J. Plasma Physics (2015), *vol.* 81, 905810104 © Cambridge University Press 2014
doi:10.1017/S0022377814000403

Numerical study of the sheath in magnetized dusty plasma with two-temperature electrons

I. Driouch†, H. Chatei and M. El Bojaddaini

Laboratory of Physics of Matter and Radiations, Department of Physics, Faculty of Science, University Mohammed I, BP. 717, 60000 Oujda, Morocco

(Received 8 December 2013; revised 22 March 2014; accepted 27 May 2014; first published online 14 July 2014)

Fluid simulations are used to investigate a multi-component magnetized dusty plasma sheath. The model consists of positive ions, dust grains, and two species of electron populations. These electrons are assumed to be a sum of two Maxwellian distributions with two different temperatures (cold and hot). According to multi-fluid equations and some dimensionless variables, the dimensionless equations are obtained and solved numerically. The effect of the presence of the hot electrons in the sheath is examined. A significant change is observed in the quantities characterizing the sheath with respect to one species electrons (cold) assumption.

1. Introduction

Numerous publications can be found in the literature devoted to the study of different properties of sheath in two species of negatively charged particles, including negative ions and electrons or two populations of electrons with different temperatures (Sheridan et al. 1991; Riemann 1995; Song et al. 1997; Ming et al. 2000; Bailung et al. 2004; Cavenago 2010; Aslaninejad and Yasserian 2012; Khoramabadi et al. 2013). Plasma with two-temperature electrons may occur mainly in edge plasma of magnetic fusion devices (Mizoshitaa et al. 1995; Stangeby 1995; Batishchev et al. 1997; Ou et al. 2013). For example, Ou et al. (2013) developed a one-dimensional model for studying the effect of two-temperature electron distribution on electrostatic plasma sheath. In their study, the high-temperature electrons are considered as truncated Maxwellian distribution (TMD), while the lower temperature electrons are described by Maxwellian distribution (MD).

In recent years, a sheath containing dust particles has become of considerable interest. It can be found in various technological devices, such as the device for plasma-assisted material and chemical processing, plasma discharges, as well as controlled fusion devices (Bouchoule 1999; Shukla and Mamum 2002). Moreover, the problem of plasma-wall interaction in the presence of dust particles has been investigated by several authors (Yu et al. 1992; Arnas et al. 2000; Nitter 1996; Xiu 2006). Therefore, the study of plasma sheath with multi-species ions, positive or negative, and dust particles, are available (Liu et al. 2000; Kalita and Das 2003; Wang et al. 2005, 2006; Xiu 2006; Foroutan 2010; Driouch et al. 2011a).

On the other hand, a number of studies have been carried out on the sheath plasma containing two species of electrons. For example, Chutov et al. (2003) and Kravchenko et al. (2003) considered the case of dusty sheaths in non-equilibrium plasmas with

† Email address for correspondence: driouch_ismael@yahoo.fr

https://doi.org/10.1017/S0022377814000403 Published online by Cambridge University Press

two-temperature electrons using kinetic computer simulation but they assumed that the plasma is collisionless and without magnetic field. The electrostatic nonlinear waves for dusty plasma in the presence of two electron populations at thermal equilibrium are considered in a recent work by Djebli and Marif (2009). Foroutan and Akhoundi (2012) considered the electrostatic sheath region of a low-pressure plasma consists of two species of electrons. The main purpose of the current work is to investigate the magnetized sheath containing two species of electrons, i.e. cold and hot electrons in thermal equilibrium by using a multi-fluid model.

In this work, we have also accounted for the collisions between the immobile neutral particles with dust particles via the constant collision frequency method. Based on the models presented for a plasma sheath containing one species of electrons in refs. Liu et al. (2004), Masoudi et al. (2009), Driouch et al. (2011b), our model is extended to include the effect of two populations of electrons. A multi-fluid approach is applied to study the effects of the hot electron component on the sheath structure as well as the characteristics of dust particles in the sheath.

The paper is organized as follows. In Sec. 2, we present our assumptions and the governing equations of the model. In Sec. 3, we solve the obtained equations numerically and analyze the obtained results. In Sec. 4, conclusions are presented.

