On HAP triangles

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1. *Introduction*

It is a well known and easily verifiable fact that not all integer triangles have integer areas. Consider the triangles with sides {9, 10, 17}, {13, 14, 15}, {5, 7, 8} and {6, 7, 9} with respective areas 36, 84, $10\sqrt{3}$ and $2\sqrt{110}$. The first two, whose areas are integers, are called *Heronian* triangles. The second triangle also has the additional property that its sides are consecutive integers and is an example of a *Brahmagupta* triangle, named after an Indian mathematician, born in AD 598. These are called Super-Heronian triangles in [1] and a method is developed there for generating examples of such triangles.

Both [2] and [3] investigate the more general problem of finding all Heronian triangles whose sides form an arithmetic progression (AP) and in each case, the problem is reduced to that of solving a Diophantine equation of degree 2. Our method involves developing results known to be valid for all Heronian triangles in the particular case where the sides also form an AP. We shall refer to such triangles as HAP triangles. It is, however, important to note that this notation is also used in a slightly different context in [3], where the triangles involved may have rational sides.

Throughout this Article, given an integer triangle T and a rational r, we shall use rT to denote the triangle similar to T whose sides are r times the lengths of those of T. An integer triangle will be called *primitive* if the gcd of its sides is 1.

2. *Use of previous results*

Let J be an HAP triangle. Since its sides form an AP, its angles must be distinct and we shall assume these to be $\theta < \phi < \sigma$. The following result arises from the fact that J is Heronian.

Theorem 1

(i) There exist unique rationals β, δ with $0 < \beta < 1$, $0 < \delta < 1$ such that $\cos\theta = \dfrac{1-\beta^2}{1+\beta^2}$ and $\cos\phi = \dfrac{1-\delta^2}{1+\delta^2}$.

(ii) J is similar to the triangle $U(\beta, \delta)$ with sides $\beta(1+\delta^2)$, $\delta(1+\beta^2)$, $(\beta+\delta)(1-\beta\delta)$, where $\beta(1+\delta^2)$ is opposite to θ, $\delta(1+\beta^2)$ is opposite to ϕ and $(\beta+\delta)(1-\beta\delta)$ is opposite to σ, and thus

$$\beta(1+\delta^2) < \delta(1+\beta^2) < (\beta+\delta)(1-\beta\delta).$$

(iii) $\beta < \frac{1}{\sqrt{3}}$.

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Proof:

Parts (i) and (ii) are particular cases ($m = 1$) of results proved in Theorems 1 and 2 in [4]. In order to prove (iii), we note that θ, being the smallest angle in J, must be less than $60°$ and thus $\cos\theta > \frac{1}{2}$. The result now follows from the expression given for $\cos\theta$ in (i).

Let us now return to the fact that the sides of J form an AP. Since the sides of any triangle similar to J will also form an AP, it follows from Theorem 1 above that

$$\beta(1 + \delta^2) + (\beta + \delta)(1 - \beta\delta) = 2\delta(1 + \beta^2),$$

which implies that $\delta = \dfrac{2\beta}{1 + 3\beta^2}$.

The sides of $U(\beta, \delta)$ now become

$$\beta\left\{1 + \frac{4\beta^2}{(1 + 3\beta^2)^2}\right\}, \frac{2\beta(1 + \beta^2)}{1 + 3\beta^2}, \left\{\left(\beta + \frac{2\beta}{1 + 3\beta^2}\right)\left(1 - \frac{2\beta^2}{1 + 3\beta^2}\right)\right\}.$$

Simple algebraic manipulation shows that $U(\beta, \delta)$, and thus J, is similar to the triangle $V(\beta)$ with corresponding sides $1 + 9\beta^2, 2(1 + 3\beta^2), 3(1 + \beta^2)$.

If we put $\beta = \frac{\beta_1}{\beta_2}$, where $\gcd(\beta_1, \beta_2) = 1$, we see that $V(\beta)$ is itself similar to the integer triangle $T(\beta)$ with sides $a = \beta_2^2 + 9\beta_1^2$, $b = 2(\beta_2^2 + 3\beta_1^2)$, $c = 3(\beta_2^2 + \beta_1^2)$. Since $T(\beta)$ is similar to J, its sides must form an AP, and any integer triangle similar to a Heronian triangle must itself be Heronian (see [5, p 105]). Thus $T(\beta)$ is an HAP triangle similar to J.

