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How Do Frictions Affect Corporate Investment? A Structural Approach

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Abstract

This paper provides a structural approach to testing investment equations based on the log-likelihood function of a nonlinear investment rule. The analysis integrates the predictions of the q-theory for the commonly studied active region of investment and provides new inferences on how real and financing frictions affect the probability that a firm invests. The empirical findings are consistent with the macro-finance literature suggesting that q-theory models with nonconvex investment frictions better explain the data. I also find that both real and financing costs of investment are related to the capital intensity of the industry in which firms operate.

I. Introduction

Although the neoclassical investment model or q-theory of investment has been heavily used in the economics literature for decades, its empirical performance remains highly controversial.[1] Throughout the years, scholars have provided multiple arguments to explain why the shadow price of capital, or q, is not a sufficient statistic for investment in the empirical tests. Yet these arguments are not integrated into a general approach to compare how the alternative adjustments to the benchmark q-theory model lead to significant improvements in its explanatory power. Consequently, different strands of the literature focus on different sets of investment frictions to derive the optimal investment rule, and yet most empirical studies refer to the same subset of firms tracked by the Center for Research in Security Prices (CRSP) and Compustat.

A relevant strand in the literature on the q-theory relates to the use of nonconvex costs of investment. Nonconvex costs of investment are usually neglected in empirical corporate finance studies and are used in various branches of the

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[1] See the surveys by Caballero (1997) and Bond and Van Reenen (2007) for a discussion of the empirical literature on neoclassical investment models.

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macro-finance literature. The widely used linear regression of investment on average Q and cash flows in empirical corporate finance relies on the underlying theoretical assumption that firms are subject to linear marginal costs of investment.[2] This benchmark regression also assumes that firms invest actively at all times. In contrast, the macroeconomics literature incorporates nonconvex costs of investment to explain plant-level data and real business cycles. Similarly, asset pricing studies rely on models with nonconvex adjustment costs to explain the value premium.[3]

This paper bridges the gap between the empirical corporate finance and macro-finance literatures in exploring the role of nonconvex costs of investment in investment equations and the interaction between nonconvex costs of investment and financing frictions. In a nutshell, I provide a structural approach to testing investment equations that integrates the predictions of the q-theory for the commonly studied active region of investment and for the less explored inertia region in which firms find it optimal not to invest. The main idea behind my theoretical analysis is that firms subject to fixed costs of investment decide to invest only if their q is sufficiently high, such that realized investment is explained by a truncated likelihood function. The truncated likelihood function is obtained once I restate the predictions on the inertia region in terms of the probability that a firm invests. I then find that the predictions of the model are supported by the data, and I quantify firms' real and financing frictions by industry group.

The analysis therefore motivates the reinterpretation of the predictions of the q-theory on the inaction region for the sake of empirical tests. A likely reason why inaction regions have been neglected in corporate finance may relate to the fact that investment is infrequently 0 or negative in CRSP/Compustat data. However, in strictly applying the rationale of the q-theory, the fact that realized investment is usually positive in CRSP/Compustat firms does not mean that these firms do not face fixed costs or face a single investment regime, but rather that the q of large U.S. public firms is unlikely to fall below their minimum q required to invest.[4] With this economic intuition, I conjecture and later verify that a truncated regression model improves the goodness of fit of the q-theory in CRSP/Compustat data.

My analysis also bridges the gap between the corporate finance and the macro-finance studies by considering the interaction between financing frictions, fixed costs of investment, and adjustment costs. Most macro-finance studies on the q-theory consider fixed costs of investment assuming that firms are all equity financed. However, only a small fraction of firms in the CRSP/Compustat data set are unlevered. In my model, the probability that a firm invests is lower

[2] Average Q is the ratio of market value of assets to replacement value of capital. The benchmark ordinary least squares (OLS) regression is discussed by Hayashi (1982).

[3] See Caballero and Leahy (1996), Cooper and Haltiwanger (2006), and Bloom (2009) for examples of studies in macroeconomics using nonconvex costs. See Zhang (2005) and Cooper (2005) for references on models with nonconvex costs explaining the value premium. I acknowledge that baseline q-theory models without nonconvex costs have had some empirical success in explaining the cross section of returns. See, for instance, Liu, Whited, and Zhang (2009).

[4] In my working sample of CRSP/Compustat firms, 2.16% and 0.09% of the observations show negative and 0 investment, respectively.

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if the cost of external financing increases. Moreover, due to biases in focus, most structural estimation studies quantify either real or financing frictions in isolation. In contrast, I estimate both types of frictions jointly by industry group, and I show that firms' real and financing costs of investment relate significantly to the capital intensity of the industry.

The contributions of the paper are three. The first is the proposed structural approach. I derive in closed form the log-likelihood function of the optimal investment policy of a q-theory model in which firms are subject to fixed and convex costs of investment and convex costs of external financing. The methodology accounts for a wedge between q and average Q due to measurement error. The approach integrates the arguments of nonlinear investment rules, inertia regions of investment, and measurement error in q. Moreover, the corresponding estimation of firms' real and financing frictions does not rely on simulated data; it is carried out through direct inference.

The second contribution is the evidence of the reduced-form tests. Consistent with my main economic intuition, the adjusted R^2 in reduced-form tests goes from 0.145 in the benchmark OLS regression of investment on Q and cash flows to an adjusted R^2 of 0.755 in a truncated regression of investment on Q. Moreover, investment is positively skewed and concave in Q. In the model, the ratio of investment to capital is concave in Q because firms rely on higher levels of external financing as they increase their investment, which in turn leads to higher marginal costs of external financing. In the data, the explanatory power of investment equations increases significantly when I include higher-order moments of Q and cash flows.

The third contribution is the joint estimation of firms' real and financing frictions. For all firms on average, firms' average adjustment costs of investment are 8.82% of their capital, firms' fixed costs are 2.48% of their capital, and firms' average costs of external financing are 10.9% of their capital. I find significant cross-sectional variation in firms' real and financing frictions by industry group. I then show that firms in more capital-intensive industries have higher adjustment costs of investment, higher fixed costs of investment, and lower costs of external financing. This is consistent with the empirical corporate finance literature showing that investment and capital structure decisions depend on the tangibility of assets and that leverage ratios have a significant industry component.[5]

The theme of my paper is most closely related to the study by Barnett and Sakellaris (1998), whose work stands as an exception in the literature testing for the existence of an inaction region. Barnett and Sakellaris (1998) find no evidence of an inaction region in a sample of CRSP/Compustat firms, and yet their methodology implicitly requires that the observations on *realized* investment in their sample cover the full spectrum of alternative investment regimes. I argue instead that CRSP/Compustat firms are usually in their active region, and I then use the predictions of the q-theory on inaction to control for the *probability* of observing positive investment. I also contribute to the work of Barnett and Sakellaris (1998) because they assume that firms are all equity financed.

[5]See, for instance, Lemmon, Roberts, and Zender (2008), Campello and Giambona (2013), and Campello and Hackbarth (2012).

The theoretical framework combines the setups of Abel and Eberly (1994) and Hennessy, Levy, and Whited (2006) to elaborate on the interaction between real and financing frictions. The model also relates to the settings by Abel, Eberly, Dixit, and Pindyck (1996), Gomes (2001), and Bolton, Chen, and Wang (2011). The empirical findings relate to several studies suggesting that nonlinear investment equations have better explanatory power, including those by Caballero and Leahy (1996), Abel and Eberly (2002), Gourio and Kashyap (2007), Bloom (2009), Cooper (2005), Belo, Xue, and Zhang (2013), and Gala and Gomes (2015). In particular, Belo et al. (2013) show that a restricted model with linear marginal adjustment costs is rejected when using the q-theory as a tool for valuation. The results of my structural estimation relate to the works of Cooper and Haltiwanger (2006) and Hennessy and Whited (2007). My empirical findings also complement the evidence given by Erickson and Whited (2000) and Schennach and Hu (2013) of significant measurement error in q.

The paper is structured as follows: Section II provides the model. Section III derives the econometric strategy. Section IV provides the supporting empirical evidence and structural estimation results. Section V concludes.

II. Model

I derive the optimal corporate policies of a q-theory model with real and financing frictions. The model builds upon the frameworks developed by Abel and Eberly (1994) and Hennessy et al. (2006) and adopts their notation where possible.

A. Main Assumptions

The firm is run by a manager who decides on the investment policy to maximize the market value of the existing shares of the firm. Investors are risk neutral and discount cash flows at a constant risk-free rate of $r > 0$, where r is the constant yield of a tax-exempt risk-free bond. In contrast to the unlevered firm described by Abel and Eberly (1994), the firm has issued a consol bond paying an infinite stream of constant payments b. The firm may also raise additional external funds in the form of equity issuance to finance its future investments, subject to convex flotation costs of equity.

The assumptions on the capital structure of the firm are done for tractability and without loss of generality for the sake of empirical tests. I elaborate on how firms' net increases in external financing relate to investment, but I make no specific claim about equity or debt issues in particular. Whereas the coupon b captures the effect of senior commitments on current investment, the convex flotation costs capture the stylized fact that firms' marginal costs of external financing are increasing in their funding needs. The financing frictions literature observes increasing marginal costs in both equity and debt issues.[6] I henceforth describe the firm as subject to convex costs of net external financing. For the sake of tractability, I further assume throughout this section that the senior consol bond

[6]Equity issues are subject to convex flotation costs of underwriting, as documented by Altınkılıç and Hansen (2000). Bond and Meghir (1994) and Gomes, Yaron, and Zhang (2006) consider a convex interest schedule on the debt obligations that is increasing in book leverage.

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b is risk-free, such that the firm operates in perpetuity. I discuss the extension with risky debt in Section IV.

The gross operating profits $\pi(K_t, \epsilon_t)$ are a linear function of the current capital stock K_t and some diversifiable shock ϵ_t. The firm is a price-taker, and its production function exhibits constant returns to scale, such that its gross profits are linear in capital.[7] The function $\pi(K_t, \epsilon_t)$ is twice continuously differentiable and strictly increasing in all its arguments. The state variable ϵ_t captures innovations in both input and output prices and evolves according to a diffusion process:

$$d\epsilon_t = \mu(\epsilon_t)\,dt + \sigma(\epsilon_t)\,dW_t, \tag{1}$$

where W_t is a standard Wiener process.

Capital is acquired by undertaking gross investment at a rate I_t, and the capital stock depreciates at a fixed proportional rate δ. The capital stock K_t evolves according to the following equation:

$$dK_t = [I_t - \delta K_t]\,dt, \quad K_0 > 0. \tag{2}$$

When the firm undertakes gross investment, it incurs different types of costs. First, the firm incurs a direct cost of purchasing capital. I set the price of capital equal to 1 such that this cost is equal to I_t. Second, the firm incurs both variable adjustment costs of investment and fixed costs of investment, which I jointly denote by $C(I_t, K_t)$. The function $C(I_t, K_t)$ is twice continuously differentiable, strictly convex, and homogeneous degree 1 in I_t and K_t such that the following holds:

$$C(I_t, K_t) = \frac{\alpha}{2} K_t \left(\frac{I_t}{K_t} - \delta\right)^2 + f K_t \mathbf{1}_t^I. \tag{3}$$

The parameter $\alpha > 0$ is the adjustment cost parameter, f is a fixed cost that is proportional to capital, and $\mathbf{1}_t^I$ indicates nonzero investment. The quadratic form in equation (3) provides tractability.

The firm funds its investment through either internal cash flows or external financing. Whereas internal financing is costless, the firm is subject to convex costs of external financing if it raises external funds. I denote the external funding requirement of the firm X_t such that

$$X_t = I_t + C(I_t, K_t) + b - \pi(K_t, \epsilon_t). \tag{4}$$

At any time t, the firm either issues new securities to fund its investment when $X_t > 0$ or distributes a dividend $D_t > 0$ to its shareholders such that $D_t \equiv -X_t$. The decision to raise additional external financing is determined as a residual claim. I denote the *net* increase in the external funding requirement of the firm in each period as $\widetilde{X}_t = X_t - b$. I assume that the costs of external financing

[7] Although I assume profits to be linear in K for analytical convenience, the empirical evidence suggests that constant returns to scale is also a reasonable assumption. See Baily, Hulten, and Campbell (1992), Olley and Pakes (1996), and Cooper and Haltiwanger (2006).