2. Basic equations and assumptions

Following some correlative works (Liu et al. 2004; Masoudi et al. 2009; Driouch et al. 2011b; Driouch and Chatei 2013), we consider a magnetized stationary ($\partial/\partial t = 0$) dusty plasma sheath in contact with a planar wall (Fig. 1). The sheath is studied in a one-dimensional coordinate space system and a three-dimensional speed space system. The external magnetic field, which is spatially uniform and constant in time, lies in the $x-z$ plane and makes an angle θ with the x-axis. The region of interest lies between $x = 0$ (the sheath edge), and the wall which can be located anywhere in the region $x > 0$. We assume that the physical parameters change only along the x-direction (normal to the wall, see Fig. 1) in the sheath region.

At the sheath edge $x = 0$, the potential is assumed to be zero, and the ion, the dust, and the cold and hot electron species densities n_{i0}, n_{d0}, n_{ec0} and n_{eh0} satisfy the quasi-neutrality condition:

$$e(n_{i0} - n_{eh0} - n_{ec0}) + q_{d0}n_{d0} = 0. \tag{2.1}$$

At the wall surface $x = D$, all particles reaching the wall are assumed to be absorbed, where e is the electron charge and q_{d0} is the dust charge at the sheath edge. The two groups of electrons are assumed to be in thermal equilibrium state, so their density obeys the Boltzmann equation:

$$n_{\mathrm{ec}} = n_{\mathrm{ec0}} \exp\left(\frac{e\phi}{T_{\mathrm{ec}}}\right), \tag{2.2}$$

$$n_{\mathrm{eh}} = n_{\mathrm{eh0}} \exp\left(\frac{e\phi}{T_{\mathrm{eh}}}\right), \tag{2.3}$$

where n_{ec}, n_{eh} are densities of cold and hot electrons respectively, T_{ec} and T_{eh} are their corresponding temperature, and ϕ is the spatial electric potential in the sheath.

The ions are treated as a fluid; the basic equations of ions are the continuity:

$$\frac{\partial(v_{ix}n_i)}{\partial x} = 0 \tag{2.4}$$

https://doi.org/10.1017/S0022377814000403 Published online by Cambridge University Press

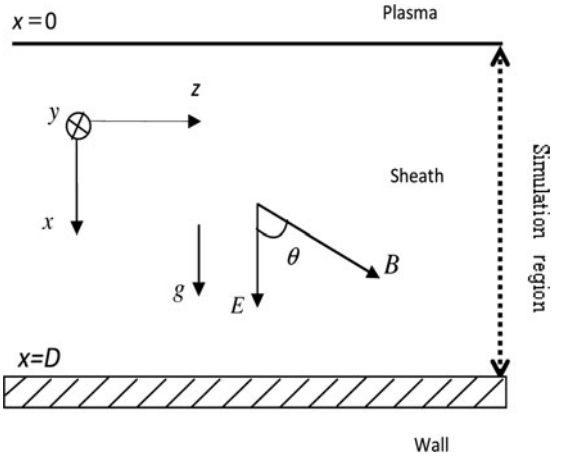


FIGURE 1. The geometry of the magnetized sheath model.

and the momentum conservation equations

$$m_i v_{ix} \frac{\partial v_{ix}}{\partial x} = -e \frac{\partial \phi}{\partial x}, \tag{2.5}$$

where v_{ix}, n_i, and m_i are the velocity in the depth direction, density, and mass of the positive ion species in the sheath, respectively. The dust grains are treated as cold fluid obeying the sources free. The electric, magnetic, gravitational, and collision dust-neutral forces are considered. The dust fluid equations are written as follows:

$$\frac{\partial (v_{dx} n_d)}{\partial x} = 0, \tag{2.6}$$

$$m_d v_{dx} \frac{\partial \vec{v_d}}{\partial x} = -q_d \frac{\partial \phi}{\partial x} \vec{x} + q_d \vec{v_d} \wedge \vec{B} + m_d g \vec{x} - \vec{F}_{\mathrm{cd}}, \tag{2.7}$$

where $q_d = e z_d$ is the charge of dust grain, z_d is the charge number of the dust, m_d, n_d and $\vec{v}_d$ are the mass, the density and velocity of the dust grain respectively, and F_{cd} is the drag force on the dust charged grain due to their collisions with neutrals. It is of the form:

$$\vec{F}_{\mathrm{cd}} = -m_d \nu(v_d) \vec{v}_d. \tag{2.8}$$

The dust collision frequency can be expressed as a function of neutral gas density n_n, the momentum transferring cross-section for collisions between dust and neutrals σ, and the total drift velocity v_d as follows:

$$\nu(v_d) = n_n \sigma(v_d) v_d. \tag{2.9}$$

The dependence between the momentum transferring cross-section and the dust velocity is given by (Mahanta and Goswami 2001; Masoudi and Taherparvar 2010)

$$\sigma(v_d) = \sigma_s (v_d / c_d)^p, \tag{2.10}$$

where $c_d = (T_e/m_d)^{1/2}$ is the dust acoustic speed, $\sigma_s = \pi r_d^2$ is the collision cross-section for collisions between the dust and neutrals at dust acoustic speed. p is a dimensionless parameter which can take various values ranging from 0 to -1. Therefore, the collision force can be considered as follows:

$$\vec{F}_{cd} = -m_d n_n \sigma_s \frac{v_d^{p+1}}{c_d^p} \vec{v}_d. \tag{2.11}$$

https://doi.org/10.1017/S0022377814000403 Published online by Cambridge University Press

In the existing literature there are usually two special cases: constant dust mean-free path (constant cross-section) and constant dust mobility corresponding to the two parameter values, $p = 0$ and $p = -1$, respectively. Here, we are interested only about the constant dust mobility case ($p = -1$).

In (2.5), absence of magnetic force is due to fact that the ions are excessively magnetized (because of their high velocity with respect to dust) (see Ping et al. 2007). Because similarity between our model and some correlative works, the equations are the same as basic equations for dusts and ions in plasma sheath like the references (Liu et al. 2004; Ping et al. 2007).

The system of equations is completed by the Poisson's equation which relates the electric potential to the density of dust grains, two electrons, and ions as follows:

$$\frac{\partial^2 \phi}{\partial x^2} = -4\pi(e(n_i - n_{ec} - n_{eh}) + q_d n_d). \tag{2.12}$$

In order to solve the system of (2.1)–(2.12), we introduce new dimensionless variables. We normalize each charged particle density to its value at the sheath edge: $N_i = n_i/n_{i0}$, $N_d = n_d/n_{d0}$, $N_c = n_{\mathrm{ec}}/n_{\mathrm{ec0}}$, $N_h = n_{\mathrm{eh}}/n_{\mathrm{eh0}}$. Also, we normalize the ion velocity to the ion sound speed $c_i = (T_{\mathrm{ec}}/m_i)^{1/2}$, and the dust velocity to the dust-acoustic speed $c_d = (T_{\mathrm{ec}}/m_d)^{1/2}$ respectively, i.e. $u_{ix} = v_{ix}/c_i$ and $\overrightarrow{u}_d = \overrightarrow{v}_d/c_i$. Furthermore, we scale the x-coordinate and the sheath potential as follows: $\xi = \frac{x}{\lambda_D}$ and $\eta = e\phi/T_{\mathrm{ec}}$, where $\lambda_D = (T_{\mathrm{ec}}/4\pi n_{e0}e^2)^{1/2}$ is the electron Debye length. The degree of collisionality in sheath β is given by the number of collisions in a Debye length: $\beta = \lambda_D/\lambda = \lambda_D n_n \sigma_s$, where λ is the mean free path of the dust. Taking the magnetic field of the form $\overrightarrow{B} = B_0(\cos(\theta)\overrightarrow{x} + \sin(\theta)\overrightarrow{z})$ and by substituting the above dimensionless variables into (2.2)–(2.12), we obtain:

$$N_c = \exp(\eta), \tag{2.13}$$

$$N_h = \exp\left(\frac{\eta}{T_{hc}}\right), \tag{2.14}$$

$$N_i = \frac{M_i}{u_{ix}}, \tag{2.15}$$

$$N_d = \frac{M_d}{u_{dx}}, \tag{2.16}$$

$$\frac{\partial u_{ix}}{\partial \xi} = \frac{-1}{u_{ix}} \frac{\partial \eta}{\partial \xi}, \tag{2.17}$$

$$u_{dx}\frac{\partial u_{dx}}{\partial \xi} = z_d \frac{\partial \eta}{\partial \xi} + z_d \gamma \sin(\theta) u_{dy} + f_g - \beta u_{dx}, \tag{2.18}$$

$$u_{dx}\frac{\partial u_{dy}}{\partial \xi} = z_d \gamma (u_{dz} cos(\theta) - u_{dx}\sin(\theta)) - \beta u_{dy}, \tag{2.19}$$