Table 1 below gives the values of the sides a, b, c of the triangle $T(\beta)$ for four simple values of β. It is left to the reader to verify that these four triangles are in fact HAP triangles and also to generate $T(\beta)$ for further values of $\beta < \frac{1}{\sqrt{3}}$.

β	β_1	β_2	$a = \beta_2^2 + 9\beta_1^2$	$b = 2(\beta_2^2 + 3\beta_1^2)$	$c = 3(\beta_2^2 + \beta_1^2)$
$\frac{1}{3}$	1	3	18	24	30
$\frac{2}{5}$	2	5	61	74	87
$\frac{3}{7}$	3	7	130	152	174
$\frac{1}{6}$	1	6	45	78	111

TABLE 1

3. *Properties of the triangle $T(\beta)$*

It is also instructive to consider the following first principles proof that $T(\beta)$ is a HAP triangle.

First of all, we note that $d = b - a = c - b = \beta_2^2 - 3\beta_1^2$

Also, denoting the semiperimeter of $T(\beta)$ by $s = \frac{1}{2}(a + b + c)$, we

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see that $s = 3(\beta_2^2 + 3\beta_1^2)$, $s - a = 2\beta_2^2$, $s - b = \beta_2^2 + 3\beta_1^2$, $s - c = 6\beta_1^2$, and thus, by Heron's theorem, the area of $T(\beta)$ is given by $6\beta_1\beta_2(\beta_2^2 + 3\beta_1^2)$, and is an integer.

We note that this proof does not in any way depend on the value of β and we may therefore extend the definition of $T(\beta)$ to all positive rationals β. In the particular case $\beta < \frac{1}{\sqrt{3}}$, $d > 0$ and the shortest side is $a = \beta_2^2 + 9\beta_1^2$. A simple application of the the cosine rule tells us now that $T(\beta)$ is an HAP triangle whose smallest angle has cosine $\dfrac{1 - \beta^2}{1 + \beta^2}$.

Thus we have proved the following result.

Theorem 2

(i) $\left\{T(\beta) \mid \beta < \frac{1}{\sqrt{3}}\right\}$ is a set of non-similar HAP triangles.

(ii) Every HAP triangle J is similar to one such $T(\beta)$.

The question now arises: what role does $T(\beta)$ play when $\beta > \frac{1}{\sqrt{3}}$?

Consider the following examples.

It may be easily verified that $T\left(\frac{2}{1}\right)$ has sides 37, 26, 15 and is indeed an HAP triangle. We also note from Table 1 above that it is similar to $T\left(\frac{1}{6}\right)$. In the same way, it may be shown that $T\left(\frac{3}{2}\right)$ is similar to $T\left(\frac{2}{9}\right)$, that $T\left(\frac{1}{1}\right)$ is similar to $T\left(\frac{1}{3}\right)$ and that $T\left(\frac{4}{3}\right)$ is similar to $T\left(\frac{1}{4}\right)$. We note that

$$\frac{2}{1} \times \frac{1}{6} = \frac{3}{2} \times \frac{2}{9} = \frac{1}{1} \times \frac{1}{3} = \frac{4}{3} \times \frac{1}{4} = \frac{1}{3}.$$

It may also easily be verified that in each of the cases above, the lengths of the sides of one of the triangles generated are three times those of the other triangle. This suggests the following possible result.

Lemma 1

Let β, μ be positive rationals such that $\beta\mu = \frac{1}{3}$. Then $T(\mu) = kT(\beta)$, where $k = 3$ or $\frac{1}{3}$ and thus, in particular, $T(\mu)$ and $T(\beta)$, are similar HAP triangles.

Proof

Write $\beta = \frac{\beta_1}{\beta_2}, \mu = \frac{\mu_1}{\mu_2}$, where $\gcd(\beta_1, \beta_2) = \gcd(\mu_1, \mu_2) = 1$.

Then $\frac{\beta_1}{\beta_2} \times \frac{\mu_1}{\mu_2} = \frac{1}{3}$ and thus $\beta_2\mu_2 = 3\beta_1\mu_1$.

We note that 3 must divide one of β_2, μ_2 but it cannot divide both since otherwise 3 would be a common factor of either β_1 and β_2 or μ_1 and μ_2.

