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(WEAK) COMPACTNESS CRITERIA IN NONCOMMUTATIVE SYMMETRIC SPACES

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This thesis is devoted to the study of norm compactness and weak compactness criteria of sets in the so-called noncommutative symmetric spaces of τ-measurable operators. It provides some new compactness criteria even in the classical case of commutative symmetric spaces. Following the Introduction and Preliminaries that include a historical overview of the research area and relevant definitions, we present three chapters that form the core of this research thesis, each corresponding to one of the three publications [1–3].

In Chapter 3, we strengthen and simplify the weak Grothendieck compactness principle for symmetric sequence spaces, and extend it to both symmetric function spaces and noncommutative symmetric spaces of measurable operators.

In Chapter 4, we present two new compactness criteria in noncommutative quasi-Banach symmetric spaces associated to a finite von Neumann algebra, with a focus on the noncommutative torus. The first result is novel even in the commutative setting, while the second resembles the classical Kolmogorov–Riesz compactness theorem.

Finally, in Chapter 5, we explore weak convergence of sequences in noncommutative symmetric spaces and related properties. In particular, we fully characterise noncommutative symmetric spaces $E(\mathcal{M},\tau)$ affiliated with a semifinite von Neumann algebra $\mathcal{M}$ equipped with a faithful normal semi-finite trace τ on a (not necessarily separable) Hilbert space that have the Gelfand–Phillips property and the WCG-property (weakly compactly generated, that is, there is a weakly compact set whose norm-closed linear span generates the whole space). A complete list of relations with other classical

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structural properties (such as the Dunford–Pettis property, the Schur property and their variations) is obtained in the general setting of noncommutative symmetric spaces.

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