$$u_{dx}\frac{\partial u_{dz}}{\partial \xi} = -z_d \gamma u_{dy}\cos(\theta) - \beta u_{dz}, \tag{2.20}$$

$$\frac{\partial^2 \eta}{\partial \xi^2} = \delta N_i - (1-\alpha)\exp(\eta) - \alpha\exp(\frac{\eta}{T_{hc}}) + N_d(1-\delta), \tag{2.21}$$

where $\gamma = eB_0\lambda_D/(m_d T_e)^{1/2}$, $T_{\mathrm{hc}} = T_{\mathrm{eh}}/T_{\mathrm{ec}}$, $f_g = g\lambda_D/c_d^2$, $\alpha = n_{h0}/n_{c0} + n_{h0}$, $\delta = n_{i0}/n_{c0} + n_{h0}$. M_i is the ion Mach number and M_d is the dust Mach number. Here the boundary conditions are as follows: at the wall ($\xi = D/\lambda_D$), $\eta(D/\lambda_D) = \eta_w$, where D is the sheath length; at the sheath edge ($\xi = 0$), the boundary conditions are

https://doi.org/10.1017/S0022377814000403 Published online by Cambridge University Press

$\eta(\xi = 0) = 0$, $u_{ix}(\xi = 0) = M_i$, $u_{dx}(\xi = 0) = M_d$ and $u_{dy}(\xi = 0) = u_{dz}(\xi = 0) = 0$. To better understand the effect of the presence of the hot electrons on the characteristics of the sheath, we definite the density ratio of hot electrons n_{h0} to all electrons $n_{e0} = n_{c0} + n_{h0}$ as $\alpha = n_{h0}/n_{c0} + n_{h0}$, where n_{c0} and n_{h0} are the hot and cold electron densities in the plasma region. Also, we definite the temperature ratio of hot and cold electrons as $T_{\rm hc} = T_{\rm eh}/T_{\rm ec}$. From the definition of α it is clear that $\alpha = 0$ is the case where magnetized collisional plasma sheath consists of only one species of electrons (cold electrons), so the increase of α leads to an increase of the concentration of hot electron populations in the plasma sheath. Furthermore, from the Poisson equation (2.21), we can verify that if ($\alpha = 0$ or $T_{\rm hc} = 1$) the sheath consists of only one electron population (cold electrons), but in the case ($\alpha \neq 0$ and $T_{\rm hc} \neq 1$) the sheath consists of two electron populations.

3. Numerical results and discussions

To solve numerically the multi-fluid equations (2.13)–(2.21) we use $n_{e0} = 10^9$ (cm^{-3}), $T_e = 2$ (eV), $M_i = 3$, $M_d = 5$, $\delta = 1.01$, $N_{i0} = 1$, $N_{d0} = 1$, $N_{c0} = 1$, $N_{h0} = 1$, and the normalized electric field is considered as a small finite value $\partial\eta/\partial\eta(\xi = 0) = 0.01$ as boundary conditions at the plasma–sheath interface ($\xi = 0$). These parameters are the same that those used in previous papers (Masoudi et al. 2009; Driouch et al. 2011a; Driouch and Chatei 2013). Also, in our calculations, we consider the dust as a spherical particle of $r_d = 4$ μm radius with uniform mass density $\rho_d = 2$ (g/cm^3) and $z_d = -1000$. The normalized magnetic field intensity and the angle of incidence are respectively, $\gamma = 0.3$ and $\theta = 30^\circ$. The position of the wall is determined from the value of its potential, considered to be $\eta_w = -30$ in this work. Equations (2.13)–(2.21) with appropriate boundary conditions are solved using a fourth-order Runge–Kutta method.

The parameters α and $T_{\rm hc}$ are used as the major input parameters and varied in the numerical calculations to investigate the properties of the sheath.

Figure 2 shows the distribution of the normalized sheath potential and plasma densities in the sheath and their dependence on the presence of the hot electron populations. From this figure, we can observe that by increasing the hot electrons density (i.e. the increase of α), the sheath width increases considerably as proved by the η profile in Fig. 2(a). Also, it is seen that the densities of ions, cold and hot electrons decrease monotonically toward the wall with increasing the ratio α. This effect can be explained as follows: the increase of α gives rise to the growth of negative charge inside the sheath; and as a result, decreases the electric force. So for large values of α, the electric force is less effective; as a result, the velocity of ions in x-depth direction becomes smaller. Because of the inverse relation between the velocity in the depth direction and the ion density equation (2.15), this latter becomes larger (Fig. 2(b)). Therefore, the net space charge density becomes larger too which leads to a broadened plasma sheath. We note that our results show a similarity with the behavior of the sheath containing the ions and two species of electron temperature (Ou et al. 2013).

