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Dynamic model and performances of 2-DOF manipulators

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SUMMARY

Tools that make it possible to measure the performances of manipulators are essential in many technical applications, for instance, when the optimal path to accomplish a task has to be chosen, or when different manipulator architectures have to be compared. This paper proposes some indices that fully describe the passive dynamic performances of manipulators with two degrees of freedom (dof). The proposed indices make it possible to compare the passive dynamic performances of different manipulator architectures, which can perform the same tasks, and can be used to build diagrams which highlight the effects of variations in the manipulator geometry on the manipulator dynamics. These features make them easy to be used in a design context. Finally, some applications of the proposed indices will be presented and discussed.

KEYWORDS: Dynamics; Performance indices; Mechanism design; Manipulator architecture.

I. INTRODUCTION

The availability of tools that make it possible to measure the performances of manipulators is essential in many technical applications, for instance, when the optimal path to accomplish a task has to be chosen, or when different manipulator architectures, that can accomplish the same task, have to be compared.

Manipulators' performances can be grouped into two categories: kinetostatic performances[1–4] and dynamic performances.[5–17] The kinetostatic performances are related to the measure of qualities like positioning precision, capacity of distributing the loads applied to the end effector among the links and the actuators, attitude to produce high velocities of the end effector in different directions at parity of joint rates, etc. On the other hand, the dynamic performances are related to the measure of the attitude of a given architecture to obtain high end-effector accelerations without considering the active forces (passive dynamic performances), and the capacity of impressing high end-effector accelerations at parity of active forces (active dynamic performances).

In order to measure the performances of a manipulator with given geometry and mass distribution, many indices that are built starting from the kinematic and/or dynamic relationship between actuated joint space and end-effector space (operational space),[2,4–6,8–17] have been proposed in the literature. In particular, with reference to the dynamic performances, Asada[5] proposed the "generalized inertia ellipsoid," which is a geometric representation of the generalized inertia matrix of the manipulator, and showed that the geometric properties of the ellipsoid are related to some dynamic performances of the manipulator. Yoshikawa[6] introduced the "dynamic manipulability ellipsoid," which led to the "dynamic manipulability measure." Khatib and Burdik[9] defined the "isotropic acceleration", which is the largest acceleration magnitude the end effector can reach in every direction. Graettinger and Krogh[10] proposed the "acceleration radius" to measure the isotropic acceleration over the workspace. Kim and Desa[11,12] developed the "acceleration set theory" to determine the isotropic accelerations in three-dof serial manipulators. Ma and Angeles[8] defined the "dynamic isotropy" and proposed the "dynamic conditioning index" to measure it. Doty et al.[13] pointed out that these earlier studies did not adequately deal with the inhomogeneities, or difference in units, between linear and angular coordinates; then, Khatib proposed the use of the "belted ellipsoid" to overcome the non-homogeneity problem. Furthermore, Bowling and Khatib[15,16] used the "belted ellipsoid" to propose the "motion isotropy hypersurface" (which describes the relationships between isotropic linear and angular velocities and accelerations) and, successively, they[17] proposed an "actuation efficiency measure" to characterize the imbalance (anisotropy) between the end-effector accelerations achievable in different directions. All the tools developed in these works are mainly oriented to choose the most suitable actuators for a given manipulator architecture and most of them deal with six-dof serial manipulators.

During the design stage, the architecture of the manipulator must be chosen starting from the design requirements and, then, actuators' location must be chosen (if the architecture is parallel, not all the joints are actuated and there are many alternatives). In this context, the use of indices defined for manipulators with given geometry and mass distribution is complex and not effective. In fact, the use of analytic relationships and/or diagrams which relate the performance index values to the link sizes and to the manipulator architecture is much more effective during design.

Dynamic performance indices which allow various architectures to be compared are not available for all type of architecture yet. This lack is due to the fact that synthesizing all the dynamic characteristics of any architecture in one index is very difficult. The problem of defining dynamic performance indices to compare manipulator architectures

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becomes simpler, if the class of manipulators to be compared through the indices is delimited. Following this intuition this author together with Parenti-Castelli, in a previous paper,[18] proposed three indices, two local and one global, for comparing some passive dynamic performances of different architectures with three degrees of freedom (dof). The indices, proposed in the previous paper, fully exploit the simplifications that occur when dealing with three-dof architectures; unfortunately, they cannot be used for manipulators with less than three dof and, moreover, they do not characterize the non-linear inertia forces (Coriolis and centrifugal forces).

Since the design requirements on the dynamic performances are defined in the end-effector space (operational space[19]), only the dynamic model of the manipulator defined in the operational space makes it possible to derive passive dynamic performance indices that are able to check whether the design requirements are matched. If the manipulator is serial, all the joints will be actuated and the relationship between the actuated joint rates and the velocities in the operational space will be univocal. Therefore, the dynamic model formulated in the operational space, suitably modified, can also be used to evaluate the relationship between active generalized forces, applied in the actuated joints, and end-effector's accelerations (active dynamic performances). On the contrary, if the manipulator is not serial (i.e. is parallel or hybrid), the choice of the joint to actuate will not be univocal and, as a consequence, the relationship between the actuated joint rates and the velocities in the operational space will not be univocal, too. Therefore, the dynamic model formulated in the operational space can only be considered as the starting point for the comparative evaluation of different ways to actuate a given architecture, and it does not contain all the pieces of information necessary to analyze the active dynamic performances of the manipulator.

In this paper, at first, the dynamic model in the operational space for 2-dof manipulators will be recalled, then, by analyzing the model, some indices that fully describe the passive dynamic performances of such manipulators will be proposed.

The proposed indices make it possible to compare the passive dynamic performances of different manipulator architectures that can perform the same tasks, and can be used to build diagrams which highlight the effects of variations in the manipulator geometry on the manipulator dynamics. These features make them easy to be used in a design context.

Finally, one application of the proposed indices will be presented and discussed.

II. DYNAMIC MODEL OF 2-DOF MANIPULATORS

In 2-dof manipulators, two homogeneous coordinates chosen among the six coordinates of the operational space, which define the end-effector position and orientation (pose) with respect to the frame, can always be used as generalized coordinates of the manipulator.

In what follows, two homogeneous coordinates, x_i, $i = 1, 2$, of the operational space will be assumed as generalized coordinates of the manipulator and $\mathbf{x}$ will indicate the vector $\{x_1, x_2\}^T$.