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$H(X_t, K_t)$ are given by

$$H(X_t, K_t) = X_t + \frac{\theta}{2} K_t \left(\frac{\widetilde{X}_t}{K_t} \right)^2 \mathbf{1}_t^X, \tag{5}$$

where $\theta > 0$ is the external financing cost parameter, and $\mathbf{1}^X$ assumes a value of 1 if $\widetilde{X}_t > 0$, and 0 otherwise. Consistent with Hennessy et al. (2006), the functional form of $H(X_t, K_t)$ is such that the marginal cost of external financing is increasing in X_t. It is strictly increasing in X_t, weakly convex in X_t, and decreasing and convex in K_t. I abstract from fixed costs of external financing for the sake of tractability.

The vector (K_t, ϵ_t) captures all the relevant information at each instant t. At each point in time, the manager chooses the investment policy I_t that maximizes the current value of existing shares S_t. The manager thus solves

$$S(K_t, \epsilon_t) = \max_I E_t \left[\int_0^\infty e^{-rs} \left[-H\left(X_{t+s}, K_{t+s}\right) \right] ds \right], \tag{6}$$

subject to equations (1), (2), (4), and $D_t \equiv -X_t$.

B. Optimal Investment Policy

I denote the shadow value of an additional unit of capital by $q \equiv S_K$. The optimal investment rule resembles that of Abel and Eberly (1994) because there exists a nondegenerate inaction region in which firms find it optimal not to invest. In the active region, the optimal investment policy that solves the problem of the manager in equation (6) is given by

$$q = H_X \left(1 + C_I\right), \tag{7}$$

such that firms invest up to the point at which the shadow value of a unit of installed capital equals the ratio of the marginal cost of investment and the marginal cost of external funding.

Rearranging terms, the optimal investment policy of the firm equals

$$\frac{I}{K} = \begin{cases} \delta - \dfrac{1}{\alpha} + \dfrac{1}{\alpha} \dfrac{q}{H_X} & \text{if } q \notin \Theta, \\ 0 & \text{if } q \in \Theta, \end{cases} \tag{8}$$

where $\Theta \equiv (q_1, q_2)$ determines the range of values of q for which the firm finds it optimal not to invest.

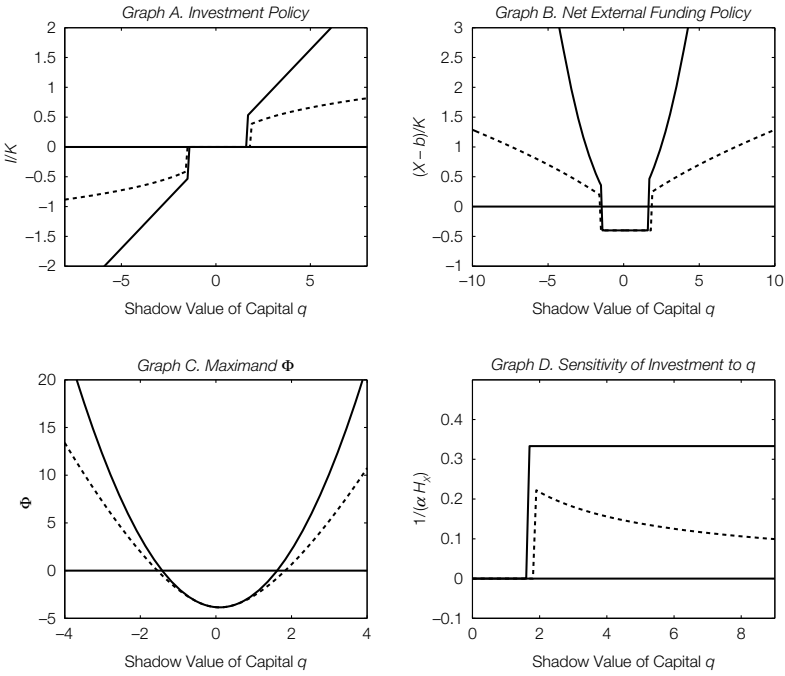
The optimal investment rule in equation (8) differs in various ways from the stylized predictions of neoclassical investment models. Figure 1 illustrates how firms' investment and net external financing policies relate to q and cash flows in the model and compares these predictions with those in the model by Abel and Eberly (1994) for unlevered firms. To ease on exposition, I discuss the main properties of the investment rule in equation (8) in terms of its testable implications. Because q is empirically unobservable, I derive an identity between q and the ratio of the total market value of the firm to its fixed assets or average Q. Considering B as the market value of debt, I define Q as the ratio of $S + B$ over K.

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FIGURE 1

Main Predictions Compared with Those of Abel and Eberly (1994)

Figure 1 illustrates the main predictions of my model for investment policies and compares them with those of Abel and Eberly (1994). The dashed lines correspond to the case in which $\theta > 0$ and $X > 0$. The solid lines correspond to the case in which firms are not subject to convex costs of external financing, either because $\theta > 0$ and $X < 0$ (as in this paper) or because $\theta = 0$ (as in Abel and Eberly (1994)). When $\theta > 0$ and $X > 0$, investment is concave in q, and firms are more reluctant to (dis)invest. The parameter choice is $\alpha = 3$; $\theta = 2$; $\delta = 0.3$; $\pi = 4$; $K = 10$; $f = 0.25$; and $b = 1$.



Lemma 1. The shadow price of capital q equals average Q.

Proof. See Appendix A. □

Proposition 1. In the active region, i) I/K is increasing in Q $\forall X$ and increasing in π/K if $X > 0$; ii) I/K is strictly concave in Q if $X > 0$, and linear in Q otherwise; iii) I/K is strictly convex in π/K if $X > 0$.

Proof. See Appendix B. □

In line with the predictions by Hayashi (1982), Lemma 1 states that the unobservable q equals average Q in the model.[8] Yet in contrast to Hayashi (1982) and Abel and Eberly (1994), Proposition 1 shows that average Q is not a sufficient statistic for investment when firms rely on external funding. When firms finance their investments with external funds, their investment policy depends on Q and the available cash flows π. The rationale behind this result is that the sensitivity of investment to Q is *itself* a function of average Q and cash flows.

[8]Lemma 1 holds under the assumption of riskless debt. In the case with risky debt, there exists a wedge between marginal q and average Q due to debt overhang effects. I elaborate on this case in Section IV.

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Proposition 1 also states that investment is nonlinear in average Q. The general observation that investment is nonlinear in Q is not novel; several studies elaborate on how real and financing frictions lead to nonlinearities in the sensitivity of investment to Q. I add to the literature in showing that, in the active region, investment is increasing and weakly concave in Q. In the basic model, the concavity of investment in Q stems from the increasing marginal costs of external financing. All else equal, when firms rely on external funds, higher investment rates imply heavier reliance on external financing and hence higher marginal costs of external financing. Figure 1 illustrates this prediction.

The model provides comparative statics on the sensitivity of investment to q and π. First, as stated in Proposition 1, investment is weakly convex in cash flows. Second, the sensitivity of investment to q is decreasing in θ; higher marginal costs of external financing reduce the optimal investment rate. Third, the sensitivity of investment to q is decreasing in α; this is consistent with Hayashi (1982) and Abel and Eberly (1994). Finally, and in contrast to Abel and Eberly (1994), the sensitivity of investment to q in the active region is decreasing in the fixed costs of investment f. When firms have to finance their investment costs with external funds, higher fixed costs of investment imply higher marginal costs of external funding.

The model predicts that fixed costs of investment induce a nondegenerate inaction region Θ in which firms find it optimal not to invest. When firms rely on external funds, I contribute to Abel and Eberly (1994) by showing how firms' marginal costs of external funding affect their willingness to invest. I make two additional assumptions on the parameter space to simplify the characterization of the inaction region of investment.

Assumption 1. $1-\alpha\delta+\alpha\left((\pi/K)-f\right)>0.$

Assumption 2. $(\pi/K)-f>2\left(\delta^2\alpha+f\right)-\delta.$

Assumption 1 ensures that the net funding requirement of the firm $\widetilde{X}$ always becomes 0 at some real value of firms' q. Assumption 2 ensures that the inaction region of investment for all equity-financed firms coincides with the inaction region in the model when firms do not rely on external funding. I prove these arguments in Appendix C.

Proposition 2. Given Assumptions 1 and 2, the inaction region $\Theta\equiv(q_1,q_2)$ is such that: i) $q_1<0$ and $q_2>0$; ii) when $\widetilde{X}\leq 0$, q_1 and q_2 are increasing in α and f such that $q_i=1-\delta\alpha\pm\sqrt{\delta^2\alpha^2+2f\alpha}$; iii) when $\widetilde{X}>0$, q_1 and q_2 are decreasing in π/K; iv) firms invest if $Q>q_2$, disinvest if $Q<q_1$.

Proof. See Appendix C. □

Proposition 2 characterizes the impact of financing frictions on the inaction region of investment. When firms rely on internal funds, the inaction region is constant over time; the thresholds q_1 and q_2 do not depend on firms' cash flows π and marginal costs of external funding H_X. Conversely, when firms rely on external financing, the inaction region shifts each period because it also depends on π and H_X. Firms are more reluctant to invest when their marginal costs of funding are high and their cash flows are low; the threshold q_2 is decreasing in

π. Figure 1 illustrates that firms have wider inaction regions of investment when they rely on external financing.

As an additional result, the model predicts that firms' financing frictions *magnify* the effect of an increase in investment costs on the thresholds q_1 and q_2. When firms rely on internal funding, increases in either the curvature of the adjustment costs of investment α or the fixed costs of investment f make firms more reluctant to reinitiate investment. When firms rely on external funding, an increase in α or f also increases firms' marginal cost of external financing, which in turn enhances the negative marginal effect of an increase in α or f on the willingness of firms to invest.

C. Net External Issues

Given an investment motive to raise external funds, the q-theory applies to both investment and financing decisions. My model characterizes firms' external financing requirements as a residual claim; I predict how firms' external financing requirements depend on q using equations (4), (5), and (8). The net external financing policy of the firm is given by

$$\frac{\widetilde{X}}{K} = \begin{cases} \frac{1}{2\alpha}\left(\frac{q}{H_X}\right)^2 + \delta + f - \frac{\pi}{K} - \frac{1}{2\alpha} & \text{if } q \notin \Theta, \\ -\frac{\pi}{K} + \frac{\alpha\delta^2}{2} & \text{if } q \in \Theta. \end{cases} \tag{9}$$

Proposition 3 summarizes the main properties of the net external financing policy in equation (9) as a function of average Q and cash flows.

Proposition 3. Firms' net external financing policy is such that: i) $\widetilde{X}/K$ depends jointly on $\pi/K\ \forall I$ and Q if and only if $Q \notin \Theta$; ii) $\widetilde{X}/K$ is increasing in $Q \forall Q \notin \Theta$, concave in Q if $X > 0$, and convex in Q otherwise; and iii) $\widetilde{X}/K$ depends mechanically on $\pi/K\ \forall I$ and $\forall Q$.

Proof. See Appendix D. □

The predictions of the model for net external financing policies are similar to the implications for investment policies. First, average Q and firms' cash flows π are both necessary to explain firms' net external financing issues $\widetilde{X}$. Firms' net external issues are significantly related to Q only if Q is sufficiently high. In the inaction region of investment, $\widetilde{X}$ is insensitive to Q, and firms may either distribute dividends or raise external funds to cover for low cash flows. Conversely, when firms are actively investing, $\widetilde{X} > 0$ is sensitive to Q, weakly concave in Q, and weakly convex in cash flows.