Let us first of all assume that 3 does not divide β_2. It follows that $\gcd(3\beta_1, \beta_2) = 1$ and therefore we must have $\mu_1 = \beta_2$ and $\mu_2 = 3\beta_1$. Simple substitution in the expressions for the sides of $T(\mu)$ now shows us that $T(\mu) = 3T(\beta)$. In the same way, it follows that if 3 does not divide μ_2 then $T(\mu) = \frac{1}{3}T(\beta)$.

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We note also that if $\beta\mu = \frac{1}{3}$, then $\beta < \frac{1}{\sqrt{3}}$ if, and only if, $\mu > \frac{1}{\sqrt{3}}$ and thus we may write down the following result, equivalent to Theorem 2, where each $T(\beta)$ $(\beta < \frac{1}{\sqrt{3}})$ is replaced by the corresponding similar triangle $T(\mu)$ $(\mu > \frac{1}{\sqrt{3}})$.

Theorem 3

(i) $\left\{T(\mu) \mid \mu > \frac{1}{\sqrt{3}}\right\}$ is a set of non-similar HAP triangles.

(ii) Every HAP triangle J is similar to one such $T(\mu)$.

It is again recommended that the reader verify that the triangles given in Table 2 below are indeed HAP triangles.

β	β_1	β_3	$a = \beta_2^2 + 9\beta_1^2$	$b = 2(\beta_2^2 + 3\beta_1^2)$	$c = 3(\beta_2^2 + \beta_1^2)$
$\frac{4}{5}$	4	5	169	146	123
$\frac{5}{6}$	5	6	261	222	183
$\frac{3}{1}$	3	1	82	56	30
$\frac{7}{3}$	7	3	450	312	174

TABLE 2

Note that the shortest side in the triangles in Table 1 is a, whereas it is c in Table 2. This will always be the case since the common difference $d > 0$ when $\beta < \frac{1}{\sqrt{3}}$ and $d < 0$ when $\beta > \frac{1}{\sqrt{3}}$.

We are now able to state the following result.

Theorem 4

(i) $\left\{T\left(\frac{y}{z}\right) \mid y, z \text{ positive integers with } \gcd(y, z) = 1 \text{ and } z \text{ not divisible by } 3\right\}$ is a set of non-similar HAP triangles.

(ii) Every HAP triangle J is similar to one such $T\left(\frac{y}{z}\right)$.

Proof

As noted in the proof of Lemma 1 above, if $\beta\mu = \frac{1}{3}$, then one of β_2, μ_2 will be divisible by 3 and the other will not. Let z denote the one *not* divisible by 3 and y the corresponding β_1 or μ_1. The result now follows from Theorems 2 and 3, noting that our choice of y, z ensures that none of the triangles generated can be similar to one another.

In all that follows, the variables y and z will always represent positive integers which are such that z is not divisible by 3 and $\gcd(y, z) = 1$, as in Theorem 4.

4. *The triangles* $T\left(\frac{y}{z}\right)$ *and* $P\left(\frac{y}{z}\right)$

First of all, we look more closely at the triangle $T\left(\frac{y}{z}\right)$ whose sides are

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now given in the above notation by $a = z^2 + 9y^2$, $b = 2(z^2 + 3y^2)$, $c = 3(z^2 + y^2)$ and where the common difference is given by $d = z^2 - 3y^2$.

Lemma 2

Let $h = \gcd(a, b, c)$. Then $h = 1$ if one of y, z is even and the other odd, and $h = 2$ if both y and z are odd.

Proof

Note that $h \mid 2a - b$ and thus $h \mid 12y^2$,

and that $h \mid 2c - b$ and thus $h \mid 4z^2$.

Since $\gcd(y, z) = 1$ and z is not divisible by 3, we see that h must be 1, 2 or 4. If one of y, z is even and the other odd then each of a and c is odd and thus $h = 1$. If both y and z are odd then each of a, b, c is even but both a and $c \equiv 2 \pmod 4$ and thus $h = 2$.

We can now write down our main result.

Theorem 5

Let y, z be as above. Define the integer triangle $P\left(\frac{y}{z}\right)$ as follows.