With these conventions, the kinetic energy of a 2-dof manipulator can be written as follows:[20]

$$T = \frac{1}{2}\dot{\mathbf{x}}^T \mathbf{M}(\mathbf{x})\, \dot{\mathbf{x}} \tag{1}$$

where $\dot{\mathbf{x}}$ is the vector $\{\dot{x}_1, \dot{x}_2\}$ of the time derivatives $\dot{x}_i$, $i = 1, 2$, of the generalized coordinates x_i, $i = 1, 2$,; whereas $\mathbf{M}(\mathbf{x})$ is the generalized inertia matrix of the manipulator, referred to the coordinates x_i, $i = 1, 2$. Matrix $\mathbf{M}(\mathbf{x})$ is a symmetric and positive-definite 2×2 matrix, and only depends on the manipulator configurations.

By using expression (1) and the Lagrangian formulation[20] of dynamics, the dynamic model of the manipulator can be written as follows

$$\mathbf{M}(\mathbf{x})\, \ddot{\mathbf{x}} + \mathbf{C}(\mathbf{x}, \dot{\mathbf{x}}) = \mathbf{Q}_e + \mathbf{G}(\mathbf{x}) \tag{2}$$

In Eq. (2), $\ddot{\mathbf{x}}$ is the time derivative of $\dot{\mathbf{x}}$. $\mathbf{G}(\mathbf{x})$ is the vector $\{-\frac{\partial V}{\partial x_1}, -\frac{\partial V}{\partial x_2}\}^T$ in which V is the potential energy of all the conservative forces that act on the manipulator (e.g. the gravitational forces). $\mathbf{Q}_e$ is the vector $\{Q_1, Q_2\}^T$ of the generalized forces, Q_i, $i = 1, 2$, which take both the external loads applied to the end effector and the torques and/or forces applied by the actuators into account. Finally, the term $\mathbf{C}(\mathbf{x}, \dot{\mathbf{x}})$ has the following expression[20]

$$\mathbf{C}(\mathbf{x}, \dot{\mathbf{x}}) = \left(\sum_{k=1,2} \dot{x}_k \frac{\partial \mathbf{M}}{\partial x_k} \right) \dot{\mathbf{x}} - \frac{1}{2} \left\{ \dot{\mathbf{x}}^T \frac{\partial \mathbf{M}}{\partial x_1}\, \dot{\mathbf{x}},\ \dot{\mathbf{x}}^T \frac{\partial \mathbf{M}}{\partial x_2}\, \dot{\mathbf{x}} \right\}^T \tag{3}$$

The term $\mathbf{C}(\mathbf{x}, \dot{\mathbf{x}})$ takes the non-linear inertia forces (centrifugal and Coriolis' forces) into account and is equal to zero in the manipulator configurations in which all the first order derivatives of matrix M with respect to x_1 and x_2 vanish.

Equation (2) can be rearranged as follows

$$\mathbf{M}(\mathbf{x})\, \ddot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}) \tag{4}$$

where $\mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}) = \mathbf{Q}_e + \mathbf{G}(\mathbf{x}) - \mathbf{C}(\mathbf{x}, \dot{\mathbf{x}})$.

III. PASSIVE DYNAMIC PERFORMANCES

The passive dynamic performances deal with the attitude of an architecture to accelerate the end effector without considering the torques and/or forces applied by the actuators. When such an attitude is considered, the vector $\mathbf{f}(\mathbf{x}, \dot{\mathbf{x}})$ which appears on the right-hand side of Eq. (4) can be taken into account as a unique external generalized force applied to the passive structure (skeleton) of the manipulator. The set of vectors $\mathbf{f}(\mathbf{x}, \dot{\mathbf{x}})$ with modulus equal to one are given by the following equation

$$\mathbf{f}^T \mathbf{f} = 1 \tag{5}$$

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Substituting for **f** the left-hand side of Eq. (4) into Eq. (5) yields

$$\ddot{\mathbf{x}}^{\mathrm{T}}\,\mathbf{M}^2\,\ddot{\mathbf{x}} = 1 \tag{6}$$

where the symmetry of matrix **M** (i.e. $\mathbf{M}^{\mathrm{T}} = \mathbf{M}$) has been exploited.

Since matrix **M** is positive-definite, Eq. (6) is the analytic expression of an ellipse in the plane $\ddot{x}_1\ddot{x}_2$ of the end-effector accelerations. Such an ellipse will be called acceleration ellipse (AE). The directions of AE's principal axes are given by the directions of the eigenvectors of matrix **M** (remind that the eigenvectors of **M** are eigenvectors of $\mathbf{M}^2$, too), and indicate the directions along which the vector $\ddot{\mathbf{x}}$ is parallel to the vector **f** and has maximum modulus (direction of the major axis) or minimum modulus (direction of the minor axis). The lengths of the AE semi-axes are equal to the inverses of the eigenvalues of matrix **M** and give the maximum and minimum magnitude of the acceleration $\ddot{\mathbf{x}}$ that the end effector can reach when the manipulator undergoes to a unit **f** (see Eq. (5)).

If the two eigenvalues of matrix **M** are equal, the AE is a circumference and all the vectors of the plane $\ddot{x}_1\ddot{x}_2$ are eigenvectors of **M**. The configurations in which the AE is a circumference will be called dynamically isotropic configurations. If any unit **f** is applied to a manipulator that assumes a dynamically isotropic configuration the acceleration $\ddot{\mathbf{x}}$ is always parallel to the applied **f** and has an magnitude which does not depend on the direction of **f**. This dynamic characteristic (dynamic isotropy) makes it possible rapid changes of the end-effector direction of motion without sharp changes in the magnitude of **f**, and is a sought-after property during design.

Moreover, the greater the AE area is, the greater the magnitude of the average acceleration, the end effector can reach at parity of **f**'s magnitude, is. The AE area increases proportionally to the inverse of the product of the eigenvalues of matrix **M**. The attitude of a manipulator architecture to produce high average accelerations of the end effector at parity of **f**'s magnitude will be called swiftness in analogy to what was done in a previous paper.[18]

Finally, expression (3) of term $\mathbf{C}(\mathbf{x}, \dot{\mathbf{x}})$, due to the non-linear inertia forces, shows that $\mathbf{C}(\mathbf{x}, \dot{\mathbf{x}})$ is roughly proportional to the quadratic monomials whose factors are $\dot{x}_1$ and/or $\dot{x}_2$. Therefore, the architecture of a fast manipulator must also be evaluated from the point of view of its capacity of reducing the term $\mathbf{C}(\mathbf{x}, \dot{\mathbf{x}})$, which can be obtained by reducing the variations of matrix **M** with the changes of configuration (see expression (3)).

In what follows, new indices, which make it possible to evaluate the dynamic isotropy, the swiftness and the non-linear inertia forces' effects of an architecture, and to compare different architectures that are able to perform the same tasks, will be defined.

III.1. Dynamic isotropy

The eigenvalues of **M** are the solutions of the following characteristic equation

$$\lambda^2 - \lambda\mathrm{Tr}(\mathbf{M}) + \mathrm{Det}(\mathbf{M}) = 0 \tag{7}$$

where λ is the unknown, $\mathrm{Tr}(\mathbf{M})$ is the trace of **M** and $\mathrm{Det}(\mathbf{M})$ is the determinant of **M**.