III. Econometric Methodology

The predictions of the model in Section II can be assessed in multiple ways. I first present the proposed estimation approach with average Q as the dependent variable. I then compare the econometric methodology to previous empirical studies on investment equations, in which the ratio of investment to capital is used as the dependent variable.

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A. Active Region

In the active region, the optimal rule in equation (7) implies that

$$q = \left[1+\alpha\left(\frac{I}{K}-\delta\right)\right]\times\left[1+\theta\mathbf{1}_t^X\left(\frac{\widetilde{X}}{K}\right)\right], \tag{10}$$

where q is empirically unobservable. Although Lemma 1 indicates that average Q is a good proxy of q, the measurement error literature suggests that there exists a significant wedge between q and Q. Blanchard, Rhee, and Summers (1993) explain that the wedge between q and average Q may relate to stock market inefficiencies or the divergence between intrinsic values and accounting reports due to accounting procedures.

I apply equation (7) and the acknowledgment that q is measured with error to equation (10) to obtain the testable implication that

$$Q = Z'\zeta+\upsilon, \tag{11}$$

where the vector of explanatory variables $Z'\zeta$ in equation (11) summarizes in stack form the set of regressors that explain q, and the error term υ captures the wedge between Q and q in the data. Consistent with Erickson and Whited (2000), I assume that the error term υ is assumed normally distributed and orthogonal to q with nonzero mean $\kappa>0$ and variance σ_υ^2. I assume homoskedasticity and time independence in υ for notational simplicity.

The model provides two alternative empirical specifications for the vector $Z'\zeta$. The first specification stems directly from multiplying the two factors on the right-hand side of equation (10). This specification of the vector $Z'\zeta$ uses investment, the observable net issuance of external funding, and the product of these as explanatory variables, such that

$$Z'\zeta = \zeta_0+\zeta_1\frac{I}{K}+\zeta_2\frac{\widetilde{X}}{K}+\zeta_3\frac{\widetilde{X}}{K}\frac{I}{K}, \tag{12}$$

where the coefficients in equation (13) are explicit functions of the parameters of the model (i.e., α, θ, δ, and f). I define the coefficients ζ_0 to ζ_3 in Appendix E. The specification of the vector $Z'\zeta$ in equation (12) is convenient for the sake of empirical tests because the variable $\widetilde{X}$ is observable and does not impose any specific definition of the budget constraint of the firm. For instance, although the model assumes that firms always distribute dividends when $X\leq 0$, in practice, firms may also repurchase shares, retain earnings, or cancel debt obligations.

An alternative specification of the vector $Z'\zeta$ is obtained by replacing the variable $\widetilde{X}\equiv X-b$ in equation (12) using the definition of X in equation (4). This yields a characterization of the optimal investment rule in terms of Q and π. The corresponding specification of the vector $Z'\zeta$ includes investment, the ratio of cash flows over capital, and the product of these as explanatory variables, such that

$$Z'\zeta = \zeta_0+\zeta_1\frac{I}{K}+\zeta_2\left(\frac{I}{K}\right)^2+\zeta_3\left(\frac{I}{K}\right)^3+\zeta_4\frac{\pi}{K}+\zeta_5\frac{\pi}{K}\frac{I}{K}, \tag{13}$$

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where the coefficients in equation (13) are explicit functions of the parameters of the model and are defined in Appendix E. Using the specification of the vector $Z'\zeta$ in equation (13), I characterize the marginal effect of an increase in firms' cash flows on the probability of investment.

B. Inaction Region

The testable implication in equation (11) applies exclusively to the active region of investment. It does not take into account the impact of the fixed costs of investment on the optimal investment rule. In this model, a firm invests if its marginal product of capital is sufficiently high. The corresponding implication is that the probability that a firm invests at any time is equal to

$$\Pr(q \geq q_2) \quad = \quad 1 - \Phi\left(\frac{q_2 - \kappa - Z'\zeta}{\sigma_\upsilon}\right), \tag{14}$$

where $q_2 > 0$ is defined as in Proposition 1, and Φ is the cumulative normal distribution function. Consistent with probit models, I derive equation (14) by noting that the probability that a firm invests given $q \geq q_2$ is equal to the probability that $Z'\zeta + \upsilon > q_2$, where the error term υ is normally distributed with mean κ and volatility σ_υ.

The probability that a firm disinvests if $q \leq q_1$ is computed in a similar way. However, the evidence presented in Section III shows that firm-level investment is both infrequently negative and infrequently equal to 0 in the CRSP/Compustat data set. This implies that the probability of disinvestment is not significantly different from 0. I incorporate this observation into the empirical strategy by making an additional assumption.

Assumption 3. Firms are subject to a fixed cost of investment f^+ and a fixed cost of disinvestment f^-, such that $f^+ \neq f^-$, $f^+ > 0$, and $f^- \longrightarrow \infty$.

In the model, investment is nonnegative under the working assumption that firms are subject to infinitely large fixed costs of disinvestment. Assumptions of asymmetric fixed costs of investment are standard in studies on irreversible investment, such as that by Abel et al. (1996). With infinitely large fixed costs of disinvestment, the optimal investment rule is such that $I \geq 0$ if $q \geq q_2$, and $I = 0$ otherwise. This implies that investment is nonnegative and positively skewed.

C. Log-Likelihood Function

I derive the conditional density of observing the average Q of a firm in my sample as the density of the optimal investment rule in the active region as in equation (11), divided by the conditional density of observing positive investment according to equation (14). The corresponding log-likelihood function resembles that of the truncated regression models regularly studied in economics textbooks such as that by Hayashi (2000).

Proposition 4. Given Assumption 3, the optimal investment rule in equation (7) implies that conditional mean of Q in the active region of investment is given by

$$\mathrm{E}(Q \mid q > q_2) \quad = \quad Z'\zeta + \kappa + \sigma_\upsilon \times \frac{\phi\left(\frac{\Delta - Z'\zeta}{\sigma_\upsilon}\right)}{1 - \Phi\left(\frac{\Delta - Z'\zeta}{\sigma_\upsilon}\right)}, \tag{15}$$

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and the corresponding log-likelihood function $\ln L(\zeta, \Delta | Z)$ equals

$$\ln L(\zeta, \Delta | Z) = \ln\left(\phi\left(\frac{Q-\kappa-Z'\zeta}{\sigma_\upsilon}\right)\right) - \ln(\sigma_\upsilon) - \ln\left(1-\Phi\left(\frac{\Delta-Z'\zeta}{\sigma_\upsilon}\right)\right), \tag{16}$$

where $\Delta \equiv q_2 - \kappa$ is the truncation threshold, ϕ is the normal distribution density, $Z'\zeta$ is defined as in equation (13), and Φ is the cumulative normal distribution function.

Proof. See Appendix E. □

Equation (15) summarizes how the investment frictions and the measurement error in q affect firms' optimal investment policies for the sake of empirical tests. The first term in equation (15) captures that investment is not a sufficient statistic for investment when firms are subject to convex costs of external financing. The second term in equation (15) allows for measurement error in q. The third term in equation (15) characterizes the upward bias that arises from considering a sample in which investment is infrequently negative. The conditional mean of Q is a nonlinear function of I/K, π/K, κ, σ_υ, and the truncation threshold Δ.

The log-likelihood function in equation (16) provides a tool to test the first-order condition of investment of a q-theory model with measurement error in q, adjustment costs, fixed costs of investment, and convex costs of external financing. The first two terms relate to the investment rule in the active region of investment. The last term controls for the fact that a significant relation between q and investment is not observed unless q exceeds the threshold q_2. I use the log-likelihood function in equation (16) to estimate the frictions α, θ, and f at the firm level, as well as the measurement error parameters κ and σ_υ.

D. Marginal Effects

Equation (15) implies that neglecting the empirical fact that investment is positively skewed affects my inferences on corporate investment. I elaborate on this argument analytically by considering the marginal effects of I/K and π/K on the optimal investment rule.

Corollary 1. The marginal effect of I/K on the conditional mean of Q in equation (15) is given by

$$\frac{\partial \mathrm{E}(Q | q > q_2)}{\partial \frac{I}{K}} = \left(\zeta_1 + 2\zeta_2\frac{I}{K} + 3\zeta_3\left(\frac{I}{K}\right)^2 + \zeta_5\frac{\pi}{K}\right) \times (1-\eta(x)) > 0, \tag{17}$$

where the vector $Z'\zeta$ is defined as in equation (13), and I derive $\eta(x)$ in Appendix F, with $x \equiv (\Delta - Z'\zeta)/\sigma_\upsilon$ and $0 < \eta(x) < 1$ for all values of x.

Proof. See Appendix F. □

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Corollary 1 characterizes the sensitivity of investment to Q in the model for the sake of empirical tests. The first factor in equation (17) indicates that investment is concave in Q when firms are subject to convex costs of external funding and that the availability of cash flows affects the sensitivity of investment to Q. This is the testable implication of Proposition 1 for the active region of investment. The second factor in equation (17) shows how the effect of the truncation in Q biases my inference about the sensitivity of investment to Q. Given that the statistic $\eta(x)$ is by construction between 0 and 1, the positive skewness in average Q leads to an attenuation bias in the estimation of the coefficients of the active region of investment.

Corollary 2. The marginal effect of π/K on the probability that a firm invests is given by

$$\frac{\partial \Pr(q \geq q_2)}{\partial \frac{\pi}{K}} = -\frac{\phi(x)}{\sigma_\upsilon} \times \mathbf{1}^X \left[\frac{\partial q_2}{\partial \frac{\pi}{K}} + \theta \left(1 - \alpha\delta + \alpha \frac{I}{K} \right) \right] \gtrless 0, \tag{18}$$

where ϕ is the normal distribution density, $x \equiv (\Delta - Z'\zeta)/\sigma_\upsilon$, and $Z'\zeta$ is defined as in equation (13).

Proof. See Appendix F. □

Kaplan and Zingales (1997) show that the sign of the investment–cash flow sensitivity predicted by a static neoclassical investment model depends jointly on the marginal costs of investment and the marginal costs of external funding. As in their paper, I find that the sign of the investment–cash flow sensitivity in the active region of investment depends on the relative magnitude of firms' marginal costs of investment and financing and also on the level of q. Corollary 2 complements the work of Kaplan and Zingales (1997) because it characterizes how firms' cash flows affect the likelihood that a firm invests.

The sign of a marginal increase in cash flows on the probability of investment is driven by the factor in square brackets in equation (18). When average Q is close to the investment threshold, the effect of an increase in cash flows on the probability of investment is positive. If the firm is close to its inaction region and requires external funding to invest, an increase in its cash flows reduces the threshold q_2 and increases the probability of investment. This is the testable implication of the result in Proposition 2 that the threshold q_2 decreases with firms' cash flows.

In contrast, if the firm is actively investing and its q is relatively high, the effect of an increase in cash flows on the probability of investment reverts. The rationale for this prediction relates to the fact that an increase in cash flows motivates firms to invest, which effectively also increases the marginal adjustment costs of the firm. The marginal costs of the firm are observed in the last term in the second factor of equation (18).

E. Comparison with Alternative Estimation Methods

My structural approach differs from that of previous studies on investment equations in three main aspects. First, the empirical strategy uses investment as

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an explanatory variable, whereas the literature regularly sets the ratio of investment to capital as the dependent variable. Second, my study considers a nonlinear investment rule and controls for measurement error in q at the same time, using a single equation. Previous studies have focused on a nonlinear investment rule without controlling for measurement error. Alternatively, the literature proposes systems of equations controlling for measurement error in an otherwise linear investment equation. Third, the structural estimation uses maximum likelihood (ML) instead of the commonly used generalized method of moments (GMM) approach.

