying P (disregarding the case in which all c_i are the universal object, which is to be treated separately in an obvious way). To such a choice there corresponds an object different from all c_i. So, we can say that the objects constitute a nondenumerable infinity in the sense that, however a succession of objects is determined by means of a plural choice, it is always possible

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to determine an object different from all components of that succession. In other words, it is impossible to attribute simultaneous actual existence to all possible objects; even if the objects are merely virtual, it is impossible to speak of them as if they were all actually existent. For, the mere virtuality of objects, as here understood, consists in the possibility of translating talk of objects into talk of plural choices. The ideal assumption of denumerably many agents grounds the possibility of simultaneously choosing infinitely many atoms, but a simultaneous performance of all possible acts of choice is in no way conceivable.

§5. The calculus of individuals with atoms (CG). The above exposition can be considered a virtual interpretation of the axiomatic theory of mereology developed by N. Goodman (1977) called "calculus of individuals" (CG) where the overlap relation is assumed as primitive. In the present paper, we consider a version of CG with atoms but without sets, where the part relation is primitive ($<$), while the other mereological notions and identity ($=$) are defined in terms of the part relation.

Precisely, we introduce the following definitions:

(CGDef.1) $x \circ y =_{df.} \exists z(z < x \wedge z < y)$ (x overlaps y)

(CGDef.2) $x = y =_{df.} x < y \wedge y < x$ (x is identical to y)

(CGDef.3) $At(x) = \forall y(y < x \rightarrow y = x)$ (x is an atom)

Two nonoverlapping individuals are said to be mutually *discrete*:

(CGDef.4) $x \jmath y =_{df.} \neg (x \mathbf{o} y)$ (x is discrete from y);

(CGDef.5) $x << y =_{df.} (x < y \wedge \neg(y < x))$ (x is proper part of y)

Finally, let us define the *sum* or *fusion* of the individuals satisfying a formula F:

(CGDef.6) $\forall y(Aty \rightarrow (y < z \leftrightarrow \exists x(Fx \wedge y < x)))$ (z is the sum of the F)

Axioms of (CG):

(CGA1) Any set of logical axioms for first-order predicate calculus without identity.

(CGA2) $x < x$

(CGA3) $x < y \wedge y < z \rightarrow x < z$

(CGA4) $\forall z(Atz \wedge z < x \rightarrow z < y) \rightarrow x < y$

(CGA5) $\forall z\,(z \mathbf{o} x \leftrightarrow z \mathbf{o} y) \rightarrow (A \rightarrow A[y/x])$

where $A[y/x]$ is any formula obtained from A by replacing some occurrence of x free in A with an occurrence of y.

(CGA6) If the formula Fx is satisfied by some individuals, there is one and only one individual which is the sum of the F.

Such an individual will be denoted by $\sigma x Fx$. In particular, taking for Fx the formula:

$$(x = y \vee x = z)$$

we get the sum of y and z which will be simply indicated by $y + z$.

Finally, it is easily seen that, if the F share a common part, there exists the product of the F defined as the individual built up from all atoms shared by all F. We shall indicate

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this product by $\pi x Fx$; in case of two individuals y and z simply by $y \cdot z$. It follows from (CGA4) that:

(Teor1) $\forall x \exists y (Aty \wedge y < x)$ (each individual has an atomic part)

Proof. Suppose, for the purpose of *reductio*, that x is an individual without atoms. It follows from (CGA4): $\forall z(Atz \wedge z < x \rightarrow z < y) \rightarrow x < y$ that x is part of every y, so x has no proper parts. Therefore, it is itself an atom, which is absurd (QED).

§6. A formal semantics for VM. Let L be the language of first-order predicate logic with just the primitive binary predicate $<$.

We define for L the interpretation of plural arbitrary choices (IPAC) by suitably modifying the above interpretation for second-order logic.

Let us imagine denumerably many agents Σ, which will be identified with mereological atoms. A plural choice c consists in the simultaneous arbitrary choice of any individual by each agent. An individual is *designated* by plural choice c if it is chosen by at least one agent. Thus, a plural choice determines the plurality of designated individuals. Define (IPAC) by fixing the truth-conditions of the L-sentences.

Let $A[x_1, \ldots, x_n]$ be a formula whose free variables are those shown. For each variable x_i consider a plural choice $c_i (i = 1, \ldots, n)$. We define the truth conditions of $A[x_1, \ldots, x_n]$ inductively relative to choices $c_1, \ldots, c_n$:

(i) an atomic formula $x_1 < x_2$ is true relative to choices c_1, c_2 if the individuals designated by c_1 are designated by c_2.

(ii) Usual clauses for propositional connectives.

(iii) $\forall x_i\, A[x_1, \ldots, x_i, \ldots, x_n]$ is true relative to choices $c_1, \ldots, c_{i-1}, c_{i+1}, \ldots, c_n$ if, however a plural choice c_i relative to x_i is performed, the formula $A[x_1, \ldots, x_i, \ldots, x_n]$ is true relative to choices $c_1, \ldots c_i, \ldots c_n$.

(iv) $\exists x_i A[x_1, \ldots, x_i, \ldots, x_n]$ is true relative to choices $c_1, \ldots, c_{i-1}, c_{i+1}, \ldots, c_n$ if it is possible to perform a choice c_i for x_i such that $A[x_1, \ldots, x_i, \ldots, x_n]$ is true relative to choices $c_1, \ldots c_i, \ldots c_n$.