The solution of Eq. (7) yields the following analytic expressions of the eigenvalues, λ_i, $i = 1, 2$, of **M**

$$\lambda_i = \frac{\mathrm{Tr}(\mathbf{M})}{2} + (-1)^i\sqrt{\frac{\mathrm{Tr}^2(\mathbf{M})}{4} - \mathrm{Det}(\mathbf{M})}, \qquad i = 1, 2 \tag{8}$$

Using expressions (8), it is easy to verify that

$$\mathrm{Tr}(\mathbf{M}) = \lambda_1 + \lambda_2 \tag{9a}$$

$$\mathrm{Det}(\mathbf{M}) = \lambda_1\lambda_2 \tag{9b}$$

Moreover, since the product of two real and positive variable is always less than or equal to the square of their arithmetic average (the equality only holds when the two variable assume the same value), the following inequalities hold

$$0 < \mathrm{Det}(\mathbf{M}) \leq \frac{\mathrm{Tr}^2(\mathbf{M})}{4} \tag{10}$$

where $\mathrm{Det}(\mathbf{M})$ is equal to $\frac{\mathrm{Tr}^2(\mathbf{M})}{4}$ if and only if a dynamically isotropic configuration occurs (i.e. when $\lambda_1 = \lambda_2$).

A consequence of (8) and (10) is that λ_1 is always less than λ_2. Therefore, the following index k_0 can be used as a measure of the proximity to the dynamically isotropic conditions of a given configuration (local index of dynamic isotropy)

$$k_0 = \frac{\lambda_1}{\lambda_2} \tag{11}$$

where the analytic expressions of λ_i, $i = 1, 2$, are given by relationships (8).

The index k_0 is directly expressible as function of the entries of matrix **M** through relationships (8). Moreover, it is always positive and less than or equal to one. Finally, the greater k_0 is, the more λ_1 is near to λ_2. One drawback of the index k_0 is its complicated analytic expression.

By exploiting the same considerations that led to inequalities (10), it is possible to show that the index k_1 defined as follows

$$k_1 = \frac{4\,\mathrm{Det}(\mathbf{M})}{\mathrm{Tr}^2(\mathbf{M})} \tag{12}$$

has the same properties as k_0 (i.e., $0 < k_1 \leq 1$; $k_1 = 1$ if and only if $\lambda_1 = \lambda_2$; the greater k_1 is, the more λ_1 is near to λ_2). Therefore, the index k_1 can be used as a local index of dynamic isotropy instead of k_0. The index k_1 has an analytic expression that is much simpler than the one of k_0.

Using relationships (9) and definition (11), the following relationship between k_0 and k_1 can be demonstrated

$$k_1 = \frac{4\,k_0}{(1 + k_0)^2} \tag{13}$$

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By using the indices k_m, $m = 0, 1$, two global indices, I_m, $m = 0,1$, of dynamic isotropy can be defined as follows

$$I_m = \frac{\int_W k_m \, dx_1 dx_2}{\int_W dx_1 dx_2}, \qquad m = 0, 1 \tag{14}$$

where W is the area of the plane x_1x_2 (configuration space) which represent the useful workspace of the manipulator.

The global indices I_m, $m = 0, 1$, only depend on the type of architecture and on the geometric and inertial parameters which identify the manipulator. Therefore, they are suitable to compare the dynamic isotropy of different architectures or of different geometries of the same architecture.

Finally, the locus of all the dynamically isotropic configurations is represented by a curve in the plane x_1x_2 (configuration space) whose analytic expression is given by the following equation (see expression (8))

$$\mathrm{Tr}^2(\mathbf{M}) - 4\,\mathrm{Det}(\mathbf{M}) = 0 \tag{15}$$

Equation (15) contains the geometric and inertial parameters of the manipulator and can be used during design to size the manipulator so that a finite number of assigned configurations are dynamically isotropic.

III.2. Swiftness

Since the area of the acceleration ellipse increases proportionally to the inverse of the product of the eigenvalues of matrix **M**, one measure of the swiftness of a manipulator in a given configuration is given by the following local swiftness index (see Eq. (9b)):

$$h = \frac{1}{\mathrm{Det}(\mathbf{M})} \tag{16}$$

The index h is always greater than zero and increases proportionally to the area ($= \pi h$) of the AE. Moreover, it is not dimensionless and depend on the configuration and on the geometric and inertial parameters of the manipulator. Finally, the analytic expression of the index h can be easily obtained from the analytic expressions of the entries of matrix **M**.

Using definition (16), the following global swiftness index can be defined

$$S = \frac{\int_W h \, dx_1 dx_2}{\int_W dx_1 dx_2} \tag{17}$$

The global index S only depends on the type of architecture and on the geometric and inertial parameters which identify the manipulator. Therefore, it is suitable to compare the swiftness of different architectures or of different geometries of the same architecture.

III.3. Non-linear inertial forces

Expression (3) of $\mathbf{C}(\mathbf{x}, \dot{\mathbf{x}})$ shows that, at parity of $\dot{\mathbf{x}}$, $\mathbf{C}(\mathbf{x}, \dot{\mathbf{x}})$ increases when the derivatives $\frac{\partial \mathbf{M}}{\partial x_k}$, $k = 1, 2$, increase. Therefore, a local index that measures the term due to the non-linear inertial forces (local index of non-linearity) can be defined as follows:

$$n = \sum_{k=1,2} \left[\left(\frac{\partial m_{11}}{\partial x_k} \right)^2 + \left(\frac{\partial m_{22}}{\partial x_k} \right)^2 + \left(\frac{\partial m_{12}}{\partial x_k} \right)^2 \right] \tag{18}$$

where m_{ij}, $i,j = 1, 2$, are the entries of matrix **M**.

The index n is equal to zero when $\mathbf{C}(\mathbf{x}, \dot{\mathbf{x}})$ vanishes and, provided that $\dot{\mathbf{x}}$ does not change, it increases when the magnitude of $\mathbf{C}(\mathbf{x}, \dot{\mathbf{x}})$ increases. It is not dimensionless and depends on the configuration and on the geometric and inertial parameters of the manipulator. Moreover, it can be expressed in analytic form by differentiating the analytic expressions of the entries m_{ij}, $i,j = 1, 2$, of **M**.

Using definition (18), the following global index of non-linearity can be defined

$$U = \frac{\int_W n \, dx_1 dx_2}{\int_W dx_1 dx_2} \tag{19}$$

The global index U only depends on the type of architecture and on the geometric and inertial parameters which identify the manipulator. Therefore, it is suitable to compare the non-linearity term of different architectures or of different geometries of the same architecture.