I begin by restating the predictions of the model in a system of equations. The first equation of the system uses investment as the dependent variable. It indicates that the ratio of investment to capital is a nonlinear function of q and π/K given the optimal investment rule in equation (8). I denote by m a nonlinear function of q and π/K such that $m_q = 0$ if $I = 0$; $m_q > 0$ if $I \neq 0$; $m_{qq} < 0$, $m_{\frac{\pi}{K}} > 0$ and $m_{\frac{\pi}{K}\frac{\pi}{K}} > 0$ if $X < 0$; and $m_{qq} = 0$ and $m_{\frac{\pi}{K}} = 0$ if $X \leq 0$. The corresponding investment equation is given by

$$\frac{I}{K} = m\left(q, \frac{\pi}{K}\right). \tag{19}$$

The second equation of the system is the measurement error equation. The purpose of this equation is to restate the unobservable marginal product of capital in equation (19) in terms of observables. The equation acknowledges that the unobservable q in equation (19) is proxied with error by average Q such that

$$q = Q + \upsilon, \tag{20}$$

where the error term υ equals the wedge between q and Q that is not explained by the model, and it is normally distributed and orthogonal to q, with nonzero mean $\kappa > 0$ and variance σ_υ^2. The orthogonality between q and the error term υ is a common assumption in the measurement error literature.

Equation (19) differs from the benchmark OLS regression given by Hayashi (1982) in several ways. Hayashi (1982) assumes that investment is linear in average Q and that Q is a sufficient statistic for investment. Equation (19) indicates that investment is nonlinear in Q and that it also depends on firms' cash flows. Second, Hayashi's (1982) model does not incorporate fixed costs of investment; the empirical tests of Hayashi (1982) assume that the sensitivity of investment to Q is significantly positive in every period.

The investment equation of my paper is also different from that given by Erickson and Whited (2000). Erickson and Whited (2000) consider a system of equations in which the investment equation is linear in q and cash flows, and the measurement error in q is also linear. In contrast, I focus on an investment equation that is nonlinear in q in cash flows, and I assume a linear measurement error term. The analysis by Erickson and Whited (2000) also builds on the working assumption that firms invest actively in every period.

My empirical strategy relates to that of Erickson and Whited (2000) to the extent that I account for measurement error in q. Because I consider Q as the dependent variable, I spare the use of a system of equations to estimate the model.

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As noted by Arellano (2003), the linear measurement error in q does not affect the consistency of the estimation; it only affects its efficiency. In contrast, Erickson and Whited (2000) consider a system of equations with investment as the dependent variable. Their methodology avoids the use of instruments by considering higher-order moments.

Barnett and Sakellaris (1998) estimate a neoclassical investment model with fixed costs in which firms may optimally decide not to invest. Their strategy relies on the GMM estimation of the unobservable investment thresholds delimiting three alternative regimes: an investment regime, an inaction region, and a disinvestment regime. The approach assumes that firms are all-equity financed such that the investment thresholds are constant over time; it also assumes no measurement error in q. When applied to CRSP/Compustat firms, a major caveat is that the methodology implicitly requires that the three alternative regimes are effectively observed in the working sample. However, CRSP/Compustat firms are usually large companies with good access to securities markets, such that their q is unlikely to fall in the inertia region.

Finally, my approach uses ML, whereas several studies on investment equations use GMM. With the exception of the work of Barnett and Sakellaris (1998), none of these studies using GMM considers a dependent variable with a truncated distribution. The use of ML to estimate truncated dependent variables is discussed in econometric textbooks such as that by Hayashi (2000). In contrast, the use of GMM to estimate models with truncated dependent variables is uncommon. Newey (2004) provides a semiparametric GMM estimator for models with truncated dependent variables. As discussed by Hansen and Singleton (1982), the main results of the estimation should remain the same using either ML or GMM.

IV. Empirical Results

A. Working Sample

The working sample is drawn from the merged CRSP/Compustat database and considers the 1980–2014 period for 5,502 U.S. public firms and 69,555 firm-year observations.[9] The average number of firms in a given year is 2,046. The sample is filtered for missing data, or for observations where total assets, the gross capital stock, or sales are either 0 or negative. All firms with less than 5 consecutive years of accounting data are not included. I also delete any firm that experienced a merger accounting for more than 15% of the book value of its assets. Consistent with the practice in asset pricing literature, I include all industry groups in the CRSP/Compustat data set.

I follow Barnett and Sakellaris (1998) and consider investment I as the difference between capital expenditures and sales of property, plant, and equipment. This stands as the difference in CRSP/Compustat items CAPX and SPPE. Total depreciation corresponds to item DP. The gross capital stock is item PPEGT. The replacement value of capital K is computed using the perpetual inventory method, which sets K equal to PPEGT for the first firm-year observation, and then for the

[9] My data set starts in 1980 because I use long-term credit ratings data that are available only starting in such year.

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remaining observations of the same firm, the subsequent values of K are adjusting using the annual deflator for nonresidential investment reported by the Bureau of Economic Analysis (BEA). The depreciation rate δ equals DP over the capital stock K.

Concerning the remaining accounting variables used in the estimation, cash flows π equal the earnings before interest, taxes, depreciation, and amortization (EBITDA) of the firm. The market value of equity S is the product of item PRCC_F times item CSHO. Net debt issuance is the difference between item DLTIS and item DLTR. Net equity issuance is the difference between item SSTK and item PRSTKC. X captures changes in external funding issues and is the sum of net debt issuance and net equity issuance. $\widetilde{X}$ captures external funding net of debt interest expenses and is computed as X minus the interest expense XINT. Total long-term debt B is the sum of long-term debt as reported in item DLT and short-term debt as reported in item DLC. Average Q is computed as the ratio of the market value of equity S, plus the book value of debt B, minus inventories INV, divided by the replacement value of capital. All variables are winsorized at 1%.

I construct the dummy variable HBLEV, which equals 1 if the book-to-leverage ratio of the firm is in the highest 25% of the distribution in the sample. I also use long-term credit ratings data to construct a dummy variable that I denote NCRAT. NCRAT equals 1 if the firm has no credit rating, and 0 otherwise. I use HBLEV and NCRAT as controls in the structural estimation.

I also construct a variable named CPINT, which is equal to 1 minus the labor intensity of the industry in which a firm operates. The labor intensity of the industry is the ratio of payroll to value added by 4-digit Standard Industrial Classification (SIC) industry. I construct the labor intensity variable using information reported by the BEA for nonmanufacturing industries and data from the Annual Survey of Manufacturers (ASM) for manufacturing industries. CPINT proxies for the capital intensity of the industry under the assumption of constant returns to scale embedded in the model.

Table 1 provides the summary statistics of the working sample. Both the ratio of investment to capital and average Q are highly skewed, with skewness of

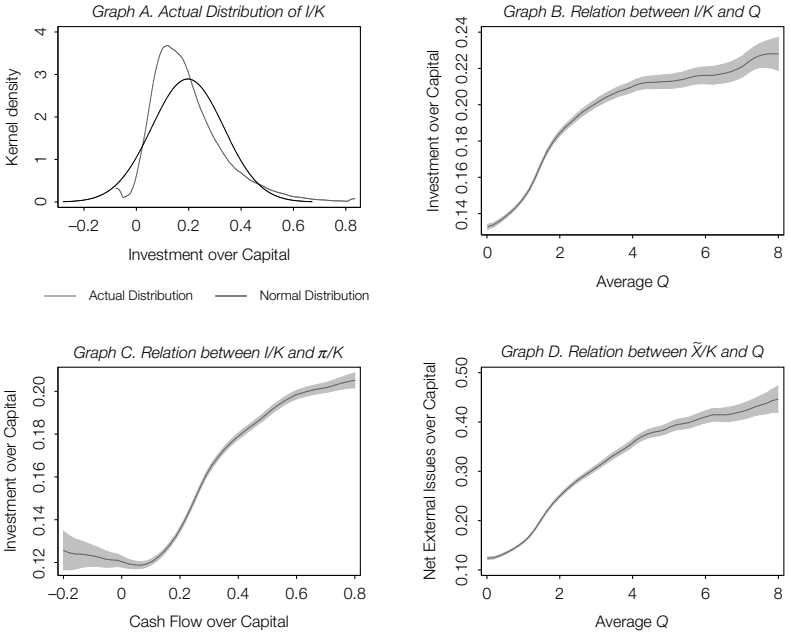
TABLE 1
Summary Statistics

Table 1 reports the distribution of firm characteristics in the working sample. I/K is the ratio of investment to capital, Q is the ratio of the market value of assets to the replacement value of capital, π/K is cash flows over capital, $\widetilde{X}/K$ is net external issues over capital, B/K is book leverage, δ is depreciation rate, HBLEV is a high-book-leverage dummy, NCRAT equals 1 if the firm has no long-term debt credit rating, and CPINT is the capital intensity of the industry in which the firm operates.

			Percentile						
Variable	Mean	Std. Dev.	P5	P25	P50	P75	P95	Skew	*N*
I/K	0.196	0.139	0.027	0.101	0.170	0.262	0.465	1.197	69,555
Q	2.148	2.410	0.354	0.727	1.226	2.517	7.433	2.477	69,555
π/K	0.446	0.558	−0.055	0.168	0.330	0.611	1.393	1.843	69,555
$\widetilde{X}/K$	0.116	0.703	−0.650	−0.062	0.087	0.302	0.975	0.434	69,555
B/K	0.326	0.244	0.000	0.131	0.297	0.480	0.801	0.580	69,555
δ	0.138	0.079	0.041	0.086	0.124	0.171	0.289	1.582	69,555
HBLEV	0.219	0.414	0.000	0.000	0.000	0.000	1.000	1.309	69,555
NCRAT	0.703	0.457	0.000	0.000	1.000	1.000	1.000	−0.884	69,555
CPINT	0.417	0.186	0.208	0.284	0.352	0.520	0.772	0.775	62,011

FIGURE 2
Nonparametric Empirical Analysis

Figure 2 elaborates on the distribution of investment and average Q in the working sample. Graph A compares the empirical distribution of investment with a normal density function with the same mean and standard deviation. The empirical distribution of investment is positively skewed. Graph B relates the empirical distribution of investment with that of Q. Investment is positively related to Q and concave in Q. Graph C relates investment to cash flows in my working sample. Investment is positively related to cash flows and nonlinear in cash flows. Graph D relates net external funding to Q in my working sample.



1.197 and 2.477, respectively. The mean ratio of investment to capital is 0.196, which is higher than the median of 0.170, and its standard deviation is 0.139. The mean Q is 2.148, which is higher than the median 1.226, and the standard deviation in 2.410. Graph A of Figure 2 compares the empirical distribution of the ratio of investment to capital with the normal density function. Investment is highly skewed toward positive values and is rarely below 0. Only 2.16% of the observations in the working sample show negative investment, and only 0.09% of the observations show 0 investment. These results are consistent with the theoretical assumption that investment is infrequently 0 or negative.

B. Descriptive Tests

As a first step, I use nonparametric tests to investigate the sensitivity of investment to average Q and cash flows and the sensitivity of net external issues to average Q. Nonparametric tests do not impose any functional form on the relation between investment, net external issues, average Q, and cash flows. Graph B of Figure 2 plots a local, nonparametric polynomial smooth-kernel function of the ratio of investment to capital on average Q. Graph C of Figure 2 shows the nonparametric relation between the ratio of investment to capital and cash flows. Graph D of Figure 2 shows the nonparametric relation between net external

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issues and average Q. Consistent with the model, investment is concave in Q and weakly convex in cash flows. Net external issues are weakly concave in Q.

As a second step, I run reduced-form parametric tests. I focus on the null hypothesis that investment is concave in Q and weakly convex in cash flows, and I restate the nonlinear investment policy in equation (19) as a bivariate Taylor expansion of order 2 in Q and π, and I estimate investment as a linear function of average $Q, \pi/K$, and higher-order terms of these variables. Table 2 provides the corresponding empirical results. In a nutshell, I find that the explanatory power of the standard OLS regression of investment on average Q and cash flows increases significantly with higher orders of average Q as additional explanatory variables. Moreover, truncated regressions significantly improve the goodness of fit of the investment equations.