Now, we turn our attention to account for the hot electron population effects on the dust dynamics. We assume that the hot electrons cannot affect the dust velocity distribution directly ((2.18)–(2.20)). From the Poisson equation, the presence of hot electrons leads to the modification of sheath potential (also electric field). And this brings about a modification in x-component of the dust velocity distribution (2.18). We illustrate in Fig. 3, the effects of the density ratio α on the distributions of dust

https://doi.org/10.1017/S0022377814000403 Published online by Cambridge University Press

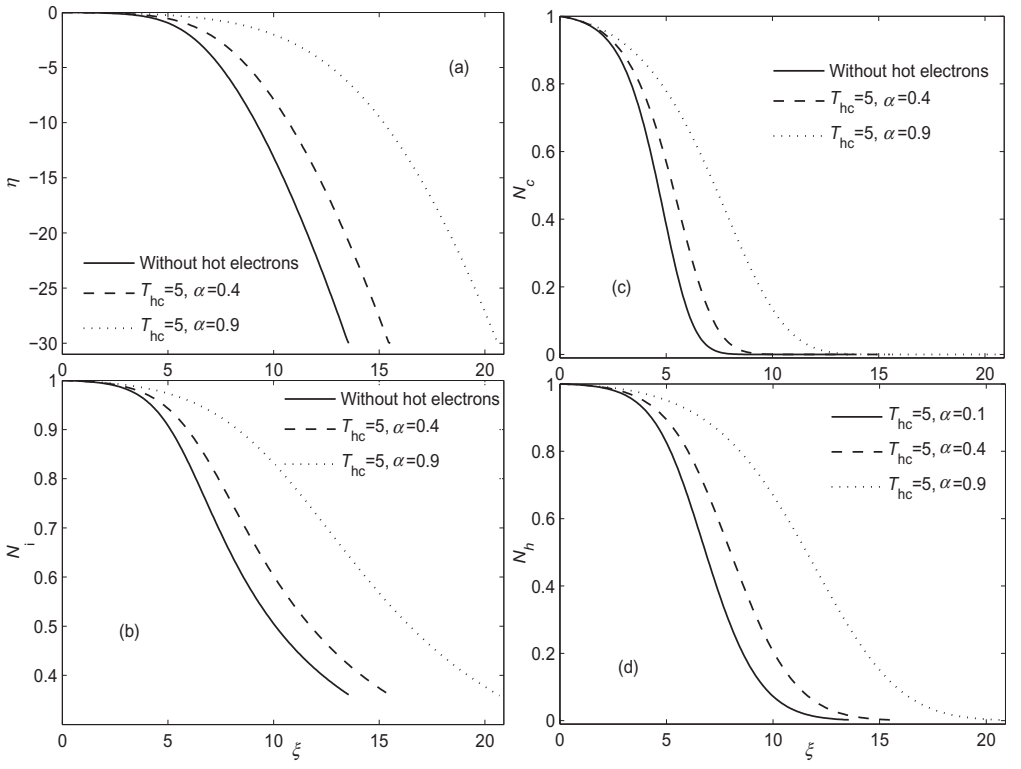


FIGURE 2. The normalized distributions of spatial electrostatic potential (a), the normalized ion density (b), the normalized cold electron density (c), and the normalized hot electron density (d) versus ξ under various α values and for the parameters: $\beta = 0.1$, $\theta = 30$, $\gamma = 0.3$, $z_d = -1000$ and $T_{hc} = 5$.