IV. USE OF THE PERFORMANCE INDICES

All the indices defined in the above section can be expressed in analytical form when the analytical form of matrix **M** is known. The availability of the analytic expressions of the indices makes it possible to size the passive structure of a manipulator so that some given design requirements are respected. In fact, the following sizing procedure can be implemented

(i) the design requirements on the dynamic performances are expressed in terms of minimum values of the indices k_m, I_m, h and S and maximum values of the indices n and U;
(ii) the analytic expressions of the indices are computed;
(iii) the geometric and inertial parameters of the manipulator are computed so that the minimum and maximum values determined in step (i) are satisfied.

Moreover, the global indices (I_m, S and U) make it possible to compare different architectures before choosing an architecture to size.

In the following subsections, some applications will be illustrated.

IV.1. Comparison of two different architectures

In this subsection, the use of the indices to compare a serial architecture with a parallel one, that are able to perform the same tasks, will be illustrated.

IV.1.1. Serial planar manipulator of type RR. Figure 1 shows a serial planar manipulator of type RR (hereafter referred as RR manipulator). The RR manipulator

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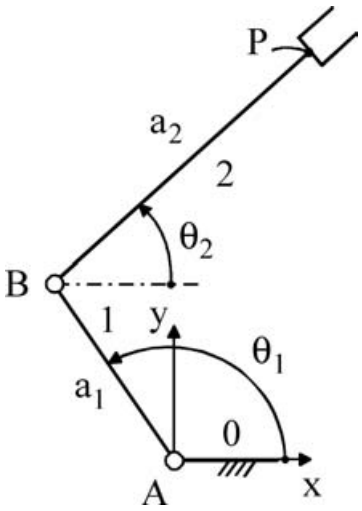


Fig. 1. Serial planar manipulator of type RR.

Table I. Values of the inertial and geometric parameters of the RR manipulator (see Fig. 1).

m_{s2} (kg)	I_{A1} (kg m^2)	I_{P2} (kg m^2)	a_1 (m)	a_2 (m)
5	3.75	3.75	1.5	1.5

is constituted of three links connected in series by two revolute pairs with parallel axes. With reference to Fig. 1, A and B are the intersections between the plane of the motion and the revolute pair axes. P is a point of the end effector (link 2). a_1 and a_2 are the lengths of the segments AB and BP, respectively. θ_1 and θ_2 are two angles that locate the orientation with respect to the frame (link 0) of the links 1 and 2, respectively. The reference system Axy is fixed to the frame. The coordinates of P in Axy are assumed as generalized coordinates of the manipulator (Table I).

The kinetic energy, T_s, of the RR manipulator can be written as follows:

$$T_s = T_{s1} + T_{s2} \tag{20}$$

where T_{s1} and T_{s2} are the kinetic energy of links 1 and 2, respectively.

The terms on the right-hand side of (20) can be written in explicit form as follows:

$$T_{s1} = \frac{1}{2} I_{A1} \dot{\theta}_1^2 \tag{21a}$$

$$T_{s2} = \frac{1}{2} I_{P2} \dot{\theta}_2^2 + \frac{1}{2} m_{s2} \dot{\mathbf{P}}^2 + m_{s2} \dot{\mathbf{P}} \cdot [\mathbf{k} \times (\mathbf{G}_{s2} - \mathbf{P})] \dot{\theta}_2 \tag{21b}$$

where $\dot{\theta}_1$ and $\dot{\theta}_2$ are the time derivative of the angles θ_1 and θ_2, respectively. $\dot{\mathbf{P}}$ is the velocity P in Axy. $\mathbf{k}$ is the unit vector $\mathbf{i} \times \mathbf{j}$, where $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors of the axes x and y, respectively. G_{s2} is the center of mass of link 2. m_{s2} is the mass of link 2. I_{A1} is the moment of inertia of link 1 with respect to the axis perpendicular to the plane of the motion and passing through A. I_{P2} is the moment of inertia of link 2 with respect to the axis perpendicular to the plane of the motion and passing through P.

The velocity analysis of the RR manipulator gives the following relationships

$$\dot{\theta}_i = \mathbf{h}_i^T \dot{\mathbf{P}}, \qquad i = 1, 2 \tag{22}$$

where $(.)^T$ stands for transpose of (.), whereas

$$\mathbf{h_1} = \frac{-(\mathbf{P} - \mathbf{B})}{\mathbf{k} \cdot (\mathbf{P} - \mathbf{B}) \times (\mathbf{B} - \mathbf{A})} \tag{23a}$$

$$\mathbf{h_2} = \frac{(\mathbf{B} - \mathbf{A})}{\mathbf{k} \cdot (\mathbf{P} - \mathbf{B}) \times (\mathbf{B} - \mathbf{A})} \tag{23b}$$

Moreover, the inverse position analysis of the RR manipulator yields the following relationship

$$\mathbf{B} = \mathbf{A} + \mu \frac{(\mathbf{P} - \mathbf{A})}{|\mathbf{P} - \mathbf{A}|} + \nu \mathbf{k} \times \frac{(\mathbf{P} - \mathbf{A})}{|\mathbf{P} - \mathbf{A}|} \tag{24}$$

where

$$\mu = \frac{(\mathbf{P} - \mathbf{A})^2 + a_1^2 - a_2^2}{2\ |\mathbf{P} - \mathbf{A}|}, \qquad \nu = \sqrt{a_1^2 - \mu^2} \tag{25}$$

The introduction of (24) and (25) into (23) gives the expressions of the vectors $\mathbf{h}_i$, $i = 1, 2$, as function of the only generalized coordinates (coordinates of P in Axy) and geometric constants of the manipulator.

The introduction of (21) and (22) into (20) yields

$$T_s = \frac{1}{2} \dot{\mathbf{P}}^T \mathbf{M}_s \dot{\mathbf{P}} \tag{26}$$

with

$$\mathbf{M}_s = I_{A1} \left(\mathbf{h}_1 \mathbf{h}_1^T\right) + I_{P2} \left(\mathbf{h}_2 \mathbf{h}_2^T\right) + m_{s2} \mathbf{1}_{2\times 2} + 2 m_{s2} \left\{[\mathbf{k} \times (\mathbf{G}_{s2} - \mathbf{P})] \mathbf{h}_2^T\right\} \tag{27}$$

where $\mathbf{1}_{2\times 2}$ is the 2×2 identity matrix.

Matrix $\mathbf{M}_s$ is the generalized inertia matrix that has to be used to compute the indices. It only depends on the coordinates of P in Axy and on the geometric constants of the manipulator.