Column 1 in Table 2 shows the estimation results of the benchmark regression of investment on Q and cash flows. Column 2 incorporates cash flows, as well as the higher-order terms of Q and cash flows corresponding to the bivariate Taylor expansion of equation (19). The coefficient on Q is significantly higher in column 2 than in column 1, and the coefficient on Q^2 is negative and highly significant. This supports the prediction that investment is concave in Q. Columns 3 and 4 add firm fixed effects and lagged investment, respectively, with similar results. The reduction on the coefficient on Q in column 4 is consistent with Eberly, Rebelo, and Vincent (2012), who find that lagged investment is a good predictor of current investment.

In column 5 of Table 2, I estimate the investment equation of column 2 using a truncated regression. Dubinin and Vardeman (2003) show that when truncation points are unknown, the most efficient truncation point is the sample minimum. I set the threshold in column 5 to the minimum investment rate, −0.089.

TABLE 2
Reduced-Form Empirical Evidence

Table 2 provides the reduced-form tests of investment and net external issues as a function of Q. Columns 1 and 5 use investment over capital I/K as the dependent variable. Columns 6–8 use the ratio of net external issuance to capital $\tilde{X}/K > 0$ as the dependent variable. All regressions include year dummies. Standard errors are clustered at the firm level. Robust standard errors are reported in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

	Investment I_t/K_t					Net External Issues $\tilde{X}_t/K_t > 0$		
	1	2	3	4	5	6	7	8
Q_t	0.017***	0.035***	0.035***	0.025***	0.039***	0.085***	0.105***	0.115***
	(0.0006)	(0.0012)	(0.0007)	(0.0008)	(0.0013)	(0.0014)	(0.0028)	(0.0027)
Q_t^2		−0.001***	−0.002***	−0.001***	−0.002***		−0.001***	−0.001***
		(0.0001)	(0.0001)	(0.0001)	(0.0001)		(0.0001)	(0.0001)
π_t/K_t	0.029***	0.042***	0.020***	0.027***	0.048***	0.205***	0.122***	0.084***
	(0.0025)	(0.0040)	(0.0027)	(0.0019)	(0.0046)	(0.0070)	(0.0127)	(0.0132)
$(\pi_t/K_t)^2$		0.004***	−0.002***	−0.002***	0.004***		0.104***	0.091***
		(0.0012)	(0.0005)	(0.0006)	(0.0013)		(0.0122)	(0.0117)
$Q_t \times \pi_t/K_t$		−0.004***	−0.001**	−0.001**	−0.005***		−0.030***	−0.0021***
		(0.0006)	(0.0003)	(0.0003)	(0.0007)		(0.0050)	(0.0054)
I_{t-1}/K_{t-1}				0.301***				
				(0.0038)				
N	69,555	69,555	69,555	69,555	69,555	46,363	46,363	46,363
R^2	0.145	0.212	0.141	0.235	0.787	0.301	0.329	0.328
Adj. R^2	0.144	0.211	0.083	0.176	0.755	0.257	0.252	0.261

Column 5 shows that truncated regressions have higher explanatory power than OLS regressions. The adjusted R^2 goes from 0.145 in the benchmark OLS regression in column 1 to an adjusted R^2 of 0.755 in the truncated regression of column 5. Similarly, the coefficient on Q in column 5 is significantly higher than the corresponding coefficient in the OLS regression in column 1.

The model predicts that the q-theory applies to external financing policies. I consider reduced-form parametric tests to assess the relation between net external issues, average Q, and cash flows as predicted by the model. I restate the nonlinear net external issuance policy in equation (9) as a bivariate Taylor expansion of order 2 in Q and π, and I estimate net external issues as a linear function of average Q and π/K and higher-order terms of Q and π/K. I report the corresponding findings in columns 6–8 of Table 2.

Column 6 in Table 2 shows the results of a panel regression of investment on average Q and cash flows. Column 7 estimates the bivariate Taylor expansion of equation (9) using OLS. The adjusted R^2 of these regressions ranges between 0.301 and 0.329. The coefficient on average Q is always positive and significant. The coefficient on Q^2 in column 7 is negative and significant, suggesting that net external issues are concave in average Q. Column 8 adds firm fixed effects to control for unobserved heterogeneity, with similar results.

C. Baseline Structural Estimation

The structural estimation of the model consists of estimating the closed-form log-likelihood function in Proposition 4 subject to the parameter constraints embedded in the coefficients of equation (12). As discussed in Section III, the specification of the explanatory variables of the optimal investment rule in equation (12) requires no explicit assumptions about how firms allocate their funds to estimate α and θ in the active region. I equate the depreciation rate δ to the median of the depreciation over capital ratios observed in the working sample.

I assume that the truncation threshold $\Delta \equiv q_2 - \kappa$ is constant and equal to the minimum value of Q in my sample when there is no equity issuance. This is consistent with the prediction in Proposition 2 that the investment threshold q_2 is constant when firms do not issue equity and the argument of Dubinin and Vardeman (2003) that when truncation points are unknown, the most efficient truncation point is the sample minimum. Once I identify the truncation point for the nonissuing firms, I solve endogenously for the corresponding investment threshold of the issuing firms in the sample. All else equal, the model predicts that the investment threshold for the issuing firms is firm-specific, time-varying, and strictly higher than the corresponding constant threshold of the nonissuing firms.

Last, the model assumes that debt is risk-free, whereas in practice, firms have the ability to default on their debt. Consistent with Hennessy (2004), I show in Appendix G that when debt is risky, there exists a wedge between q and average Q due to debt-overhang effects. I include two additional dummy variables to control for these effects. The dummy variable NCRAT is equal to 1 if the firm does not have a long-term debt credit rating. The dummy variable HBLEV equals 1 if the book leverage of the firm is above the 75th percentile of the leverage ratio in the working sample. As I prove in Appendix G, the use of these variables as controls is consistent with the predictions of the extended model; the wedge

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between q and average Q due to debt overhang equals 0 either when firms are unlevered or when debt is risk-free. The mean value of NCRAT is 0.703, and the standard deviation in 0.457. The mean value of HBLEV is 0.219, and the standard deviation is 0.414.

The baseline estimation considers all observations in my working sample. I report the corresponding results in Table 3. The estimated parameters α, f, θ, κ, and σ_υ are all significantly different from 0. The R^2 of the baseline structural estimation is 0.549.

The curvature of firms' adjustment costs of investment α equals 5.179. This estimate is closer to the lower bound of previous studies in the literature; the corresponding estimates for α range from over 20 (Hayashi (1982)) to as low as 3 (Gilchrist and Himmelberg (1995)). I also benchmark the finding by computing the average ratio of adjustment costs to capital and the average ratio of adjustment costs to cash flows in my working sample. I report the corresponding figures in Table 4. The average ratio of adjustment costs over capital is equal to 0.088%. The average ratio of adjustment cost to cash flows is equal to 0.133%. Bloom (2009) surveys the estimates of convex adjustment costs to be between 0% and 20% of cash flows.

The results in Table 3 show that firms' fixed costs of investment are equal to 2.5% of firms' capital. Cooper and Haltiwanger (2006) estimate fixed costs of investment to be larger and equal to 3.9% of capital using plant-level data from the U.S. Census of Manufacturers. I attribute the difference between my estimate and that of Cooper and Haltiwanger (2006) to the fact that the ratio of fixed costs to installed capital is presumably lower in large public firms than at the plant level in manufacturing industries. Another possible reason relates to the composition

TABLE 3
Baseline Structural Estimation

Table 3 reports the results of the baseline structural estimation of the model using all observations in the working sample from 1980–2014. The working sample includes all industry groups in the CRSP/Compustat data set. Robust standard errors are reported in parentheses. All variables are significant at the 1% level.

Description	Notation	Estimate
Curvature of adjustment costs	α	5.179 (0.2561)
Fixed cost of investment	f	0.025 (0.0016)
Curvature of external funding costs	θ	0.591 (0.0456)
Mean of measurement error	κ	1.068 (0.0527)
Dispersion of measurement error	σ_υ	2.735 (0.0063)
No-credit-rating dummy	NCRAT	−0.265 (0.0027)
High-book-leverage dummy	HBLEV	−0.162 (0.0294)
Depreciation rate	δ	0.124
Truncation threshold if no issuance	Δ	0.013
Sample size	N	69,555
R^2	R^2	0.549
Log-likelihood function	ln L	−146,228

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TABLE 4

Quantitative Implications of Baseline Structural Estimation

Table 4 reports the quantitative implications of the results of the baseline structural estimation. The implications for firms' costs of investment and financing in the active region are in line with previous studies in the literature. The marginal effect of cash flows on the probability of investment is evaluated at the 10th and 90th percentiles of the distribution of investment-to-capital ratios in the working sample.

Description	Notation	Estimate
Shadow price of capital	q	1.315
Marginal adjustment cost	C_I	0.376
Average adjustment cost over capital	$C(I,K)/K$	0.088
Average adjustment cost over cash flows	$C(I,K)/\pi$	0.133
Marginal cost of external funding	$H_X - 1$	0.145
Average cost of external funding	$H(X,K)/X - 1$	0.109
Probability of investment	$\Pr(Q > q_2)$	0.831
Marginal effect of I/K on $E(Q \mid Q > q_2)$	$\partial E(Q \mid Q > q_2)/\partial I/K$	2.853
Marginal effect of π/K on $\Pr(Q > q_2)$ at p_{10} of I/K	$\partial \Pr(Q > q_2)/\partial \pi/K\big\vert_{I/K=0.013}$	0.011
Marginal effect of π/K on $\Pr(Q > q_2)$ at p_{90} of I/K	$\partial \Pr(Q > q_2)/\partial \pi/K\big\vert_{I/K=0.329}$	−0.052

of my working sample, which includes firms operating in both manufacturing and nonmanufacturing industries.

The curvature of firms' costs of external financing θ equals 0.591 for all firms on average. I report the corresponding quantitative implications in Table 4. The mean value of the average marginal cost of external funding is 14.5%, the median is 5.11%, and the standard deviation is 2.86%. Using an alternative empirical approach, Hennessy and Whited (2007) report marginal costs of equity issuance between 5% and 11% of firms' capital. I also report an average ratio of costs of external financing to capital of 10.9% and an average ratio of cost of external financing to cash flows of 12.9%. These figures imply that, on average, firms forgo 12.9% of their operating cash flows to pay for the associated costs of external financing.

The coefficients on NCRAT and HBLEV in Table 3 suggest that firms with no credit rating and book leverage above the sample median have a significantly lower marginal product of capital. This is consistent with the finding by Hennessy (2004) that average Q overstates q due to debt-overhang effects. It is also aligned with the financial economics literature predicting that more financially constrained firms have lower investment rates and lower q.

The estimate for κ in Table 3 indicates that there exists a significant wedge between q and average Q due to measurement error. The estimate for κ in the baseline estimation equals 1.068. Similarly, whereas Table 4 shows that the predicted q equals 1.315 for all firms on average; the sample mean of average Q in Table 1 is significantly higher. These findings suggest that there exists significant measurement error in q.

D. Marginal Effects

The baseline structural estimation allows me to make inferences about the marginal effects of investment and cash flows on the optimal investment rule. The average marginal effect of an increase in I/K on the unconditional mean of Q is 2.853, which is strictly lower than the curvature of the adjustment costs of 5.179. The difference in point estimates indicates that the estimated investment rule is nonlinear. The estimated average marginal effect of investment on Q in my sample is dampened by the probability that the firm invests in a given period, and it is also

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affected by the nonlinear relation between investment and Q in the active region. The marginal effect of an increase in I/K on the conditional mean of Q is also strictly lower than the unconditional mean of Q. This illustrates how the positive skewness in Q leads to attenuation biases of the coefficients of OLS regressions.