By induction on the complexity of formulas we see that the above clauses determine the truth-value of every formula relative to the plural choices associated with the free variables. For the atomic formulas this fact follows directly by observing that a plural choice designates a well-determined plurality of individuals. For the formulas whose principal logical constant is a propositional connective, the truth-value is determined in an obvious way from the values of the components. As to quantifiers, suppose that, relative to every plural choice, Ax has a well-determined value (assuming for the sake of simplicity only one free variable). Then, it is well determined whether, however the plural choices performed, the formula turns out to be true or it is possible to perform a choice falsifying it. So, the value of $\forall x A[x]$ is well determined. Similarly for $\exists x A[x]$.

Notice that the possibility of performing a choice verifying $A[x]$ is not to be understood in an epistemic sense, but in an alethic sense: that is it is not required that the agents may follow some strategy in order to obtain some goal. A plural choice is always constituted by arbitrary acts of choice, performed by the agents independently of each other. The possibility of a plural choice verifying $A[x]$ is to be understood in the sense that it might happen that the purely casual choices performed by the agents verify $A[x]$.

In particular, the truth-value of every sentence (closed formula) will be well determined. It follows that our interpretation is in agreement with the laws of classical logic.

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Introduce for IPAC the mereological terminology. We say that a variable x, relative to a plural choice c, denotes an object. In particular, if c designates a unique individual, we say that x denotes an atom that we identify with the designated agent. We say that the object denoted by x relative to choice c is identical to the object denoted by y relative to choice c' if the individuals designated by c are the same individuals designated by c'. We say that x is part of y if every individual designated by c is designated by c'.

So, the ontological commitment reduces just to atoms. Speaking of compound objects is only a *façon de parler*. The singular L-terms pretend to denote in virtue of the choice acts associated with them.

§7. VM as a model of CG. Consider the axioms of CG:

(CGA1) Any set of logical axioms for first-order predicate calculus without identity.

(CGA2) $x < x$

(CGA3) $x < y \wedge \mathrm{y} < \mathrm{z} \rightarrow x < z$

(CGA4) $\forall z(Atz \wedge z < x \rightarrow z < y) \rightarrow x < y$

(CGA5) $\forall z(z \mathbf{o} x \leftrightarrow z \mathbf{o} y) \rightarrow (A \rightarrow A[y//x])$

(CGA6) If the formula Fx is satisfied by some individual, there is one and only one individual which is the sum of the F.

(CGA1)–(CGA4) are trivially verified in (VM).

We limit ourselves to verifying (CGA5) and (CGA6). As to (CGA5), observe that from the antecedent of the conditional—$\forall z(z \mathbf{o} x \leftrightarrow z \mathbf{o} y)$—restricting z to the atoms, it follows that x and y have the same atoms, so that they are identical, and so it is possible to substitute one for the other. As to (CGA6), let Fx be a formula satisfied by at least one object. Since every individual is built up from atoms there is at least one atom that is part of an individual satisfying Fx. Besides, as observed above, the formula determines which individuals (which plural choices) satisfy it. Therefore, for all atoms it is well determined whether it is part of at least one individual satisfying Fx. So, there is a possible plural choice whose designated atoms are precisely those satisfying the condition in question. Such a choice represents the object $\sigma x Fx$.

(VM) is inadequate to interpret Lewis' mereology because the latter uses plural reference and plural quantification over all individuals, including among them all possible sums (see Lewis, 1991, 1993). In contrast, our virtual mereology (VM) allows only plural choices of atoms, not plural choices of arbitrary virtual individuals. Accordingly, our justification of (CGA6) exploits essentially the fact that the objects whose sum is claimed to exist are determined by a formula of the language (they are not arbitrarily chosen).

§8. Conclusion. In the present paper we have tried to take at face value Lewis' claim that mereological sums are identifiable with the objects composing them. To this end, we have developed a *virtual mereology* (VM), where atoms are the sole genuine objects, while composed objects are nothing but pluralities of atoms. So, the innocence of (VM) is inherited from the innocence of the plural interpretation of second-order logic, where second-order variables range over individuals plurally.

Our virtual mereology is inadequate to interpret Lewis' use of mereology, since this essentially exploits the existence of sums as singular objects. In fact, Lewis tries to defend

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the innocence of mereology by arguing that, though sums are genuine objects, nevertheless the commitment to their existence is already implicit in the commitment to the existence of atoms. We have criticized elsewhere Lewis' argument (see Carrara & Martino, 2008). However, our virtual mereology shows that to a certain extent, if we are dealing with a suitable mereological theory weaker than the one Lewis offers, the ontological innocence of mereological sums can be vindicated.

Our version of mereology, as well as Lewis', essentially exploits the *atomic assumption*, according to which every object is built up from atoms. Observe however that such an assumption, as it is used in the present paper, does not concern the nature of the given objects. Any given objects can play the role of atoms, insofar as we are not interested in their internal structure, but in the mereological individuals built up from all possible fusions of some of them. Our claim is that, whichever objects are accepted as robust objects, their fusion can be understood as a virtual object.

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