IV.1.2. Parallel planar manipulator of type RRRRR. Figure 2 shows a parallel planar manipulator of type RRRRR (hereafter referred as RRRRR manipulator). The RRRRR manipulator is constituted of five binary links connected in a closed loop by five revolute pairs with parallel axes. With reference to Fig. 2, C, D, E, F and P are the intersections between the plane of the motion and the revolute pair axes. b_1, b_2, b_3 and b_4 are the lengths of the segments CD, DP, EF and EP, respectively. ϕ_j, $j = 1, \ldots, 4$, is the angle that locate the orientation with respect to the frame (link 0) of the j-th link. The reference system Cxy is fixed to the frame. The coordinates of P in Cxy are assumed as generalized coordinates of the manipulator.

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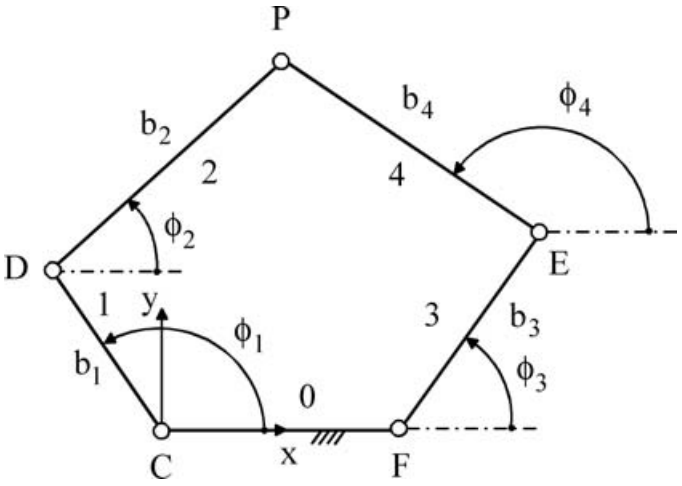


Fig. 2. Parallel planar manipulator of type RRRRR.

The kinetic energy, T_p, of the RRRRR manipulator can be written as follows:

$$T_p = \sum_{j=1,4} T_{pj} \tag{28}$$

where T_{pj}, $j = 1, \ldots, 4$, is the kinetic energy of the j-th link.

The terms on the right-hand side of (28) can be written in explicit form as follows:

$$T_{p1} = \frac{1}{2} I_{C1}\, \dot{\phi}_1^2 \tag{29a}$$

$$T_{p2} = \frac{1}{2} I_{P2}\, \dot{\phi}_2^2 + \frac{1}{2} m_{p2}\, \dot{\mathbf{P}}^2 + m_{p2}\, \dot{\mathbf{P}} \cdot [\mathbf{k} \times (\mathbf{G}_{p2} - \mathbf{P})]\, \dot{\phi}_2 \tag{29b}$$

$$T_{p3} = \frac{1}{2} I_{F3}\, \dot{\phi}_3^2 \tag{29c}$$

$$T_{p4} = \frac{1}{2} I_{P4}\, \dot{\phi}_4^2 + \frac{1}{2} m_{p4}\, \dot{\mathbf{P}}^2 + m_{p4}\, \dot{\mathbf{P}} \cdot [\mathbf{k} \times (\mathbf{G}_{p4} - \mathbf{P})]\, \dot{\phi}_4 \tag{29d}$$

where $\dot{\phi}_j$, $j = 1, \ldots, 4$, is the time derivative of the angle ϕ_j. G_{p2} and G_{p4} are the centers of mass of the links 2 and 4, respectively. m_{p2} and m_{p4} are the masses of the links 2 and 4, respectively. I_{C1} is the moment of inertia of link 1 with respect to the axis perpendicular to the plane of the motion and passing through C. I_{F3} is the moment of inertia of link 3 with respect to the axis perpendicular to the plane of the motion and passing through F. I_{P2} and I_{P4} are the moments of inertia with respect to the axis perpendicular to the plane of the motion and passing through P of the links 2 and 4.

The velocity analysis of the RRRRR manipulator gives the following relationships

$$\dot{\phi}_j = \mathbf{q}_j^T\, \dot{\mathbf{P}}, \qquad j = 1, \ldots, 4 \tag{30}$$

where

$$\mathbf{q}_1 = \frac{-(\mathbf{P} - \mathbf{D})}{\mathbf{k} \cdot (\mathbf{P} - \mathbf{D}) \times (\mathbf{D} - \mathbf{C})} \tag{31a}$$

$$\mathbf{q}_2 = \frac{(\mathbf{D} - \mathbf{C})}{\mathbf{k} \cdot (\mathbf{P} - \mathbf{D}) \times (\mathbf{D} - \mathbf{C})} \tag{31b}$$

$$\mathbf{q}_3 = \frac{-(\mathbf{P} - \mathbf{E})}{\mathbf{k} \cdot (\mathbf{P} - \mathbf{E}) \times (\mathbf{E} - \mathbf{F})} \tag{31c}$$

$$\mathbf{q}_4 = \frac{(\mathbf{E} - \mathbf{F})}{\mathbf{k} \cdot (\mathbf{P} - \mathbf{E}) \times (\mathbf{E} - \mathbf{F})} \tag{31d}$$

Moreover, the inverse position analysis of the RRRRR manipulator yields the following relationships

$$\mathbf{D} = \mathbf{C} + \alpha \frac{(\mathbf{P} - \mathbf{C})}{|\mathbf{P} - \mathbf{C}|} + \beta \mathbf{k} \times \frac{(\mathbf{P} - \mathbf{C})}{|\mathbf{P} - \mathbf{C}|} \tag{32a}$$

$$\mathbf{E} = \mathbf{F} + \gamma \frac{(\mathbf{P} - \mathbf{F})}{|\mathbf{P} - \mathbf{F}|} + \delta \mathbf{k} \times \frac{(\mathbf{P} - \mathbf{F})}{|\mathbf{P} - \mathbf{F}|} \tag{32b}$$

where

$$\alpha = \frac{(\mathbf{P} - \mathbf{C})^2 + b_1^2 - b_2^2}{2\ |\mathbf{P} - \mathbf{C}|}, \quad \beta = \sqrt{b_1^2 - \alpha^2} \tag{33a}$$

$$\gamma = \frac{(\mathbf{P} - \mathbf{F})^2 + b_3^2 - b_4^2}{2\ |\mathbf{P} - \mathbf{F}|}, \quad \delta = -\sqrt{b_3^2 - \gamma^2} \tag{33b}$$

The introduction of (32) and (33) into (31) gives the expressions of the vectors $\mathbf{q}_j$, $j = 1, \ldots, 4$, as function of the only generalized coordinates (coordinates of P in Cxy) and geometric constants of the manipulator.