Table 4 quantifies the marginal effect of an increase in cash flows on the probability that a firm invests. I evaluate the marginal effect at the 10th and 90th percentiles of firms' investment-to-capital ratio I/K, and I allow the ratio of cash flow to capital π/K to vary by firm-year. For low levels of investment (i.e., the 10th percentile of the distribution of I/K in my sample), a 1% increase in π/K increases the probability of investment by 1.10%. This suggests that firms are more financially constrained for low levels of q. In contrast, for high levels of investment (i.e., the 90th percentile of the distribution of I/K in my sample), an increase in cash flows reduces the probability of investment by 5.21%. When firms' q is more distant from the inaction threshold q_2, an increase in cash flows motivates them to invest more, which in turn increases their marginal costs of investment and reduces the likelihood of investment in the future.

Last, Table 4 reports that the average probability of investment for a firm in my working sample equals 83.1%. This indicates that the goodness of fit of the model is high but not ideal because the fraction of observations with nonnegative investment in the sample is higher.

E. Investment Frictions by Industry Group

Due to natural biases in focus, the macroeconomics literature has estimated the magnitude of firms' adjustment costs of investment assuming all-equity-financed firms. Similarly, the corporate finance literature has focused on quantifying firms' marginal costs of external financing. A natural concern is whether these practices are subject to estimation biases because firms' real and financing frictions are likely to be jointly determined by common latent variables such as the production technology.

I estimate firms' real and financing frictions jointly by industry group, using the same methodology as in the baseline estimation. The industry definition is based on the 12-industry groups proposed by Fama and French. Fama and French group 4-digit SIC codes into 12 industries. The 12-industry groups of Fama and French are posted on Kenneth French's online data library (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). The industry groups include both manufacturing and nonmanufacturing firms. I use the 4-digit SIC industry definition reported in the Compustat data set. The depreciation rate δ is set equal to the median depreciation rate of each Fama–French industry.

Table 5 reports the estimation results. I observe significant differences in the point estimates for α, f, and θ across Fama–French industry groups. The goodness of fit of the model varies across industries but remains significantly high in all cases. Table 6 reports the corresponding estimates for firms' q, average adjustment costs, and average costs of external financing by industry group. Table 6 also reports the corresponding sample means for I/K, Q, B/K, and the dummy NCRAT for the sake of comparison. The mean ratio of investment to capital ranges from 0.125 in Healthcare (group 8) to 0.219 in Energy, Oil, and Gas (group 4).

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TABLE 5

Structural Estimation for the Cross Section of Industries

Table 5 reports the results of the structural estimation for each of 12 Fama–French industries in the working sample. Robust standard errors are reported in parentheses. All variables are significant at the 1% level.

	Industry	α	f	θ	κ	σ_υ	N	R^2
1	Consumer Nondurables	2.562	0.034	1.451	0.936	2.628	5,257	0.590
		(1.062)	(0.007)	(0.395)	(0.094)	(0.798)		
2	Consumer Durables	1.638	0.036	2.148	0.874	1.995	1,780	0.662
		(0.401)	(0.016)	(0.236)	(0.132)	(0.607)		
3	Manufacturing	2.443	0.028	1.054	0.145	2.052	12,630	0.577
		(0.515)	(0.012)	(0.147)	(0.056)	(0.623)		
4	Energy, Oil, and Gas	0.723	0.002	0.975	0.092	1.068	5,283	0.589
		(0.223)	(0.001)	(0.170)	(0.023)	(0.325)		
5	Hi-Tech Equipment	6.471	0.029	0.417	0.455	2.698	9,808	0.624
		(0.711)	(0.013)	(0.068)	(0.150)	(1.122)		
6	Telecommunications	3.252	0.013	0.816	0.799	2.071	2,227	0.599
		(0.918)	(0.002)	(0.258)	(0.148)	(0.630)		
7	Retail Services	2.529	0.027	1.488	0.190	2.262	8,248	0.591
		(0.607)	(0.011)	(0.204)	(0.085)	(0.687)		
8	Healthcare	1.347	0.010	1.164	0.955	0.987	8,482	0.565
		(0.791)	(0.003)	(0.186)	(0.014)	(0.300)		
9	Utilities and Transportation	3.708	0.029	0.984	2.218	2.744	3,897	0.392
		(1.335)	(0.012)	(0.259)	(0.160)	(1.136)		
10	Financial Services	5.733	0.070	0.526	1.049	2.461	1,794	0.435
		(2.737)	(0.015)	(0.138)	(0.269)	(1.051)		
11	Chemicals	4.583	0.027	0.884	0.479	1.754	2,428	0.638
		(1.256)	(0.013)	(0.306)	(0.073)	(0.533)		
12	Other	3.905	0.022	0.891	0.944	1.732	7,718	0.582
		(0.593)	(0.004)	(0.155)	(0.103)	(0.829)		

TABLE 6

Quantitative Implications of Structural Estimation by Industry Group

Table 6 reports the quantitative implications of the structural estimation of firms' investment frictions by industry group. The table also reports summary statistics by industry group on the average investment-to-capital ratio I/K, the average Q, the average book leverage ratio B/K, and the share of firms with no credit rating by industry (average of NCRAT by industry).

	Description	$\frac{I}{K}$	Q	NCRAT	$\frac{B}{K}$	q	$\frac{C(I,K)}{K}$	$\frac{H(X,K)}{K}-1$
1	Consumer Nondurables	0.184	2.213	0.719	0.369	1.543	0.055	0.218
2	Consumer Durables	0.202	2.025	0.722	0.378	1.814	0.051	0.341
3	Manufacturing	0.173	1.721	0.719	0.336	1.395	0.046	0.160
4	Energy, Oil, and Gas	0.218	1.167	0.620	0.233	1.095	0.015	0.089
5	Hi-Tech Equipment	0.270	3.398	0.835	0.268	2.189	0.132	0.119
6	Telecommunications	0.203	1.890	0.509	0.346	1.577	0.040	0.091
7	Retail Services	0.211	1.877	0.740	0.329	1.383	0.058	0.209
8	Healthcare	0.125	1.012	0.525	0.366	1.045	0.021	0.062
9	Utilities and Transportation	0.208	3.007	0.773	0.332	2.282	0.074	0.229
10	Financial Services	0.178	3.286	0.710	0.397	2.606	0.164	0.136
11	Chemicals	0.169	1.860	0.542	0.344	1.825	0.052	0.102
12	Other	0.202	2.289	0.751	0.326	1.651	0.092	0.137

The average Q ranges from 1.012 in Healthcare (group 8) to 3.399 in Hi-Tech Equipment (group 5).

The results in Table 6 are fairly consistent with economic intuition. The Energy, Oil, and Gas sector (group 4) has the lowest average adjustment cost and also the lowest cost of external financing of all groups. The Hi-Tech Equipment sector (group 5) shows high average adjustment costs and median average costs of external financing due to its moderate leverage ratio. The fraction of firms with-

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out credit rating (NCRAT) is the highest in this sector, suggesting more limited access to debt financing. The Financial Services sector (group 10) has a high average cost of external financing, and yet the book-to-leverage ratio for this group is also the highest in the sample and equal to 0.397 on average. The median cost of external financing of financial services is 0.077.

The cross-sectional differences in $\alpha, \theta, f, \kappa$, and σ_v by Fama–French industry group in Table 6 suggest that firms' real and financing frictions are significantly related to their production technology. I test this hypothesis by regressing the estimates for firms' marginal and average costs of investment and external financing on the capital intensity of the industry in which firms operate. The mean of the variable CPINT in my sample is 0.417, the median is 0.352, and its standard deviation is 0.186. Given the changes in the SIC industry definition during the sample period being considered, I run Fama–MacBeth (1973) regressions to focus on the cross-sectional differences across industry groups. Because CPINT varies by 4-digit SIC, I run the tests at the 4-digit SIC level.

The evidence in Table 7 suggests that the technological heterogeneity across industries leads to significant cross-sectional variation in firms' real and financing costs of investment. Firms' marginal adjustment costs of investment and average adjustment costs of investment are positively related to CPINT. In untabulated tests, I also find that the point estimate for firms' fixed cost of investment f is positively and significantly related to CPINT at the 1% level. Both variable and fixed costs of investment are higher in more capital-intensive industries. Moreover, Table 7 shows that firms' costs of external financing are significantly related to CPINT. Firms in more capital-intensive industries with better collateral have lower marginal and average costs of external funding. These findings are consistent with Campello and Giambona (2013), who find that firms in industries with more salable assets have better access to credit markets.

TABLE 7

Relation between Investment Costs and the Capital Intensity of the Industry CPINT

Table 7 reports Fama–Macbeth (1973) cross-sectional regressions on the relation between firms' marginal costs of investment C_I, average costs of investment $C(I,K)/K$, marginal costs of external financing H_X, and average costs of financing $H(X,K)/K-1$ with the capital intensity of the industry in which firms operate. The capital intensity of the industry is captured by the variable CPINT. Robust standard errors are reported in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

	C_I	$C(I,K)/K$	H_X-1	$H(X,K)/K-1$
CPINT	0.095***	0.068***	−0.042*	−0.051**
	(0.023)	(0.005)	(0.019)	(0.015)
Constant	0.106***	0.024***	0.304***	0.229***
	(0.011)	(0.002)	(0.014)	(0.010)
Average R^2	0.019	0.022	0.017	0.021
N	10,931	10,931	10,931	10,931

V. Conclusion

In this paper, I provide an alternative approach to testing investment equations that puts together the implications of the q-theory for realized investment and also for the probability that investment occurs. The approach integrates predictions in the literature about how investment frictions and measurement error in

q affect the performance of the q-theory. More importantly, the analysis motivates the reinterpretation of the predictions of the q-theory on the inaction region for the sake of empirical tests.

The empirical findings in this paper have practical implications for applied research. The explanatory power of the benchmark OLS regression of investment on Q and cash flows improves significantly by adding the square of average Q as an additional regressor, and using truncated regressions controls for the fact that realized investment of CRSP/Compustat firms is usually positive. The measurement of the marginal effect of firms' cash flows on the probability of investment is an alternative way to identify how higher costs of financing prevent firms from investing. Finally, the significant differences in investment frictions by industry group should be considered in the calibration of investment and asset pricing models.

The proposed structural approach can be extended in multiple ways. For instance, I derive the log-likelihood function under the assumption of frictionless labor markets; a potential extension of my analysis could involve incorporating labor frictions and quantifying the effect of labor frictions on investment rates and the probability of investment. Similarly, my structural approach can be extended to estimate more specific financing frictions on debt and equity issues, or frictions determining the composition of debt. I believe that these are all useful avenues for future research.

Appendix A. Proof of Lemma 1

The derivation is similar to that given by Hennessy et al. (2006). I define the Dynkin operator $A(f)$ on a twice-differentiable function f as

$$A(f) = dKf_K + \mu(\epsilon) f_\epsilon + \frac{\sigma(\epsilon)^2}{2} f_{\epsilon\epsilon}, \tag{A-1}$$

such that the Bellman equation for the market value of equity S equals

$$rS = \max_I -H + A(S). \tag{A-2}$$

The Bellman equation holds pointwise, implying that partial derivatives of both sides of equation equation (A-2) must match. Differentiating equation equation (A-2) with respect to K, and applying the optimality condition on I, it follows that

$$KA(q) - \delta qK = A(qK) - qI = A(qK) - H_X(I + C_I I).$$

Rearranging terms and invoking the properties of functions π, C, and H noted in Section II, I obtain

$$rqK = -H + b + A(qK). \tag{A-3}$$

Subtracting equation (A-3) from equation (A-2) and rearranging terms, I obtain

$$A(S - qK) - r(S - qK) = b.$$

I define the Ito process ξ as

$$\xi_t \equiv e^{-rt}(S_t - K_t q_t),$$

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such that the application of Ito's lemma to ξ yields

$$d\xi_t \equiv e^{-rt} b\,dt + e^{-rt}\sigma(\epsilon)(S_\epsilon - Kq_\epsilon)\,dW_t.$$

Because the firm is has an infinite life, I integrate ξ up to infinity, and I take expectations such that

$$\lim_{T\to\infty} E\left[e^{-rT}(S_T - K_T q_T)\right] = S_t - K_t q_t + E\left[\int_t^\infty e^{-rs} b\,ds\right] + E\left[\int_t^\infty e^{-rs}\sigma(\epsilon)(S_\epsilon - Kq_\epsilon)\,dW_s\right].$$

Given the transversality conditions $\lim_{T\to\infty} E\left[e^{-rT}S_T\right]=0$ and $\lim_{T\to\infty} E\left[e^{-rT}q_T\right]=0$, the left-hand side of the previous expression equals 0. I also note that the last term on the right-hand side is a martingale with expectation 0. Hence, the expression simplifies to

$$q_t K_t = S_t + E\left[\int_t^\infty e^{-rs} b\,ds\right].$$

Denote by B_t the market value of debt. It then follows that $Q_t \equiv (S_t + B_t)/K_t$ equals q_t.