$P\left(\frac{y}{z}\right) = T\left(\frac{y}{z}\right)$ if one of y, z is even and the other odd, $P\left(\frac{y}{z}\right) = \frac{1}{2}T\left(\frac{y}{z}\right)$ if both y and z are odd. Then $P\left(\frac{y}{z}\right)$ is a primitive HAP triangle similar to $T\left(\frac{y}{z}\right)$.

Proof

All that needs to be proved is that $P\left(\frac{y}{z}\right)$ is actually an HAP triangle when both y and z are odd. Its sides certainly form an AP. Since $P\left(\frac{y}{z}\right)$ is an integer triangle which is similar to $T\left(\frac{y}{z}\right)$, it must also be Heronian (see [5, p 105]).

Before proceeding further, readers may wish to familiarise themselves with the content of this section by choosing their own values for y and z, calculating the sides of $T\left(\frac{y}{z}\right)$ and $P\left(\frac{y}{z}\right)$ and then verifying in each case that $P\left(\frac{y}{z}\right)$ actually is a primitive HAP triangle.

We shall henceforth use a', b', c' to denote the sides of $P\left(\frac{y}{z}\right)$ corresponding to a, b, c in $T\left(\frac{y}{z}\right)$ and d' to denote the common difference.

5. *Properties of* $T\left(\frac{y}{z}\right)$ *and* $P\left(\frac{y}{z}\right)$

We have already shown that the area of $T\left(\frac{y}{z}\right)$ is given by $6yz(z^2 + 3y^2)$. This fact, together with Lemma 2, yields the following result.

Theorem 6

$$\text{Area } P\left(\frac{y}{z}\right) = 6yz(z^2 + 3y^2), \text{ if one of } y, z \text{ is even and the other odd,}$$

$$= \frac{6yz(z^2 + 3y^2)}{4}, \text{ if both } y \text{ and } x \text{ are odd.}$$

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We observe that when both y and z are odd, $z^2 + 3y^2 \equiv 0 \pmod 4$ and thus in both cases, the area of $P\left(\frac{y}{z}\right)$ will always be an integer divisible by 6. This is not unexpected, since it is a well-known result for any Heronian triangle (see [5, p 105]).

Since the area of any triangle is the product of its inradius and its semi-perimeter, it follows that in our case, the inradius of $T\left(\frac{y}{z}\right)$ of is given by $\frac{6yz\left(z^2 + 3y^2\right)}{3\left(z^2 + 3y^2\right)} = 2yz$, which leads immediately to the following result for $P\left(\frac{y}{z}\right)$.

Theorem 7

$$\text{Inradius}\, P\left(\frac{y}{z}\right) = 2yz, \text{ if one of } y, z \text{ is even and the other odd,}$$

$$= yz, \text{ if both } y \text{ and } z \text{ are odd.}$$

We also have the following simple corollary.

Corollary 1

Let n be a positive integer. There exists a primitive HAP with inradius n if, and only if, n is not congruent to 2 (mod 4).

Proof

We note that the inradius will be odd if both y and z are odd and will be divisible by 4 if one of y, z is even and the other odd. If n is odd, put $y = n$ and $z = 1$.

If $n \equiv 0 \pmod 4$, put $y = \frac{1}{2}n$, $z = 1$.

On page 193 of [2], the author states that d' is always congruent to 1 or 11 (mod 12), but omits the proof. We now use our results not only to show that this is in fact true, but also to find similar expressions for a', b' and c'. We shall require the following results, the proofs of which are omitted.

Lemma 3

If y is even, then $y^2 \equiv 0 \pmod 4$, if y is odd, then $y^2 \equiv 1 \pmod 8$.

If z is even, then $z^2 \equiv 4 \pmod{12}$, if z is odd, then $z^2 \equiv 1 \pmod{24}$.

First of all, let us consider $a = z^2 + 9y^2$ and the case where z is even and y odd. Then $z^2 \equiv 4 \pmod{12}$. As $y^2 \equiv 1 \pmod 8$ it follows that $9y^2 \equiv 9 \pmod{72}$ and thus that $9y^2 \equiv 9 \pmod{12}$. This implies that $a' = a = z^2 + 9y^2 \equiv 1 \pmod{12}$.