The introduction of (29) and (30) into (28) yields

$$T_p = \frac{1}{2} \dot{\mathbf{P}}^T\, \mathbf{M}_p\, \dot{\mathbf{P}} \tag{34}$$

where

$$\begin{aligned} \mathbf{M}_p = {} & I_{C1}\left(\mathbf{q}_1 \mathbf{q}_1^T\right) + I_{P2}\left(\mathbf{q}_2 \mathbf{q}_2^T\right) + I_{F3}\left(\mathbf{q}_3 \mathbf{q}_3^T\right) \\ & + I_{P4}\left(\mathbf{q}_4 \mathbf{q}_4^T\right) + (m_{p2} + m_{p4}) \mathbf{1}_{2\times 2} \\ & + 2\left\{m_{p2} [\mathbf{k} \times (\mathbf{G}_{p2} - \mathbf{P})] \mathbf{q}_2^T \right. \\ & \left. + m_{p4} [\mathbf{k} \times (\mathbf{G}_{p4} - \mathbf{P})] \mathbf{q}_4^T \right\} \end{aligned} \tag{35}$$

Matrix $\mathbf{M}_p$ is the generalized inertia matrix that has to be used to compute the indices. It only depends on the coordinates of P in Cxy and on the geometric constants of the manipulator.

IV.1.3. Comparison of the passive dynamic performances. The two manipulators are compared at parity of weight and useful workspace. Moreover, their geometric parameters are chosen so that they have comparable overall sizes.

In particular, the RR manipulator has $\mathbf{A} = \{0, 0\}^T$, $(\mathbf{G}_{s2} - \mathbf{P}) = 0.5(\mathbf{B} - \mathbf{P})$ (all the position vectors are measure in Axy (see Fig. 1)) and the geometric and inertial parameters reported in Table I (only the inertial parameters involved in the computation of $\mathbf{M}_s$ are reported).

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Table II. Values of the inertial and geometric parameters of the RRRRR manipulator (see Fig. 2).

m_{p2} (kg)	m_{p4} (kg)	I_{C1} (kg m^2)	I_{P2} (kg m^2)	I_{F3} (kg m^2)	I_{P4} (kg m^2)	b_1 (m)	b_2 (m)	b_3 (m)	b_4 (m)
2.5	2.5	1.875	1.875	1.875	1.875	1.5	1.5	1.5	1.5

Moreover, the RRRRR manipulator has $\mathbf{C} = \{0,\ 0\}^T$, $\mathbf{F} = \{1, 0\}^T$, $(\mathbf{G}_{p2} - \mathbf{P}) = 0.5(\mathbf{D} - \mathbf{P})$, $(\mathbf{G}_{p4} - \mathbf{P}) = 0.5(\mathbf{E} - \mathbf{P})$ (all the position vectors are measure in Cxy (see Fig. 2) and the length unit is meter) and the geometric and inertial parameters reported in Table II (only the inertial parameters involved in the computation of $\mathbf{M}_p$ are reported).

Finally, the useful workspace of both the manipulators is assigned as follows: $x \in [0,1]$, $y, \in [0.5,1.5]$ where x and y are the coordinates of P (length unit is meter) in the reference system fixed to the frame (i.e., Axy for the RR manipulator (Fig. 1) and Cxy for the RRRRR manipulator (Fig. 2)).

In order to evaluate the performances, a two-dimensional grid with 400 nodes has been defined inside the workspace which is a square with side of 1 m (the elementary cell of the grid is a square with side of 0.05 m). The local indices k_m for $m = 0,1$, h and n have been evaluated at the nodes of the grid for both the manipulators. Finally, the corresponding global indices (I_m for $m = 0,1$, S and U) have been computed. The results of these computations are reported in Tables III, IV and V.

By comparing the results of the two manipulators, it comes out that the dynamic isotropy (Table III) and the non-linearity (Table V) exhibited by the RRRRR manipulator are better than the ones of the RR manipulator, whereas the swiftness (Table IV) of the RR manipulator is better than the one of the RRRRR manipulator (note that the global swiftness index, S, of the RR manipulator is equal to the constant value of h which a translating mass of 9.584 kg has; whereas the S of the RRRRR manipulator is equal to the constant value of h which a translating mass of 11.197 kg has). The results obtained for the dynamic isotropy and the non-linearity are in agreement with the feeling that symmetric architectures should be more dynamically isotropic and balanced than non-symmetric ones. The result obtained for the swiftness highlights that the serial manipulator is more agile than the parallel one; this is in agreement with the fact that serial manipulators have in general a reduced number of links.

IV.2. Use of the indices to improve the performances

In this subsection, the proposed indices will be used to derive design criteria for improving the dynamic performances of three manipulators with two degrees of freedom: two translational manipulators and one spherical manipulator. For the sake of brevity, manipulators with constant generalized inertia matrix have been chosen.

Table IV. Swiftness: minimum and maximum values of the local index, h, and values of the global index, S, for the two architectures, RR and RRRRR.

	h_{min} (kg^{-2})	h_{max} (kg^{-2})	S (kg^{-2})
RR	0.00268908	0.247018	0.010886
RRRRR	0.00431596	0.0093757	0.00797616

Table V. Non-linear inertial forces: minimum and maximum values of the local index, n, and values of the global index, U, for the two architectures, RR and RRRRR.

	n_{min} (kg^2 m^{-2})	n_{max} (kg^2 m^{-2})	U (kg^2 m^{-2})
RR	47.5554	21964.6	984.173
RRRRR	35.2802	5151.7	372.042

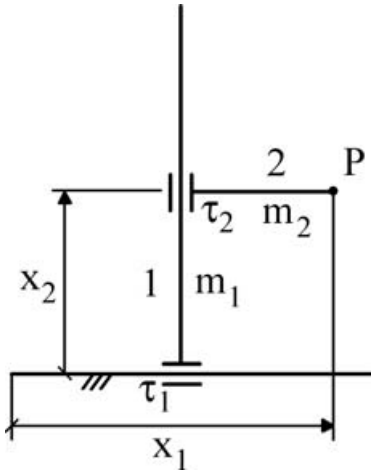


Fig. 3. Two-dof translational manipulator of type PP.

IV.2.1. Translational manipulators. Figure 3 shows a translational manipulator of type PP (P stands for prismatic pair) where the sliding directions of the two prismatic pairs are mutually perpendicular. With reference to Fig. 3, m_1 and m_2 are the masses of the links 1 and 2, respectively. Link 2 is the end effector and point P is the end-effector's reference point used for assigning the task (workspace, etc.), the manipulator has to accomplish in the operational space (plane of motion). The coordinates x_1 and x_2 locate the position of P in the plane of motion and are assumed as generalized coordinates of the manipulator. Finally, τ_1 and τ_2 are the forces applied to links 1 and 2, respectively, by the linear actuators mounted in the prismatic pairs.

With these notations, it is easy to demonstrate that the generalized inertia matrix, $\mathbf{M}_{PP}$, of this manipulator has the

Table III. Dynamic isotropy: minimum and maximum values of the local indices, k_0 and k_1, and values of the global indices, I_0 and I_1, for the two architectures, RR and RRRRR.