Appendix B. Proof of Proposition 1

Given equations (4) and (8), in the active region investment, the total derivative of I/K yields

$$d\frac{I}{K} = \frac{\partial \frac{I}{K}}{\partial q} dq + \frac{\partial I}{\partial \frac{\pi}{K}} d\frac{\pi}{K}\mathbf{1}^X. \tag{A-4}$$

Given equations (4) and (8), the sensitivity of I/K to q is such that

$$\frac{\partial \frac{I}{K}}{\partial q} = \left[\alpha H_X + H_{XX}\left(\frac{q}{H_X}\right)^2\right]^{-1} > 0, \tag{A-5}$$

$$\frac{\partial^2 \frac{I}{K}}{\partial q^2} = -\alpha H_{XX}\left(\frac{\partial \frac{I}{K}}{\partial q}\right)^2\left(1 + 2\frac{\partial \frac{I}{K}}{\partial q}\right)\mathbf{1}^X \leq 0, \tag{A-6}$$

where equation (A-6) equals 0 if $X \leq 0$ and is strictly negative if $X > 0$. Given equations (8) and (4), the sensitivity of I/K to π/K is such that

$$\frac{\partial \frac{I}{K}}{\partial \frac{\pi}{K}} = \frac{\theta \mathbf{1}^X}{\alpha H_X} \geq 0, \tag{A-7}$$

$$\frac{\partial^2 \frac{I}{K}}{\partial \left(\frac{\pi}{K}\right)^2} = \frac{\theta \mathbf{1}^X}{\alpha H_X^2}\frac{\partial \frac{I}{K}}{\partial \frac{\pi}{K}} \geq 0. \tag{A-8}$$

Appendix C. Proof of Proposition 2

The derivation of the investment thresholds q_1 and q_2 follows Abel and Eberly (1994) for the extended case of my model. The case of their paper is a special case of my model when firms do not rely on external funding and $X \leq 0$. Because this case is simpler, I begin by considering the case in which $X \leq 0$ and then move forward to the case in which $X > 0$.

Assume first that $X \leq 0$ such that $\mathbf{1}^X = 0$. This assumption holds for a specific range of values of q in my model, which I will characterize later on in this proof. For now, I follow

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Abel and Eberly (1994) and note that to derive the optimal investment rule in the active region, the only relevant terms of the Bellman equation in equation (A-2) are those that include I. The optimal investment rule in the active region thus maximizes the function

$$\Psi(q,K) \equiv \max_{I}\left[(q-1)I - C(I,K)\right],$$

where Ψ captures the rewards to investment for shareholders. Replacing by the specific functional form of $C(I,K)$ and applying the optimal investment policy in the active region, I obtain that the maximand Ψ is quadratic and homogeneous of degree 1 in K such that

$$\Psi(q,K) = \frac{1}{2\alpha}(q-1)^2 + \delta(q-1) - f. \tag{A-9}$$

The firm invests $I \neq 0$ when the rewards to investment Ψ are greater than 0. Hence, the roots of the maximand in equation (A-9) yield the investment thresholds q_1 and q_2. For the case in which $X \leq 0$ such that $\mathbf{1}^X = 0$ in my model, the investment thresholds q_1 and q_2 are the same as those of Abel and Eberly (1994) such that

$$q_i = 1 - \delta\alpha \pm \sqrt{\delta^2\alpha^2 + 2f\alpha}. \tag{A-10}$$

The thresholds in equation (A-10) are constant for all firms.

Consider now the more general case in which firms may raise external funding to invest. Following Abel and Eberly (1994), I first note that to solve the maximization problem on the optimal investment policy, the only relevant terms of the Bellman equation in equation (A-2) are those that include I. As a result, to obtain the thresholds q_1 and q_2, I define the maximand Ψ reflecting the rewards to investment as

$$\Psi(q,\pi,K) = \max_{I}(q-1)I - C(I,K) - \mathbf{1}^X\frac{\theta}{2}K\left[\left(\frac{\widetilde{X}}{K}\right)^2 - \left(\frac{\pi}{K}\right)^2\right], \tag{A-11}$$

where the last term incorporates the terms in the Bellman equation that relate to the costs of external funding and also depend on investment.

I solve for the investment thresholds q_1 and q_2 with external funding in three steps. I first need to understand for which levels of q the firm needs to raise external funds. Denote $\widetilde{q} \equiv q/H_X$. First, I note that $\widetilde{X}/K$ is a function of $\widetilde{q}$ and π/K such that

$$\frac{\widetilde{X}}{K} = \frac{1}{2\alpha}(\widetilde{q}-1)^2 + \frac{1}{\alpha}(\widetilde{q}-1) - \left(\frac{\pi}{K} - f - \delta\right). \tag{A-12}$$

This implies that $\widetilde{X}/K$ equals 0 at the funding thresholds $\mathbf{q}_i^x$, given by

$$\mathbf{q}_i^x = \pm\sqrt{1 - \alpha\delta + \alpha\left(\frac{\pi}{K} - f\right)}, \tag{A-13}$$

such that $H_X > 1$ if and only if $q > \mathbf{q}_2^x$ or $q < \mathbf{q}_1^x$. Assumption 1 ensures that the thresholds $\mathbf{q}_i^x$ exist for any parameter value.

Reconsider the case in which $X \leq 0$ such that $\mathbf{1}^X = 0$ and $q_i \in (\mathbf{q}_1^x, \mathbf{q}_2^x)$. Assumption 2 is a sufficient yet not necessary condition such that the investment thresholds in equation (A-10) are defined within the range $(\mathbf{q}_1^x, \mathbf{q}_2^x)$, namely

$$\begin{aligned} q_2 &< \mathbf{q}_2^x, \\ q_1 &> \mathbf{q}_1^x. \end{aligned}$$

Assume now that $X > 0$ such that $\mathbf{1}^X = 1$ and $q \notin (\mathbf{q}_1^x, \mathbf{q}_2^x)$. When $X > 0$, Ψ is a polynomial of order 4 in $\widetilde{q}$, and the investment thresholds are implicit functions of α, f, θ, and

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π/K that solve $\Psi=0$. Given that the investment policy is now a function of $\widetilde{q}$, I solve for the investment thresholds in terms of q in two steps. I first solve for the roots of the rewards to investment Ψ as a polynomial of order 4 in $\widetilde{q}$. This gives the investment thresholds $\widetilde{q}_i$. Once I obtain $\widetilde{q}_i$, I replace it in equation (A-12) to obtain the corresponding $\widetilde{X}$, and infer $q_i \equiv \widetilde{q}_i \times (1+\theta \widetilde{X}/K)$.

I express Ψ as function of $\widetilde{q}$ and $\widetilde{X}$. I equate equation (A-11) to 0, divide equation (A-11) by K, and replace I/K with the optimal investment rule such that

$$(q-1)\left[\frac{1}{\alpha}(\widetilde{q}-1)+\delta\right]-\frac{1}{2\alpha}(\widetilde{q}-1)^2-f-\frac{\theta}{2}\left(\frac{\widetilde{X}}{K}\right)^2+\frac{\theta}{2}\left(\frac{\pi}{K}\right)^2 \quad = \quad 0.$$

I replace q with $\widetilde{q}\left(1+\theta\widetilde{X}/K\right)$ in the first term of the previous expression and operate to obtain

$$\begin{aligned}\widetilde{q}^2\frac{1}{2\alpha}\left(1+2\theta\frac{\widetilde{X}}{K}\right)+\widetilde{q}\left[\left(\delta-\frac{1}{\alpha}\right)\left(1+\theta\frac{\widetilde{X}}{K}\right)\right]\\ -\left[f+\delta-\frac{1}{2\alpha}+\frac{\theta}{2}\left(\frac{\widetilde{X}}{K}\right)^2-\frac{\theta}{2}\left(\frac{\pi}{K}\right)^2\right] \quad = \quad 0.\end{aligned} \tag{A-14}$$

Lastly, I use the expression in equation (A-12) to infer that

$$\begin{aligned}\left(\frac{\widetilde{X}}{K}\right)^2-\left(\frac{\pi}{K}\right)^2 \quad = \quad & \frac{1}{4\alpha^2}\widetilde{q}^4-\frac{1}{2\alpha}\widetilde{q}^2\left(\frac{\pi}{K}-f-\delta+\frac{1}{2\alpha}\right)\\ & -2\left(f+\delta-\frac{1}{2\alpha}\right)\frac{\pi}{K}+\left(f+\delta-\frac{1}{2\alpha}\right)^2.\end{aligned} \tag{A-15}$$

Putting equations (A-11), (A-12), and (A-15) together, I obtain an expression for the rewards to investment Ψ as a function of $\widetilde{q}$ and π/K such that

$$\begin{aligned}0 \quad = \quad & \frac{3}{8}\frac{\theta}{\alpha^2}\widetilde{q}^4\\ & +\left(\delta-\frac{1}{\alpha}\right)\frac{\theta}{2\alpha}\widetilde{q}^3\\ & +\widetilde{q}^2\left[\frac{1}{2\alpha}-\frac{\theta}{2\alpha}\left(\frac{\pi}{K}-f-\delta+\frac{1}{2\alpha}\right)\right]\\ & +\widetilde{q}\left(\delta-\frac{1}{\alpha}\right)\left(1-\theta\left(\frac{\pi}{K}-f-\delta+\frac{1}{2\alpha}\right)\right)\\ & -\left(f+\delta-\frac{1}{2\alpha}\right)\left(1+\theta\left[\frac{1}{2}\left(f+\delta-\frac{1}{2\alpha}\right)-\frac{\pi}{K}\right]\right).\end{aligned} \tag{A-16}$$

The roots of the polynomial in equation (A-16) are not constant because they depend on π/K. The comparative statics of the positive investment threshold when $X>0$ are provided in Figure 1.

To compare the investment thresholds when $X\leq 0$ with those when $X>0$, we may consider a first-order Taylor approximation of the net costs of external issuance around the vector $(\widetilde{x},k)$, such that

$$\frac{\theta}{2}K\left(\frac{\widetilde{X}}{K}\right)^2 \quad \approx \quad (h-1)\,\widetilde{X}-\eta K, \tag{A-17}$$

where equation (A-17) is homogeneous of degree 1, and $h\equiv H_X(\widetilde{x},k)>0$, $\eta\equiv -H_K(\widetilde{x},k)>0$. Using this Taylor expansion, I redefine $\widetilde{q}\equiv q h^{-1}$, and I express the rewards to investment

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when firms are issuing such that

$$\text{(A-18)} \qquad \frac{1}{hK}\Psi \approx \frac{1}{2\alpha}\widetilde{q}^2 + \left(\delta - \frac{1}{\alpha}\right)\widetilde{q} - \left(f + \delta - \frac{1}{2\alpha} + \frac{\eta}{h}\right).$$

The corresponding cutoff values q_i when $X > 0$ are given by

$$\text{(A-19)} \qquad q_i \approx 1 - \delta\alpha \pm \sqrt{\delta^2\alpha^2 + 2\alpha f + 2\alpha h^{-1}\eta}.$$

The expression in equation (A-19) reflects that when $X > 0$, the investment threshold q_2 is strictly higher than the corresponding threshold when $X \leq 0$. Furthermore, the investment thresholds in equation (A-19) are not constant and depend on π/K through h and η.