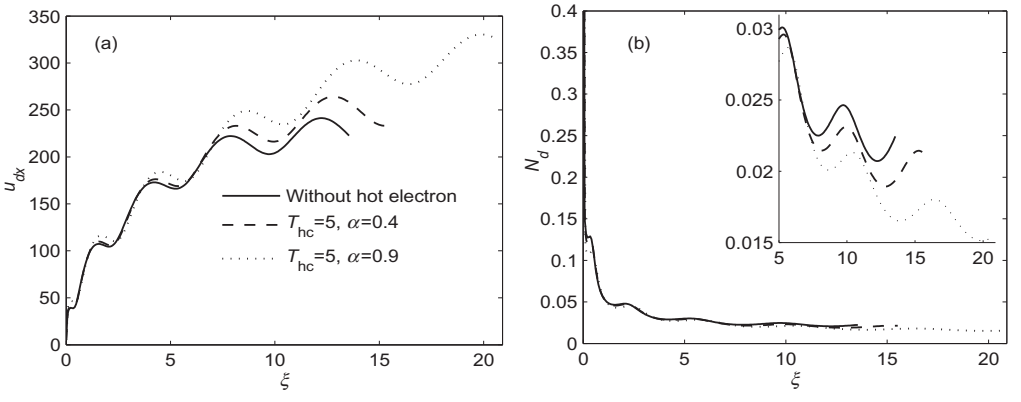


FIGURE 3. The normalized distributions of spatial dust velocity in the depth direction (a) and the normalized distributions of spatial dust density (b) versus ξ for different α values, the other parameters are the same as Fig. 2.

density and dust velocity in depth direction. The figure shows that the presence of hot electron populations has a similar effect on dust velocity. For a fixed value of T_{hc}, by increasing α (increasing the hot electron populations) the velocity of dust in depth direction increases. Because of the inverse relation between the velocity in the depth

https://doi.org/10.1017/S0022377814000403 Published online by Cambridge University Press

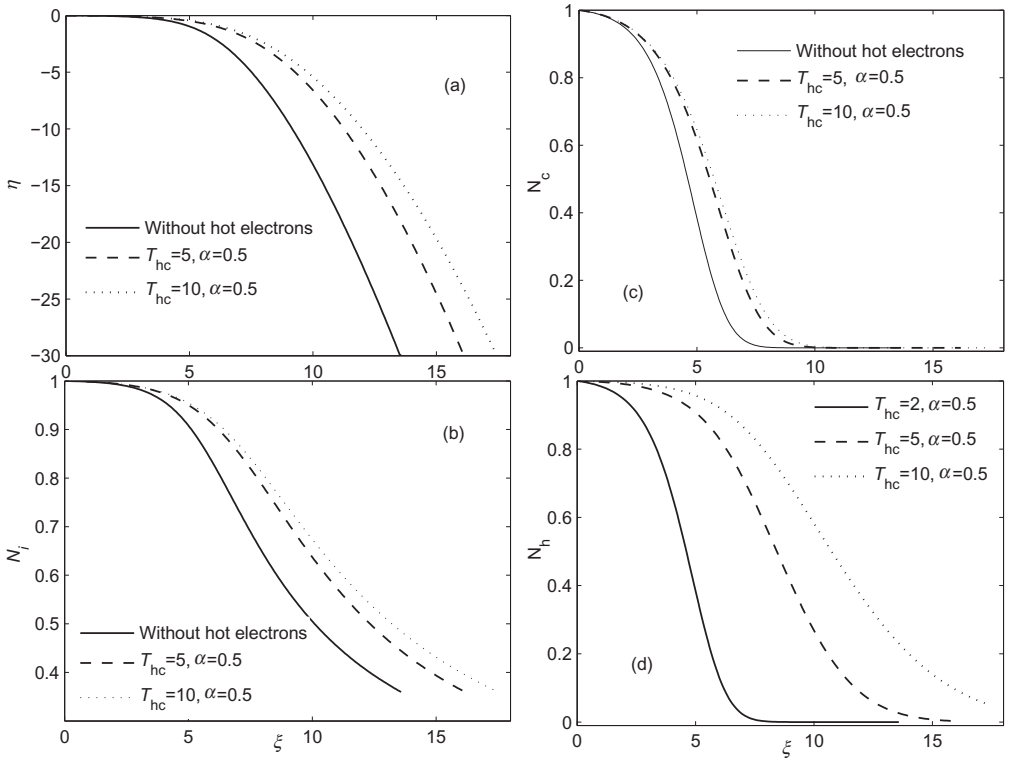


FIGURE 4. The normalized distributions of spatial electrostatic potential (a), the normalized ion density (b), the normalized cold electron density (c) and the normalized hot electron density (d) versus ξ under various T_{hc} values and $\alpha = 0.5$, the other parameters are the same as Fig. 2.

direction and the dust density equation (2.16), the results can be related to the density of dust, thus the dust density decreases by increasing α. The reason is for large values of α the electric force becomes less effective in favor of the gravitational force which leads to the acceleration of the dust grains. Consequently, the dust grains fall on the wall farther from the sheath edge.