On the other hand, if z is odd and y even, then $z^2 \equiv 1 \pmod{24}$ and

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$y^2 \equiv 0 \pmod 4$. It follows that $9y^2 \equiv 0 \pmod{36}$. Thus $z^2 \equiv 1 \pmod{12}$ and $9y^2 \equiv 0 \pmod{12}$ which means that $a' = a \equiv 1 \pmod{12}$.

Finally for a', if both z and y are odd, then $z^2 \equiv 1 \pmod{24}$ and $y^2 \equiv 1 \pmod 8$. It follows that $9y^2 \equiv 9 \pmod{72}$ and thus $9y^2 \equiv 9 \pmod{24}$. This means that $a \equiv 10 \pmod{24}$ and since $a' = \frac{1}{2}a$, we have $a' \equiv 5 \pmod{12}$

The calculations involving $b = 2(z^2 + 3y^2)$, $c = 3(z^2 + y^2)$ and $d = z^2 - 3y^2$ follow a similar pattern and are omitted. The results are given in Table 3 below.

	a'	b'	c'	d'
z even, y odd	$\equiv 1 \pmod{12}$	$\equiv 14 \pmod{24}$	$\equiv 3 \pmod{12}$	$\equiv 1 \pmod{12}$
z odd, y even	$\equiv 1 \pmod{12}$	$\equiv 2 \pmod{24}$	$\equiv 3 \pmod{12}$	$\equiv 1 \pmod{12}$
z odd, y odd	$\equiv 5 \pmod{12}$	$\equiv 4 \pmod{24}$	$\equiv 3 \pmod{12}$	$\equiv 11 \pmod{12}$

TABLE 3

6. *Acute–angled, right-angled or obtuse-angled*

In triangle $T\left(\frac{y}{x}\right)$, $d = b - a = c - b = z^2 - 3y^2$. If $\frac{y}{z} < \frac{1}{\sqrt{3}}$, then $d > 0$ and c is the longest side. It may easily be verified that

$$a^2 + b^2 - c^2 = 4(9y^2 - z^2)(3y^2 + z^2)$$

and it follows that $T\left(\frac{y}{z}\right)$ will be acute-angled if $\frac{y}{z} > \frac{1}{3}$, that is if $\frac{1}{3} < \frac{y}{z} < \frac{1}{\sqrt{3}}$. In the same way, $T\left(\frac{y}{z}\right)$ will be obtuse-angled if $\frac{y}{z} < \frac{1}{3}$. Note that we do not need to consider $\frac{y}{z} = \frac{1}{3}$, since $3 \nmid z$.

On the other hand, if $\frac{y}{z} > \frac{1}{\sqrt{3}}$, then $d < 0$ and a is the longest side. As above, it may easily be verified that $c^2 + b^2 - a^2 = 12(z^2 - y^2)(z^2 + 3y^2)$ and it follows that $T\left(\frac{y}{z}\right)$ will be acute-angled if $\frac{y}{z} < 1$, that is if $\frac{1}{\sqrt{3}} < \frac{y}{z} < 1$. In the same way, $T\left(\frac{y}{z}\right)$ will be obtuse-angled if $\frac{y}{z} > 1$. We do in this case get a right-angled triangle when $\frac{y}{z} = 1$.

These results are displayed in Figure 1 below.

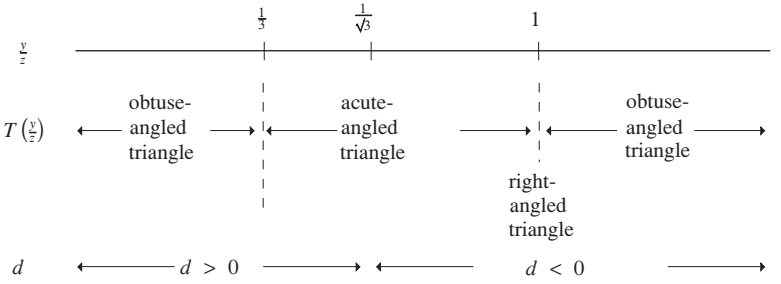


FIGURE 1

The above Figure, although calculated and expressed in terms of $T\left(\frac{y}{z}\right)$,

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is clearly valid for any triangle similar to $T\left(\frac{y}{z}\right)$ and in particular it is valid for $P\left(\frac{y}{z}\right)$. We may also now write down the following well-known result which can easily be proved using other methods.