	$k_{0,min}$	$k_{0,max}$	I_0	$k_{1,min}$	$k_{1,max}$	I_1
RR	0.00333595	0.646251	0.334708	0.0132552	0.953826	0.690773
RRRRR	0.265639	0.599972	0.39525	0.663334	0.937489	0.798852

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following expression:

$$\mathbf{M}_{PP} = \begin{bmatrix} m_1 + m_2 & 0 \\ 0 & m_2 \end{bmatrix} \tag{36}$$

Moreover, the dynamic model can be written as follows:

$$\tau_1 = (m_1 + m_2)(\ddot{x}_1 - g_1) \tag{37a}$$

$$\tau_2 = m_2(\ddot{x}_2 - g_2) \tag{37b}$$

where g_1 and g_2 are the components of the gravity-acceleration vector along the sliding directions of the two prismatic pairs.

Since $\mathbf{M}_{PP}$ is constant in all the workspace the non-linear inertia terms are not present and the values of the local indices are constant and coincide with the values of the corresponding global indices. In particular, the following relationships hold:

$$k_0 = I_0 = \frac{m_2}{m_1 + m_2} \tag{38a}$$

$$h = S = \frac{1}{m_2\,(m_1 + m_2)} \tag{38b}$$

$$n = U = 0 \tag{38c}$$

Taking into account relationship (38a), the ratio of Eq. (37b) to Eq. (37a) yields

$$\frac{\tau_2}{\tau_1} = k_0 \frac{\ddot{x}_2 - g_2}{\ddot{x}_1 - g_1} \tag{39}$$

Provided the gravity-acceleration components are not considered, relationship (39) shows that, if k_0 was much less than 1, actuators greatly different would be necessary for contemporarily obtaining the same maximum acceleration in the two sliding directions ($(\tau_2/\tau_1) \cong k_0 \ll 1$). Therefore, increasing k_0 as much as possible by reducing the ratio m_1/m_2 is convenient for mounting two near equal actuators (it is worth noting that this manipulator cannot have dynamically isotropic configurations (see (38a)).

By taking into account relationship (38b), the product of (37a) by (37b) yields

$$(\ddot{x}_1 - g_1)(\ddot{x}_2 - g_2) = h(\tau_1\,\tau_2) \tag{40}$$

By neglecting the gravity-acceleration components and taking the square root of relationship (40), the following condition comes out

$$\ddot{x}_{g.av.} \cong \sqrt{h}\,\sqrt{\tau_1 \tau_2} \tag{41}$$

where $\ddot{x}_{g.av.}$ is the geometric average of the accelerations along the two sliding directions (i.e., $\ddot{x}_{g.av.} = \sqrt{\ddot{x}_1 \ddot{x}_2}$).

Relationship (41) shows that, at parity of maximum forces, τ_{1max} and τ_{2max}, the actuators can carry, the maximum average acceleration $\ddot{x}_{g.av.\,max}$, along the sliding directions, increases proportionally to the square root of the index h. Thus, the dynamic performances can be improved by increasing the value of h, which is obtained by reducing the masses of the mobile links (see (38b)).

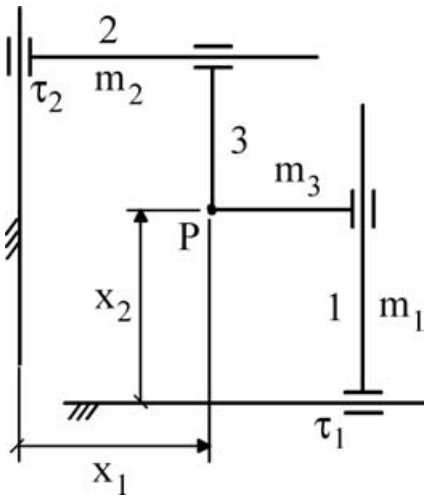


Fig. 4. Two-dof translational parallel manipulator of type 2PP.

Another type of two-dof translational manipulator is the one shown in Fig. 4. Figure 4 shows a parallel manipulator of type 2PP, where link 3 is the end effector, the adjacent prismatic pairs have sliding directions perpendicular to each other and the two prismatic pairs of the frame are the actuated pairs. With reference to Fig. 4, point P is the end-effector's reference point used for assigning the task in the operational space (plane of motion). The coordinates x_1 and x_2 locate the position of P in the plane of motion and are assumed as generalized coordinates of the manipulator. τ_1 and τ_2 are the forces applied to links 1 and 2, respectively, by the linear actuators mounted in the actuated prismatic pairs. Finally, m_1, m_2 and m_3 are the masses of links 1, 2 and 3 respectively.

With these notations, the generalized inertia matrix, $\mathbf{M}_{2PP}$, of the 2PP manipulator of Fig. 4 is

$$M_{2PP} = \begin{bmatrix} m_1 + m_3 & 0 \\ 0 & m_2 + m_3 \end{bmatrix} \tag{42}$$

Moreover, the dynamic model of this translational manipulator is

$$\tau_1 = (m_1 + m_3)(\ddot{x}_1 - g_1) \tag{43a}$$

$$\tau_2 = (m_2 + m_3)(\ddot{x}_2 - g_2) \tag{43b}$$

where g_1 and g_2 are the components of the gravity-acceleration vector along the sliding directions of the two actuated prismatic pairs.

Since $\mathbf{M}_{2PP}$ is constant in all the workspace the non-linear inertia terms are not present, and the values of the local indices are constant and coincident with the values of the corresponding global indices. In this case, the following relationships hold:

$$k_0 = I_0 = \frac{m_2 + m_3}{m_1 + m_3} \tag{44a}$$

$$h = S = \frac{1}{(m_1 + m_3)\,(m_2 + m_3)} \tag{44b}$$

$$n = U = 0 \tag{44c}$$

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Taking into account relationship (44a), the ratio of Eq. (43b) to Eq. (43a) yields

$$\frac{\tau_2}{\tau_1} = k_0 \frac{\ddot{x}_2 - g_2}{\ddot{x}_1 - g_1} \tag{45}$$

Relationship (45) is formally equal to relationship (39), but, now, the expression of k_0 is changed. The new expression of k_0 (relationship (44a)) makes it possible to have k_0 equal to 1 in the whole workspace (i.e., all the manipulator configurations are dynamically isotropic) by imposing, at the design stage, that m_1 is equal to m_2. So doing, the ratio of the accelerations, $\ddot{x}_2$ and $\ddot{x}_1$, along the sliding directions practically coincides with the ratio of the forces, τ_2 and τ_1, applied by the actuators. Thus, actuators, carrying the same maximum force, can be mounted in the two actuated prismatic pairs to match the design requirement that the same value of maximum acceleration must be reached at the same time along the two sliding directions.