Figure 4 shows the effect of T_{hc} ratio on the sheath parameters such as the normalized potential, the ion density, and the cold and hot electron densities. It is shown that in comparison with the case in which the sheath consists of only one group of electrons (cold electrons), i.e. $T_{hc} = 1$, the presence of both electrons ($T_{hc} \neq 1$) has a considerable effect on these parameters. It is clearly seen that the increase of the temperature ratio of the hot electrons to cold electrons (i.e. increasing T_{hc}) increases the sheath width, while the absolute potential increases monotonically. The densities of ions and two species of electrons are slowly rarefied at large distance from the sheath edge for the larger T_{hc} compared with the case of one species of electrons ($T_{hc} = 1$).

Figure 5 displays the spatial distribution of the dust density and dust velocity for different values of the temperature ratio T_{hc}. It is shown that by increasing the temperature ratio of the hot electrons to cold electrons (T_{hc}), the dust density decreases and the dust velocity increases. Also, the dust grains move at a large distance from the sheath edge for larger T_{hc}. To explain this effect, we note that the higher T_{hc}

https://doi.org/10.1017/S0022377814000403 Published online by Cambridge University Press

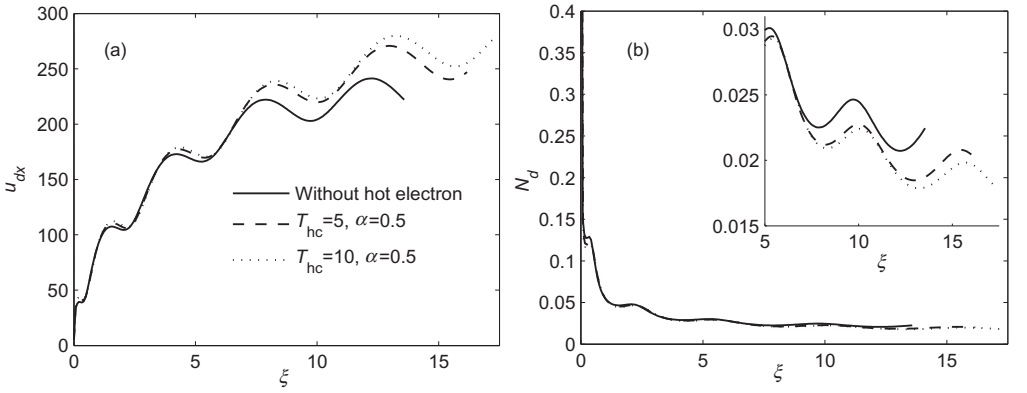


FIGURE 5. The normalized distributions of spatial dust velocity in the depth direction (a) and the normalized distributions of spatial dust density (b) versus ξ for different T_{hc} values, the other parameters are the same as Fig. 4.

induces more energetic electrons inside the sheath, thus decreases in the electric field. Consequently, the effect of the decelerating electric force decreases for the higher T_{hc}.

4. Conclusion

In this paper, we have investigated a magnetized sheath of dusty plasma composed of two groups of electrons with different temperatures (cold and hot). We have assumed that both the electrons are distributed according the Boltzmann relation. The study focuses on the effect of hot electron populations on some quantities characterizing the magnetized plasma sheath such as the electrostatic potential, the sheath thickness, and dust velocity. Based on multi-fluid model, the structures of the sheath have been investigated for different values of the density ratio of the hot electrons to the total densities of all electrons and temperature ratios of hot-to-cold electron populations. The numerical results show that, with the increase in the hot electron populations or the temperature ratio of hot-to-cold electrons the sheath thickness increases. Also, for higher α or T_{hc} the densities of ions and two species of electrons are slowly rarified. Furthermore, our numerical results reveal that the higher values of the density ratio of the hot electrons to the total densities α and the temperature ratio of hot-to-cold electrons α, cause the dust grains to move farther to the sheath edge.

Acknowledgement

This work is supported by the National Center for Scientific and Technical Research (CNRST), under contract: URAC-07.