Corollary 2

The only primitive right-angled HAP triangle is the 3, 4, 5 triangle.

Proof

From the above work, we see that $T\left(\frac{y}{x}\right)$ will be right-angled if, and only if, $y = z = 1$. In this case we have $a = 10$, $b = 8$, $c = 6$ and thus $P\left(\frac{1}{1}\right) = \frac{1}{2}T\left(\frac{1}{1}\right)$ has sides 5, 4, 3.

7. *Dividing an HAP triangle into two right-angled triangles*

Figure 2 below shows the acute-angled HAP triangle $T\left(\frac{y}{z}\right)$ with sides $a = z^2 + 9y^2$, $b = 2\left(z^2 + 3y^2\right)$, $c = 3\left(z^2 + y^2\right)$. D is the foot of the perpendicular from B to AC.

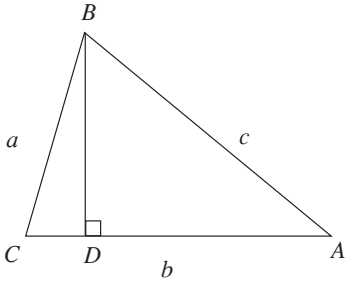


FIGURE 2

We have already shown that the area of $T\left(\frac{y}{z}\right)$ is given by $6yz\left(z^2 + 3y^2\right)$, from which it follows that $BD = 6yz$. If we now apply Pythagoras' theorem to triangles BCD and BDA in turn, we see that $CD^2 = \left(z^2 - 9y^2\right)^2$ and $AD^2 = 9\left(z^2 - y^2\right)^2$ and thus both triangles will be integer-sided right-angled triangles. Since $T\left(\frac{y}{z}\right)$ is acute-angled, we see from Figure 1 that $\frac{1}{3} < \frac{y}{z} < 1$. It now follows immediately that $CD = 9y^2 - z^2$ and $AD = 3\left(z^2 - y^2\right)$ and thus we have $AD - CD = 4\left(z^2 - 3y^2\right) = 4d$. The fact that $P\left(\frac{y}{z}\right)$ and $T\left(\frac{y}{z}\right)$ are similar triangles implies that the corresponding result, with d' replacing d, is also true for $P\left(\frac{y}{z}\right)$.

This aspect of the work is covered in [2, pp 193,194], together with a suggestion as to what happens when $T\left(\frac{y}{z}\right)$, and thus $P\left(\frac{y}{z}\right)$, are obtuse-angled triangles. The proofs involved, all of which follow the same lines as the above, are omitted and left as an exercise for the reader.

8. *Calculating y and z when a′, b′ and c′ are known*

Finally, we consider the following problem. Given a primitive HAP

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triangle, what are the values of y and z which generate it in the sense of Theorems 4 and 5? This question may be resolved quickly as follows.

In the proof of Lemma 2, we have already used the fact that $2a - b = 12y^2$ and that $2c - b = 4z^2$. Thus $\frac{y^2}{z^2} = \frac{1}{3}\left(\frac{2a-b}{2c-b}\right) = \frac{1}{3}\left(\frac{2a'-b'}{2c'-b'}\right)$, which enables us to find y and z once we have identified which sides are represented by a' and c' respectively. This is easily done since c' is clearly divisible by 3 and a' is not (see e.g. Table 3).

A table containing all primitive HAPs with $b' < 150$ is given on page 193 of [2]. In order to illustrate the above result, we shall consider two of the triangles which appear in this table.

First of all, let us look at the HAP triangle with sides 51, 98 and 145.

Then we have $a' = 145$, $b' = 98$ and $c' = 51$.

Then $\frac{y^2}{z^2} = \frac{1}{3}\left(\frac{2 \times 145 - 98}{2 \times 51 - 98}\right) = \frac{192}{3 \times 4} = \frac{16}{1} = 16$, which implies that $y = 4$ and $z = 1$.

For the triangle with sides 101, 148 and 195 we have $a' = 101$, $b' = 148$ and $c' = 195$. Then $\frac{y^2}{x^2} = \frac{1}{3}\left(\frac{2 \times 101 - 148}{2 \times 195 - 148}\right) = \frac{54}{3 \times 242} = \frac{9}{121}$, giving us $y = 3$ and $z = 11$.

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