Regarding the index h, through Eqs. (43) and relationship (44b), it can be demonstrated that condition (41) still holds. Thus, for this 2PP manipulator, the conclusion obtained for the above-analyzed PP manipulator still holds.

IV.2.2. Spherical manipulator. Figure 5 shows a manipulator of type RR where the revolute-pair axes z_1 and z_2 intersect each other in O and are mutually perpendicular. The axis z_2 of the second revolute pair is axis of symmetry of link 2 (which is the end effector). The center of gravity of link 2 coincides with O and the center of gravity of link 1 falls on z_1 (i.e. the manipulator is balanced). Hereafter $J_{2\parallel}$ and $J_{2\perp}$ will denote the moments of inertia of link 2 with respect to the axis z_2 and with respect to an axis perpendicular to z_2 respectively; moreover, J_1 will denote the moment of inertia of link 1 with respect to the axis z_1. With reference to Fig. 5, θ_1 and θ_2 are the joint variables, whereas τ_1 and τ_2 are the torques applied by the actuators to links 1 and 2 respectively (both the revolute pairs are actuated).

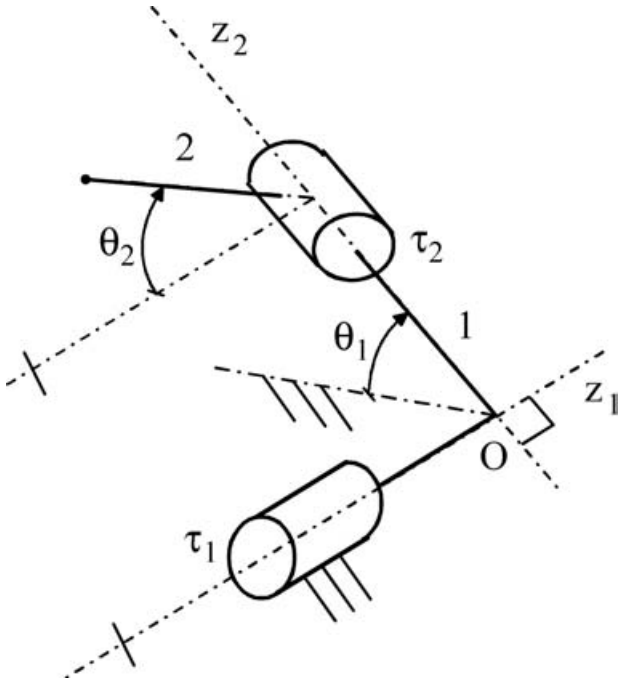


Fig. 5. Two-dof spherical manipulator of type RR.

With this geometry, the end effector is constrained to perform spherical motions with center O that do not involve rotations around the axis through O and perpendicular to the plane located by the axes z_1 and z_2 (this restriction to the end-effector spherical motion make the RR manipulator of Fig. 5 a spherical manipulator with only 2 dof). Thus, the operational space of this manipulator is the set of all the orientations the end effector can assume.

The orientation of the end effector with respect to the frame can be easily expressed through the angles θ_1 and θ_2 since the axes z_1 and z_2 are mutually perpendicular. Therefore, θ_1 and θ_2 can also be considered a set of coordinates of the operational space and will be assumed as generalized coordinates of the manipulator.

With these notations, it is easy to demonstrate that the generalized inertia matrix, $\mathbf{M}_{RR}$, of the RR manipulator of Fig. 5 is

$$\mathbf{M}_{RR} = \begin{bmatrix} J_1 + J_{2\perp} & 0 \\ 0 & J_{2\parallel} \end{bmatrix} \tag{46}$$

Moreover, its dynamic model is

$$\tau_1 = (J_1 + J_{2\perp})\,\ddot{\theta}_1 \tag{47a}$$

$$\tau_2 = J_{2\parallel}\,\ddot{\theta}_2 \tag{47b}$$

Since $\mathbf{M}_{RR}$ is constant in all the workspace the non-linear inertia terms are not present and the values of the local indices are constant and coincident with the values of the corresponding global indices. In particular, the following relationships hold:

$$k_0 = I_0 = \frac{J_{2\parallel}}{J_1 + J_{2\perp}} \tag{48a}$$

$$h = S = \frac{1}{J_{2\parallel}(J_1 + J_{2\perp})} \tag{48b}$$

$$n = U = 0 \tag{48c}$$

Taking into account relationship (48a), the ratio of Eq. (47b) to Eq. (47a) yields

$$\frac{\tau_2}{\tau_1} = k_0 \frac{\ddot{\theta}_2}{\ddot{\theta}_1} \tag{49}$$

Relationship (49) shows that, if $\ddot{\theta}_1$ and $\ddot{\theta}_2$ are required (design requirement) to reach the same maximum value contemporarily, the ratio of the maximum torques carried by the actuators is equal to k_0. Therefore, by imposing $k_0 = 1$ at the design stage (i.e., manipulator dynamically isotropic in all the workspace), the two actuators can be equal to each other. Relationship (48a) shows that $k_0 = 1$ is obtained by imposing that $(J_1 + J_{2\perp})$ be equal to $J_{2\parallel}$.

Finally, Eqs. (47) and relationship (48b) make it possible to deduce the following condition

$$\ddot{\theta}_{g.av.} = \sqrt{h}\,\sqrt{\tau_1 \tau_2} \tag{50}$$

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where $\ddot{\theta}_{g.av.}$ is the geometric average of $\ddot{\theta}_1$ and $\ddot{\theta}_2$ (i.e., $\ddot{\theta}_{g.av.} = \sqrt{\ddot{\theta}_1 \ddot{\theta}_2}$).

Relationship (50) shows that, at parity of maximum torques, τ_{1max} and τ_{2max}, the actuators can carry, the maximum value of $\ddot{\theta}_{g.av.}$ increases proportionally to the square root of the index h. Therefore, the dynamic performances can be improved by increasing the value of h, which is obtained by reducing the moments of inertia of the mobile links (see (48b)).

V. CONCLUSIONS

Four local indices and four global indices that fully describe the passive dynamic performances of manipulators with two degrees of freedom have been proposed. In particular, among the proposed indices, two local and two global indices measure the dynamic isotropy, one local and one global indices measure the swiftness, and, finally, one local and one global indices measure the non-linear inertia forces.

The proposed indices can be easily expressed in explicit form when the explicit expressions of the entries of the generalized inertia matrix of the manipulator are known. They make it possible to compare the passive dynamic performances of different manipulator architectures and can be used to build diagrams which highlight the effects of variations in the manipulator geometry on the manipulator dynamics. These features make them easy to be used in a design context.

Finally, some applications of the proposed indices have been presented and discussed to show their effectiveness in manipulator evaluation and design.

Acknowledgments

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