REFERENCES

Arnas, C., Mikikian, M., Bachet, G., Doveil, F., Ma, J. X. & Yu, C. X. 2000 *Phys. Plasmas* **7**, 4418.
Aslaninejad, M. & Yasserian, K. 2012 *Phys. Plasmas* **19**, 033504.
Bailung, H., Boruah, D., Pal, A. R. & Chutia, J. 2004 *Pramana-J. Phys.* **62**.
Batishchev, O. V., et al. 1997 *Phys. Plasmas* **4**, 1672.
Bouchoule, A. 1999 *Dusty Plasma: Physics, Chemistry and Technical Impact in Plasma Processing*. New York: Wiley.

https://doi.org/10.1017/S0022377814000403 Published online by Cambridge University Press

Cavenago, M. 2010 *Rev. Sci Instrum.* **81**, 02B501.
Chutov, Yu. I., Kravchenko, O. Yu., Pshenychnyj, A. F., Smirnov, R. D., Asano, K., Ohno, N., Takamura, S. & Tomita, Y. 2003 *Phys. Plasmas* **10**, 546.
Djebli, M. & Marif, H. 2009 *Phys. Lett*. A **373**, 2572.
Driouch, I. & Chatei, H. 2013 *J. Appl. Fluid. Mech.* **6**, 511.
Driouch, I., Chatei, H. & Elkaouini, M. 2011a *Int. Rev Physics.* **5**, 306.
Driouch, I., Chatei, H., Elkaouini, M., El Bojaddaini, M. & EL Hammouti, M. 2011b *Phys. Chem News.* **62**, 50.
Foroutan, G. 2010 *Phys. Plasmas* **17**, 123711.
Foroutan, G. & Akhoundi, A. 2012 *Phys. Lett*. A **376**, 2444.
Kalita, P. & Das, G. C. 2003 *J. Plasma Phys.* **69**, 551.
Khoramabadi, M., Ghomi, H. & Shukla, P. K. 2013 *J. Plasma Phys.* **79**, 267.
Kravchenko, O. Yu., Chutov, Yu. I., Pshenychnyj, A. F., Smirnov, R. D. & Takamura, S. 2003 *J. Nucl. Mater*. **313-316**, 1109.
Liu, J. Y., Wang, D. & MA, T. C. 2000 *Vacuum.* **59**, 126.
Liu, J. Y., Zhang, Q., Zou, X. X. Wang Z. & Gong, Y. 2004 *Vacuum.* **73**, 687.
Mahanta, M. K. & Goswami, K. S. 2001 *Pramana-J. Phys.* **56**, 579.
Masoudi, S. F., Jafari, G. R. & Shorakaee, H. A. 2009 *Vacuum.* **83**, 1031.
Masoudi, S.F. & Taherparvar, P. 2010 *J. Fusion Energy* **29**, 240.
Ming, Li., Vyvoda, M. A., Dew, S. K. L. & Brett, M. J. 2000. *IEEE Trans. Plasma Sci.* **28**, 248.
Mizoshitaa, S., Shiraishib, K., Ohnoa, N. & Takamura, S. 1995 *J. Nucl. Mater*. **220**, 488.
Nitter, T. 1996 *Plasma Source Sci. Technol.* **5**, 93.
Ou, J., Xiang, N., Gan, C. & Yang, J. 2013 *Phys. Plasmas* **20**, 063502.
Ping, D., Liu, G., Gong, Y., Liu, Y. & Xiaogang, W. 2007 *Plasma. Sci. Technol.* **9**, 394.
Riemann, K. U. 1995 *IEEE Trans. Plasma Sci.* **23**, 709.
Sheridan, T. E., Goeckner, M. J. & Goree, J. 1991 *J. Vac. Sci. Technol.* **9**, 688.
Shukla, P. K. & Mamum, A. A. 2002 *Introduction to Dusty Plasma Physics*. Bristol: Institute of Physics.
Song, S. B., Chang, C. S. & Choi, Duk-In. 1997 *Phys. Rev* E **55**, 1213.
Stangeby, P. C. 1995 *Plasma Phys. Control Fusion* **37**, 1031.
Wang, Z. X., Liu, J. Y., Liu, Y. & Wang, X. 2005 *Phys. Plasmas* **12**, 012104.
Wang, Z. X., Liu, Y., Ren, L. W., Liu, J. Y. & Liu, Y. 2006 *Thin Solid Films* **506-507**, 637.
Xiu, Z. 2006 *Chin. Phys Lett*. **23**, 396.
Yu, M. Y., Seleem, H. & Luo, H. 1992 *Phys. Fluids* **B4**, 3427.

https://doi.org/10.1017/S0022377814000403 Published online by Cambridge University Press