Last, to obtain the derivative of the threshold q_2 with respect to π/K, I follow two steps. I first apply the implicit function theorem to equation (A-16) to obtain

$$\text{(A-20)} \qquad \frac{\partial \widetilde{q}_2}{\partial \frac{\pi}{K}} = \frac{\theta\left[\frac{1}{2\alpha}(\widetilde{q}-1)^2 + \delta(\widetilde{q}-1) - f\right]}{\left[\frac{1}{\alpha}(\widetilde{q}-1) + \delta\right]\left[\frac{3\theta}{2\alpha}\widetilde{q}^2 + 1 - \theta\left(\frac{\pi}{K} - f - \delta + \frac{1}{2\alpha}\right)\right]} < 0.$$

The derivative in equation (A-20) has a negative sign because the denominator is strictly positive, and the numerator is strictly negative. To see why the numerator is strictly negative, note that the polynomial in square brackets corresponds to the same equation at which Ψ equals 0 in equation (A-9) when firms are not issuing. Hence, because $\widetilde{q} < q$ for any $\widetilde{X} > 0$, then equation (A-9) is negative at $\widetilde{q} < q$, and the numerator in equation (A-20) is strictly negative. The denominator is strictly positive because it is the product of two factors that are strictly positive. The first factor equals I/K according to the optimal investment rule. The second factor is strictly positive given Assumptions 1 and 2.

The derivative of $q_2 \equiv \widetilde{q}_2 \times H_X$ with respect to π/K equals

$$\text{(A-21)} \qquad \frac{\partial q_2}{\partial \frac{\pi}{K}} = \frac{\partial \widetilde{q}_2}{\partial \frac{\pi}{K}} H_X - \theta \widetilde{q}_2 < 0,$$

which is strictly negative due to the result in equation (A-20). This result proves the statement in Proposition 2 that the investment threshold q_2 decreases with cash flows when $X > 0$.

Appendix D. Proof of Proposition 3

Given equation (9), the sensitivity of $\widetilde{X}/K$ to q is such that

$$\text{(A-22)} \qquad \frac{\partial \frac{\widetilde{X}}{K}}{\partial q} = \frac{1}{\alpha}\frac{q}{H_X}\frac{\partial \frac{I}{K}}{\partial q} > 0,$$

$$\text{(A-23)} \qquad \frac{\partial^2 \frac{\widetilde{X}}{K}}{\partial q^2} = \left(\frac{\partial \frac{I}{K}}{\partial q}\right)^2 + \frac{1}{\alpha}\frac{q}{H_X}\frac{\partial^2 \frac{I}{K}}{\partial q^2} \leqslant 0.$$

When $X \leq 0$, equation (A-23) is strictly positive because equation (A-6) is 0. When $X > 0$, equation (A-23) is strictly negative if and only if

$$\text{(A-24)} \qquad \frac{qH_{XX}}{H_X} - 1 + 2\frac{\partial \frac{I}{K}}{\partial q}\frac{qH_{XX}}{H_X} > 0.$$

Rearranging terms, equation (A-24) is equivalent to

$$\left(\frac{qH_{XX}}{H_X} - 1\right)\left(\alpha H_X + \frac{q}{H_X}\right) + \frac{q}{H_X}\left(\frac{qH_{XX}}{H_X} - 1\right)^2 + 2\left(\frac{qH_{XX}}{H_X}\right) > 0.$$

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A sufficient condition such that equation (A-24) holds is then

$$\left(\frac{qH_{XX}}{H_X}-1\right)\left(\alpha H_X+\frac{q}{H_X}\right)+\frac{q}{H_X}\left(\frac{qH_{XX}}{H_X}-1\right)^2+2\left(\frac{qH_{XX}}{H_X}-1\right) \quad > \quad 0,$$

which holds for any parameter value because

$$\alpha H_X+H_{XX}\left(\frac{q}{H_X}\right)^2+2 \quad > \quad 0,$$

such that equation (A-23) is strictly negative when $X>0$. The sensitivity of $\widetilde{X}/K$ to π/K is such that

$$\text{(A-25)} \qquad \begin{aligned} \frac{\partial\frac{X}{K}}{\partial\frac{\pi}{K}} &= \frac{\theta}{\alpha^2}\frac{q}{H_X^2}\mathbf{1}^I\mathbf{1}^X-1 \quad \leqslant \quad 0; \quad \text{and} \\ \frac{\partial^2\frac{X}{K}}{\partial\left(\frac{\pi}{K}\right)^2} &= \frac{\theta^2 q}{\alpha^2 H_X^3}\mathbf{1}^I\mathbf{1}^X \quad \geq \quad 0. \end{aligned}$$

Appendix E. Proof of Proposition 4

I begin by deriving the preliminary equations (11) and (14). Consider first equation (11) such that

$$Q \quad = \quad Z'\zeta+\upsilon,$$

where the vector $Z'\zeta$ has two alternative specifications. Taking as given the budget constraint of the firm, the specification of $Z'\zeta$ in equation (13) is such that the coefficients are functions of α, θ, f, and δ such that

$$\begin{aligned} \zeta_0 &= (1-\alpha\delta)+\theta(1-\alpha\delta)\left(2f+\alpha\delta^2\right)\mathbf{1}^X, & \zeta_3 &= \tfrac{1}{2}\alpha^2\theta\mathbf{1}^X, \\ \zeta_1 &= \alpha+\theta\left(1+f\alpha-2\alpha\delta+\tfrac{3}{2}\alpha^2\delta^2\right)\mathbf{1}^X, & \zeta_4 &= -\theta(1-\alpha\delta)\mathbf{1}^X, \\ \zeta_2 &= \tfrac{3}{2}\alpha\theta(1-\alpha\delta)\mathbf{1}^X, & \zeta_5 &= -\alpha\theta\mathbf{1}^X. \end{aligned}$$

The alternative specification in equation (12) is such that the coefficients are functions of α, θ, and δ:

$$\begin{aligned} \zeta_0 &= (1-\alpha\delta), & \zeta_2 &= \theta(1-\alpha\delta)\mathbf{1}^X, \\ \zeta_1 &= \alpha, & \zeta_3 &= \alpha\theta\mathbf{1}^X. \end{aligned}$$

Consider equation (14). Using equation (11), the probability that $q\geq q_2$ equals $Z'\zeta+\upsilon>q_2$, where the measurement error is assumed to be normally distributed with mean κ and volatility σ_υ. Using standard techniques, I conclude that

$$\Pr\left(\mathbf{z}\geq\frac{q_2-\kappa-Z'\zeta}{\sigma_\upsilon}\right) \quad \equiv \quad 1-\Phi\left(\frac{q_2-\kappa-Z'\zeta}{\sigma_\upsilon}\right),$$

where $\mathbf{z}$ follows a standard normal distribution with mean 0 and volatility 1.

Given equations (11) and (14) and also assuming $f^+\neq f^-$, $f^+>0$, and $f^-\longrightarrow\infty$, I derive the density of the conditional mean of Q in my sample as the density of the optimal investment rule in the active region as in equation (11), divided by the conditional density of observing positive investment according to equation (14). The posttruncation density of the average Q of any firm at time t, defined over (Δ,∞), is given by

$$\text{(A-26)} \qquad f(Q|Z;\zeta,\Delta) \quad = \quad \frac{\frac{1}{\sigma_\upsilon}\phi\left(\frac{Q-\kappa-Z'\zeta}{\sigma_\upsilon}\right)}{1-\Phi\left(\frac{\Delta-Z'\zeta}{\sigma_\upsilon}\right)},$$

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such that equation (A-26) is the density (conditional on Z) of observing the average Q of any firm at time t in the working sample. The expression in equation (15) equals the conditional mean of variable whose density is given by equation (A-26). The formula is consistent with the conditional mean of a truncated regression as discussed by Hayashi (2000). The conditional log likelihood in equation (16) is obtained by taking logs to equation (A-26).

Appendix F. Proof of Corollaries 1 and 2

The computation of the marginal effect of I/K on Q follows the econometric theory for marginal effects of truncated regressions. Denote the inverse Mills's ratio $\lambda(x) \equiv \phi(x)/(1-\Phi(x))$, where $x \equiv (\Delta - Z'\zeta)/\sigma_\upsilon$. Using the definition of the vector $Z'\zeta$ in equation (13), I obtain that

$$\begin{aligned}\frac{\partial \mathrm{E}(Q \mid q > q_2)}{\partial \frac{I}{K}} &= \zeta_1 + 2\zeta_2 \frac{I}{K} + 3\zeta_3 \left(\frac{I}{K}\right)^2 + \zeta_5 \frac{\pi}{K} + \sigma_\upsilon \times \frac{\partial \lambda}{\partial x} \times \frac{\partial x}{\partial \frac{I}{K}}, \\ &= \left(\zeta_1 + 2\zeta_2 \frac{I}{K} + 3\zeta_3 \left(\frac{I}{K}\right)^2 + \zeta_5 \frac{\pi}{K}\right)[1 - \lambda(x)(\lambda(x) - x)],\end{aligned}$$

which yields the expression in Corollary 1 because $\eta(x) \equiv \lambda(x)(\lambda(x) - x)$.

The computation of the marginal effect of π/K on the probability of investment follows the econometric theory for marginal effects of probit models. I obtain the result in Corollary 2 by taking the derivative of the probability in investment in equation (14) with respect to π/K. The derivative $\partial q_2/\partial \frac{\pi}{K}$ is provided in Appendix C.

Appendix G. Debt-Overhang Effects

The derivation is similar to that in Appendix A. When debt is risky, the problem of the manager consists of determining the optimal investment policy and the optimal time to default T. We assume that a fraction $\gamma \in (0,1)$ of the assets of the firm is kept by bondholders upon default. Using the definition of the Dynkin operator $A(f)$ on a twice-differentiable function f as in equation (A-1), the Bellman equation of the firm with risky debt equals

$$rS = \max_{I,T} -H + A(S). \tag{A-27}$$

The Bellman equation holds pointwise, implying that partial derivatives of both sides of equation (A-27) must match. Differentiating equation (A-27) with respect to K and applying the optimality condition on I, it follows that

$$KA(q) - \delta qK = A(qK) - qI = A(qK) - H_X(I + C_I I).$$

Rearranging terms and invoking the properties of functions π, C, and H noted in Section II, I obtain

$$rqK = -H + b + A(qK). \tag{A-28}$$

Subtracting equation (A-28) from equation (A-27) and rearranging terms, I obtain

$$A(S - qK) - r(S - qK) = b.$$

I define the Ito process ξ as

$$\xi_t \equiv e^{-rt}(S_t - K_t q_t),$$

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such that the application of Ito's lemma to ξ yields

$$d\xi_t \equiv e^{-rt} b\, dt + e^{-rt} \sigma(\epsilon)(S_\epsilon - K q_\epsilon)\, dW_t.$$

I integrate ξ up to the optimal time to default T, and I take expectations such that

$$E\left[e^{-rT}(S_T - K_T q_T)\right] = S_t - K_t q_t + E\left[\int_t^T e^{-rs} b\, ds\right] + E\left[\int_t^T e^{-rs} \sigma(\epsilon)(S_\epsilon - K q_\epsilon)\, dW_s\right].$$

The smooth pasting conditions for optimal default are $S_T = q_T = 0$. Given these conditions, the left-hand side of the previous expression equals 0. I also note that the last term on the right-hand side is a martingale with expectation 0. Hence the expression simplifies to

$$q_t K_t = S_t + E\left[\int_t^T e^{-rs} b\, ds\right].$$

The wedge between q_t and Q_t relates to debt-overhang effects as noted by Hennessy (2004) because

$$q_t \equiv Q_t + \frac{1}{K_t} E_t\left[e^{-r(T-t)} \gamma K_T\right]. \tag{A-29}$$

Average Q overstates q by incorporating the undepreciated value of assets allocated to bondholders upon default.

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